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OSCILLATIONS IN MULTI-COMPONENT DENSE PLASMA

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A dispersion relation for a multi-component highly-degenerate plasma is investigated by applying the RPA to an idealised system in which carriers are assumed to be free charged particles with isotropic effective mass. The dispersion relation is expanded up to the sixth-order terms in the wave vector and the frequencies of optical and acoustic plasma modes are calculated.

1. *Introduction*

The concept of solid state plasma was first introduced by Kronig and Korringa[1,2]. A more rigorous treatment of electron plasma in metals was later developed by Bohm and Pines[3−6]. They investigated plasma oscillations of a highly-degenerate electron gas by applying the Random Phase Approximation (RPA). Further understanding of plasma effect in metals has been achieved owing to the studies by Nozieres and Pines[7−11], Hubbard[12], Kleinman[13], Langreth[14], Singwi et al.[15] and many others (a more complete list of references may be found in, for instance, the review article by Glicksman[16]).

The problem of collective behaviour in plasma formed by electrons and holes in semiconductors and semimetals was considered by Pines and Schrieffer[17,18]. In two-component plasma there exist two collective modes. The high-frequency mode (optical branch) corresponds to the out-of-phase motion of electrons and holes, while the low-frequency mode (acoustic branch) describes the oscillation in which electrons and holes move in phase. Frohlich[19,20] and Salustri[21] studied oscillations of two-component plasma composed of s- and d-band electrons in metals. Various properties of N-component highly-degenerate plasmas were considered by Cambescot and Nozieres[22], Brinkman and Rice[23], Vashista et al.[24], Bhattacharyya et al.[25], Wünsche[26], Su and Song[27], Kirch and Wolfe[28], Schmidt and Röpke[29], Kraeft et al.[30], Mahanty and Das[31] and Held[32].

The aim of the present paper is to investigate the long-wavelength dielectric function of a multi-component highly-degenerate Fermi gas with disparate classical plasma frequencies and disparate Thomas-Fermi wave vectors. Using the effective mass approximation and confining ourselves to the RPA, we calculate the long-wavelength frequencies of optical and acoustic plasma modes after expanding the dispersion relation in power series of the wave vector.

2. Constitutive equations

We consider an N-component dense plasma consisting of free carriers with effective masses m_j, charges e_j and concentrations n_j, $j = 1,\ 2...\ N$. In the RPA the response function of each group of carriers is[33]

$$Q(\mathbf{k},\omega) = \frac{4\pi e^2}{\varepsilon_0 k^2} \sum_{s,\vec{\kappa}} \frac{N_s(\vec{\kappa}) - N_s(\vec{\kappa}+\mathbf{k})}{\hbar\omega - E(\vec{\kappa}+\mathbf{k}) + E(\vec{\kappa}) + \mathrm{i}\delta}\,, \tag{1}$$

where ε_0 is the dielectric constant of the medium, $\vec{\kappa}$ and $\mathbf{k}$ are the wave vectors, s is the spin (which is assumed to be 1/2), ω is the frequency, $N_s(\vec{\kappa})$ is the occupation number operator in the state of the wave vector $\vec{\kappa}$ and spin s, $\hbar$ is the Planck's constant, δ is the positive infinitesimal and, in the effective mass approximation, the energy of the particle is

$$E(\vec{\kappa}) = \frac{\hbar^2\kappa^2}{2m}\,. \tag{2}$$

Expression (1) enables one to calculate the wave-vector and the frequency-dependent longitudinal dielectric function of a multi-component plasma

$$\varepsilon(\mathbf{k},\omega) = 1 - \sum_{j=1}^{N} Q_j(\mathbf{k},\omega)\,. \tag{3}$$

For a high-density quantum plasma, the response function can be calculated in the zero temperature approximation. This leads to the well-known Lindhard's expression[34]

$$\mathrm{Re}Q(\mathbf{k},\omega) = -\frac{k_s^2}{2k^2}\left\{1 + \frac{k_s}{2k}\left[1 - \left(\frac{\omega + \hbar k^2/2m}{kv_F}\right)^2\right] \ln\left|\frac{\omega + kv_F + \hbar k^2/2m}{\omega - kv_F + \hbar k^2/2m}\right|\right.$$

$$\left. + \frac{k_s}{2k}\left[1 - \left(\frac{\omega - \hbar k^2/2m}{kv_F}\right)^2\right] \ln\left|\frac{\omega - kv_F + \hbar k^2/2m}{\omega + kv_F + \hbar k^2/2m}\right|\right\}, \tag{4}$$

$$\mathrm{Im}Q(\mathbf{k},\omega)=\begin{cases} -\dfrac{\pi\omega}{2kv_F}\left(\dfrac{k_s}{k}\right)^2, & \text{if } \omega\le kv_F-\hbar k^2/2m,\\ -\dfrac{\pi k v_f}{4k}\left(\dfrac{k_s}{k}\right)^2\times & \\ \times\left[1-\left(\dfrac{\omega-\hbar k^2}{kv_F}\right)^2\right], & \text{if } kv_F-\hbar k^2/2m\le\omega\le kv_F+\hbar k^2/2m,\\ \qquad 0, & \text{if } kv_F+\hbar k^2/2m\le\omega. \end{cases} \tag{5}$$

In Eqs. (4) and (5), k_F is the Fermi wave number

$$k_F=\sqrt[3]{3\pi^2 n}\,. \tag{6}$$

v_F the Fermi velocity

$$v_F=\frac{\hbar k_F}{m}, \tag{7}$$

and k_s the Thomas-Fermi screening wave number. With the help of the long-wavelength plasma frequency

$$\omega_P=\sqrt{\frac{4\pi e^2 n}{\varepsilon_0 m}}, \tag{8}$$

it can be expressed in the form

$$k_s=\frac{\omega_P}{v_F}\sqrt{3}\,. \tag{9}$$

Studying the long-wavelength plasma waves, we confine ourselves to the small-k limit ($k\ll k_s$). Then in the high-frequency region Eqs. (4) and (5) reduce to

$$Q(\mathbf{k},\omega)=\frac{\omega_P^2}{\omega^2}\left(1+\frac{3k^2v_F^2}{5\omega^2}+\frac{\hbar^2k^4}{4m^2\omega^2}+\frac{3k^4v_F^4}{7\omega^4}+\frac{v_F^2\hbar^2k^6}{2m^2\omega^4}+\frac{v_F^6k^6}{3\omega^6}+\ldots\right),$$

$$\omega\gg kv_F, \tag{10}$$

while in the low-frequency region the following expansion is valid,

$$Q(\mathbf{k},\omega)=-\frac{k_s^2}{k^2}\left(1-\frac{\omega^2}{k^2v_F^2}-\frac{\hbar^2\omega^2}{6m^2v_F^4}-\frac{\hbar^2k^2}{12m^2v_F^2}-\frac{\omega^4}{3k^4v_F^4}-\frac{\hbar^4k^4}{240m^4v_F^2}\right.$$

$$\left.-\frac{\omega^6}{5k^6v_F^6}-\frac{\hbar^2\omega^4}{4k^2v_F^6}-\frac{3\hbar^4\omega^2k^2}{80m^4v_F^6}-\frac{\hbar^6k^6}{2240m^6v_F^6}+\mathrm{i}\frac{\pi\omega}{2kv_F}\right),\quad \omega\ll kv_F. \tag{11}$$

3. Plasma eigenfrequencies

Normal modes of the plasma are determined by the zeros of the dielectric function

$$\varepsilon(\mathbf{k},\omega) = 0\,. \tag{12}$$

By virtue of Eq. (3), the dispersion relation (12) can be rewritten as

$$1 = \sum_{j=1}^{N} Q_j(\mathbf{k},\omega)\,. \tag{13}$$

For the sake of simplicity, we impose the following conditions on frequencies and screening wave vectors,

$$\omega_{p1}^2 \gg \omega_{p2}^2 \gg \omega_{p3}^2 \gg \ldots, \tag{14}$$

$$k_{s1}^2 \ll k_{s2}^2 \ll k_{s3}^2 \ll \ldots\,. \tag{15}$$

As will be demonstrated later on, these conditions ensure that the damping of plasma waves is not strong.

Generally speaking, the plasma frequency is of the complex form

$$\omega(k) = \Omega(k) - \mathrm{i}\gamma(k), \tag{16}$$

where the imaginary part $\gamma(k)$ describes the Landau damping of the waves.

The optical plasma mode is determined by applying the high-frequency expansion (10) to all plasma components. In the k^6 approximation, we arrive at

$$\Omega_1^2(k) = \omega_{p1}^2 \left\{ 1 + \frac{9}{5}\left(\frac{k}{k_{s1}}\right)^2 + \left(\frac{k}{k_{s1}}\right)^4 \left[\frac{108}{175} + \frac{3}{4}\left(\frac{k_{s1}}{k_{F1}}\right)^2\right] \right. \\ \left. \left(\frac{k}{k_{s1}}\right)^6 \left[-\frac{144}{875} + \frac{9}{5}\left(\frac{k_{s1}}{k_{F1}}\right)^2\right] \right\}, \tag{17}$$

which agrees with the corresponding expression for the single-component plasma. Note that $\gamma_1(k)$ is zero, i.e., in this simplified model, optical plasma waves are not damped. This is the consequence of the fact that the phase velocity $\Omega_1(k)/k$ is much larger then the Fermi velocity of the first species, v_{F1}.

The remaining $N-1$ modes are of the acoustic type. Like the acoustic lattice frequency, the frequencies of the acoustic plasma modes are proportional to k for $k \to 0$. The frequency of the first acoustic mode is calculated by applying the low-frequency expansion (11) to the first plasma component and the high-frequency expansion (10) to $j = 2,\ 3...\ N^{th}$ components. The second acoustic frequency is determined using Eq. (11) for $j = 1,\ 2$ and Eq. (10) for $j = 3,\ 4...\ N$ etc. As it is seen from Eq. (11), in the low-frequency regime the response function is complex indicating that acoustic modes are Landau damped. In our model, the damping of the j^{th} mode is due to the $(j-1)^{th}$ carriers.

Considering the waves whose phase velocity lies between two Fermi velocities, $v_{F\,j-1}$ and v_{Fj},

$$v_{Fj-1} \gg \Omega_j(k)/k \gg v_{Fj}, \tag{18}$$

we shall suppose that in the long-wavelength limit the imaginary part of the acoustic frequency is much less than the real one

$$\gamma(k) \ll \Omega(k). \tag{19}$$

After straightforward manipulations one finds that up to the sixth-order terms in the wave vector the solution to the dispersion relation (13) takes the form

$$\Omega_j^2(k) = \omega_{pj}^2 \left(\frac{k}{k_{sj-1}}\right)^2 \left\{1 + \left(\frac{k}{k_{sj-1}}\right)^2 \left[1 - \frac{1}{12}\left(\frac{k_{sj-1}}{k_{Fj-1}}\right)^2\right]\right.$$

$$\left. - \frac{1}{240}\left(\frac{k}{k_{sj-1}}\right)^4 \left(\frac{k_{sj-1}}{k_{Fj-1}}\right)^4 - \frac{1}{2240}\left(\frac{k}{k_{sj-1}}\right)^6 \left(\frac{k_{sj-1}}{k_{Fj-1}}\right)^6\right\}^{-1}, \tag{20}$$

$$\gamma_j(k) = \frac{\pi\sqrt{3}\,\Omega_j^4(k)}{12\omega_{pj-1}\omega_{pj}^2}\left(\frac{k_{sj-1}}{k}\right)^3. \tag{21}$$

The acoustic waves describe the oscillations of the j^{th} group of carriers which are screened by the static dielectric function of the faster $(j-1)^{th}$ carriers.

It can be easily verified that the Landau damping of the long-wavelength acoustic plasma waves is relatively weak. Starting from Eqs. (20) and (21), with the help of Eq. (14) we find for the ratio between the imaginary and the real part of the acoustic frequency in the $k \to 0$ limit

$$\frac{\gamma_j(k)}{\Omega_j(k)} = \frac{\pi\sqrt{3}\,\omega_{pj}}{12\omega_{pj-1}} \ll 1, \tag{22}$$

which verifies the assumption (19).

The results derived are applied to a three-component plasma. This case may be realized in high-density systems composed of heavy holes, light holes and electrons. The conditions (14) and (15) then transform into

$$\frac{n_1}{m_1} \gg \frac{n_2}{m_2} \gg \frac{n_3}{m_3}, \tag{23}$$

$$m_1 n_1^{1/3} \ll m_2 n_2^{1/3} \ll m_3 n_3^{1/3}. \tag{24}$$

Supposing that all components are of approximately equal densities, Eqs. (23) and (24) may be satisfied if the effective mass of electrons is much smaller then the mass of light holes which, in its turn, is much smaller then that of heavy holes. In other words, the electrons play the role of the first species, the light holes of the second species and the heavy holes of the third species.

We add that the above criteria are partially fulfilled in III-V semiconductors. In a typical III-V semiconductor, such as InSb, GaSb, GaAs etc., the effective mass of heavy holes is much larger than that of light holes, but the mass of light holes differs only slightly from the electronic effective mass. Hence, under the condition of equal densities, in III-V semiconductors, the matching between the optical and the upper acoustic branch becomes strong.

In order to illustrate the behaviour of the real parts of frequencies, we have performed the calculation by choosing

$$\frac{\omega_{p1}}{\omega_{p2}} = \frac{\omega_{p2}}{\omega_{p3}} = 5\,, \tag{25}$$

$$\frac{k_{s1}}{k_{s2}} = \frac{k_{s2}}{k_{s3}} = \frac{1}{5}\,, \tag{26}$$

$$k_{Fj} = k_{sj}\ , \qquad j = 1, 2, 3\,. \tag{27}$$

The corresponding graphs are shown in Figs. 1 and 2. The acoustic branch is represented by two curves, the upper and the lower curve, which have very different slopes for small wave vectors. As it is seen from the dispersion relation (20), the ratio between the acoustic frequency and the wave number is constant in the long-wavelength limit and then decreases with increasing k. This is in qualitative agreement with the results derived for a classical three-component plasma[35].

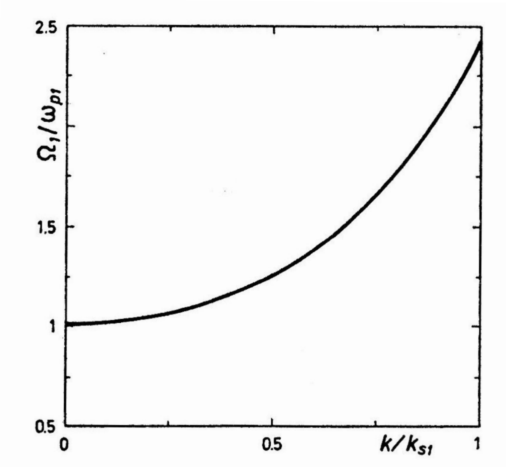

Fig. 1. Optical plasma frequency Ω_1 as a function of wave vector k (in relative units) for $\omega_j/\omega_{j-1} = 5$, $k_{sj}/k_{s\,j-1} = 1/5$.

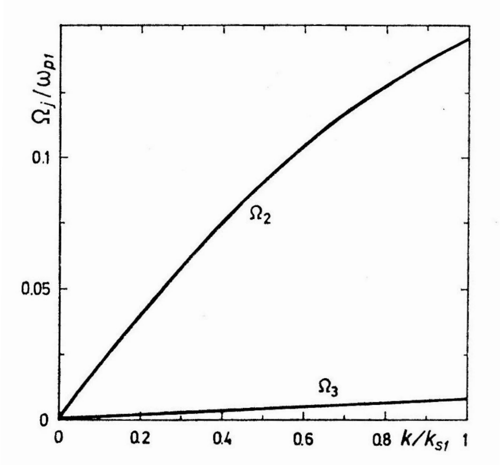

Fig. 2. Acoustic plasma frequencies Ω_j, $j = 2,\ 3$ as a function of the wave vector (in relative units) for $\omega_j/\omega_{j-1} = 5$, $k_{sj}/k_{s\,j-1} = 1/5$.

4. Conclusions

We have studied the collective motion in highly-degenerate quantum plasmas composed of N components. Assuming that each component is formed of free charged particles with isotropic effective masses, we had approximated the dielectric function of the plasma using Lindhard's expression. Confining the consideration to the model in which both the eigenfrequencies and the Thomas-Fermi screening wave vectors of the components are quite different, we have expanded the dispersion relation up to the k^6 terms and determined the real and the imaginary parts of the plasma frequencies. The highest frequency represents the optical mode, while the remaining $N-1$ frequencies belong to the acoustic part of the spectra. The general feature of all acoustic modes is that for $k \to 0$ the frequency depends linearly on the wave number.

Contrary to the optical mode, the acoustic modes are Landau damped. However, in the long-wavelength limit, this damping is not strong.

Although our consideration has been based on an oversimplified model, we believe that the obtained results may qualitatively describe the main features of the collective motion of particles in dense multi-component solid state plasmas.

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OSCILACIJE U GUSTOJ VIŠEKOMPONENTNOJ PLAZMI

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Primjenjujući RPA na sistem sastavljen od slobodnih naboja s izotropnom efektivnom masom razmotrena je disperzivna relacija višekomponentne degenerirane plazme. Razvojem disperzivne relacije do članova šestog reda u valnom vektoru izračunate su optičke i akustičke frekvencije plazme.