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MOTION OF DISTANT OBJECTS IN THE UNIVERSE

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We give a brief survey of investigations of the structure of the Universe and of its expansion, for educational purposes. As we know it today, the size of the Universe is immense, enormous numbers of galaxies have been found in it, and in each galaxy enormous numbers of stars. The galaxies move from us at very high speeds, some of the more distant ones at speeds comparable to the velocity of light. Many theories have been developed to explain the vast amount of data that have been obtained by astronomical observations. We give a simplified theory of the cosmic expansion and present results of three models for the cosmic scale function $R(t)$: the constant rate of expansion, the accelerated expansion and the constant Hubble parameter models.

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1. Introduction

In a dark and clear night, far from the street and other lights, one can clearly see the Milky Way, the pale, irregular band arching the sky and dividing it into two nearly equal hemispheres. It appears entirely different from the bright point-like stars. Its name says it clearly - it was considered as a continuous area in the sky that somehow shines the pale light that we see. Besides the large-area Milky Way, one can see, also by naked eyes, small patches of pale light, most notably in the constellations Andromeda and Orion. These patches have been called nebulae, what also shows how simple ideas were conceived for what they are. Construction of larger and larger telescopes and exploration of the night sky brought about the cognition that these seemingly continuous sources of light consist of immense numbers of point-like sources. About 90 years ago, Öpic [1] in 1922 and Hubble

[2] in 1929 determined distances to some nebulae that showed decisively that they are far too distant to be a part of the Milky Way. It was realized that the nebulae are very distant huge assemblies of thousands of millions of stars, and that the Milky Way is one of them with our Sun about half way from the centre to the edge. The Milky Way has the old Latin name Via Lactea (lactus meaning milk) that was translated from the Greek Galaxias. So all these systems of stars are named galaxies and the Milky Way the Galaxy or the Milky Way galaxy. The diameter of the Milky Way galaxy is about 100 kca (thousands light years), the closest galaxy is the Andromeda galaxy, that is about three times bigger then the Milky Way galaxy, at the distance of about 3 Mca, while the so far observed very distant ones are at distances of over 500 Mca.

It is hard to understand these distances. A light year is the distance travelled by a light signal in one year, 300 000 km/s$\times$3.16 10^7 s (1a= 3.16 10^7 s). For a light signal, it takes about 1 s to reach the Moon, about 8 minutes to reach the Sun and about 4.24 years to reach the closest star, Proxima-Centaury. Imagine, that star had an Earth-like planet, a highly developed civilization, and we and them succeeded to realize communication the fastest way, using the electromagnetic waves (say light). Suppose we send them a question, say what do they look like. We would have to wait 8.5 years for their answer! What about the much more distant objects?

In the past several hundred years, especially in the more recent times, our knowledge about the Universe has dramatically changed. An old idea was that the Earth was at the centre of the World, a plate that floats on a big ocean, that the sky with the stars, Moon and planets is a hollow sphere that rotates around the Earth, the stars and Moon and planets dip into the ocean in the West and reappear in the East in the morning, that the verticals are parallel in all places on Earth, etc. When the idea that the Earth was a body of a spherical shape was accepted, some people would say – it is impossible – people on the opposite side would drop from the Earth. It took quite a time that the relativity of the vertical was generally accepted. Of course, even more dramatic was the acceptance of the Copernicus's idea of the solar planetary system, displacing the Earth from the throne of the centre of the Universe.

Investigations in astronomy, astrophysics and cosmology enhanced tremendously our knowledge about the Universe, such as distances, luminosities, spectra of emitted radiation, their motion, types of stars, binary stars, etc. It is a huge amount of data. The subject of this work is the large-scale structure related to the expansion of the Universe. As initially discovered by Hubble [2], the galaxies move radially from us at velocities that are proportional to their distance. Various methods are applied to measure the distances (mostly light intensity) and velocities (mostly wavelength shifts of characteristic atomic or molecular lines). Looking back in time, it was realized that the galaxies, i.e. the whole Universe, must have originated at one moment of time and at the same point. So the idea of a terrific explosion was created in 1927 (Lemaître [3]), the Big Bang hypothesis. From the distance-velocity data for many galaxies, present results are that it occured about t_0 = 13.7 Ga ago, the time that is usually called the age of the Universe. The Hubble parameter at the present time, H_0 = 70 km/s/Mpc, is just the inverse of that age (1 pc = 3.2616 ca).

Very exciting results have been obtained in the last about twelve years by the observations of the supernovae (SN) Ia explosions. Two teams of astronomers, the High-z Supernova Team and the Supernova Cosmological Project [4] obtained rather accurate data on distances in addition to the improved results on relative wavelength shifts (velocities) of SN Ia. In these explosions of stars, enormous energy of light is emitted, so they are visible even when originating from extremely large distances, but more important is that the light intensity as a function of time (what is carefully observed for several days) is related to the total intensity, so the variability in intensities of emitted light of explosions can be normalized.

2. *A simple presentation of the FRW metric*

The Einstein's general theory relativity (GTR) is basic to our understanding of effects of gravitation. While his special theory relativity (STR) is based on the principle that physical laws are the same in all inertial reference frames (from which follows the principle of constancy of the velocity of light irrespectively of the motion of the source and of the observer), the GTR is based on the principle that physical laws are the same in all (inertial and noninertial) reference frames (resulting in local constancy of the velocity of light) and the principle of equivalence of effects of acceleration and of the gravitational field.

The Einstein's equations of the GTR are nonlinear differential equations, rather difficult to solve. Many of their solutions have been found under various assumptions. The solutions that have been mostly used for many years in theories of the origin, evolution, present state and future of the Universe, the branch of science called cosmology, are based on the theory introduced by Friedmann in 1922 and 1924 [5], when the expansion of the Universe was not yet known. His idea was that objects in the Universe (galaxies and other) have fixed positions in space, while the space expands or shrinks (called the Friedmann model [6]). Starting from the Einstein's field equations, assuming both the flat and curved space and various ways of evolution of the expansion, Robertson in 1935 and Walker in 1936 [7] extended the theory. The metric is given by the equation [5−8]

$$\mathrm{d}s^2 = \mathrm{d}t^2 - R^2(t)\left(\frac{\mathrm{d}r^2}{1-kr^2} + r^2\mathrm{d}\theta^2 + r^2\sin^2\theta\mathrm{d}\phi^2\right), \tag{1}$$

where $\mathrm{d}s$ is the invariant "proper" time differential, t the "cosmic" time, $R(t)$ the cosmic scale factor, r the relative position of the cosmic object under consideration (for a galaxy r does not change in time), $k = +1$, -1 or 0 (corresponding to positive curvature, negative curvature of the space and flat space, respectively) and θ and ϕ are the polar angles of the object. The units are chosen so that the velocity of light is $c = 1$. The cosmic scale factor describes the history of the cosmic expansion. One of the main goals of the cosmology is to find this function.

It is known in STR that the time is a relative concept. Two simultaneous events at separate points in one inertial reference frame are not simultaneous in another inertial reference frame if the frames move relatively one to another. So there is no "absolute" time. This effect is even more complex in GTR, when gravitaion or acceleration are in effect, because, as predicted by Einstein and experimentally proved, clocks run faster in a stronger than identical clocks in a weaker gravitational field. To make cosmological theories tractable, the so-called "cosmic time" was introduced. The basic idea is derived from the cosmological principle which states that the space in the Universe is homogeneous and isotropic. The Universe and all physical and other quantities, on a large scale, are independent of the position of the observer (the origin of the coordinate system) and of the orientation (directions of coordinate axes). So the basic assumptions of the theory that is used for the description of the evolution of the Universe is based on the cosmological principle and the principle of equivalence (the basis of GTR). Some physical quantities, in particluar the temperature of the cosmic radiation background and the cosmic energy density, change with the age of the Universe. According to the cosmological principle, they change the same way as observed at any point in the Universe. Although not very precisely determined as functions of time, they may be taken as the universal basis for the definition of a universal time, called the "cosmic time". As it is based on the cosmological principle, it is an invariant physical quantity. Very detailed presentation of the idea of the cosmic time and the cosmic coordinate systems is given in Ref. [8], p. 407 and following pages.

The idea of Eq. (1) is to describe the Universe as a system of objects (galaxies) which have fixed positions in space, and the space expands. The expansion is given by the "cosmic scale function", $R(t)$. One can visualize the expansion by a balloon with tiny elastic fibers attached diametrically inside it, some specs attached to the fibers at various distances from the centre of the balloon. As we inflate the balloon, relative positions of the specs do not change, but the distances do in a proportional way.

Equation (1) is analogous to the invariant of the STR, $\mathrm{d}s^2 = \mathrm{d}t^2 - (\mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2)$. For a light signal in vacuum, because $c = constant$ and invariant, $\mathrm{d}s^2 = 0$. Limiting our considerations to radial motion only, i.e. $\mathrm{d}\theta^2 = \mathrm{d}\phi^2 = 0$, for the light signal, according to Eq. (1), we have

$$-\mathrm{d}t^2 + R^2(t)\frac{\mathrm{d}r^2}{1-kr^2} = 0\,, \quad \text{or} \quad \mathrm{d}t = \pm\frac{\mathrm{d}r}{R(t)\sqrt{1-kr^2}}\,. \tag{2}$$

Accurate measurements of the cosmic radiation background strongly favour flat space, i.e. $k = 0$. Assuming this, the calculations of the space expansion, for a selected function $R(t)$, are much easier. We adapt it, so

$$\mathrm{d}r = \pm\frac{c\mathrm{d}t}{R(t)}\,. \tag{3}$$

We introduced the velocity of light c in this expression, so in the following, t is expressed in units of time and $R(t)$ in units of length. Assuming a function $R(t)$, one can calculate the change of r of the light signal travelling radially from or towards the origin (signs + or − in Eq. (3)). We see that a light signal changes its relative position r faster when the expansion scale factor is small.

To visualize relation Eq. (3), consider the terrestrial thought experiment of motion on an elastic string that is fixed at one end and is being uniformly stretched by pulling it at the other end. On the string, an object (say, shelter of a bug) is fixed at a relative position $0 < r < 1$ and the length of the string is $R(t)$. So the distance of the object from the fixed end is $rR(t)$. At a later moment $t+\mathrm{d}t$, the length of the string is $R(t+\mathrm{d}t)$ and the home of the bug is at the position $rR(t+\mathrm{d}t)$. The condition of uniform stretching requires that relative position of the home of the bug does not change, i.e. $rR(t+\mathrm{d}t)/R(t+\mathrm{d}t) = rR(t)/R(t) = r = \textit{constant}$. Now imagine the bug leaves the shelter and moves at a constant speed v_0 relatively to the string towards the pulled end of the string (see Fig. 1). In the time interval $\mathrm{d}t$, the bug will change the position on the string by $v_0\mathrm{d}t$ and its relative position to $r+\mathrm{d}r$. The condition of uniform stretching requires that the new distance of the bug from the fixed end, $(r+\mathrm{d}r)(R+\mathrm{d}R)$, decreased by the shift of the bug, $v_0\mathrm{d}t$, be proportional to the new total length of the string $R+\mathrm{d}R$. So, we should have

$$\frac{(r+\mathrm{d}r)(R(t)+\mathrm{d}R) - v_0\mathrm{d}t}{R(t)+\mathrm{d}R} = \frac{R(t)+\mathrm{d}R}{R(t)} \,.$$

From this equation follows

$$\mathrm{d}r = \frac{v_0\mathrm{d}t}{R(t)} \,.$$

a relation anagolous to Eq. (3). Actually, by making proper substitutions, the above considerations apply to the radial travelling of light signals in a uniformly stretching space, the Friedmann space.

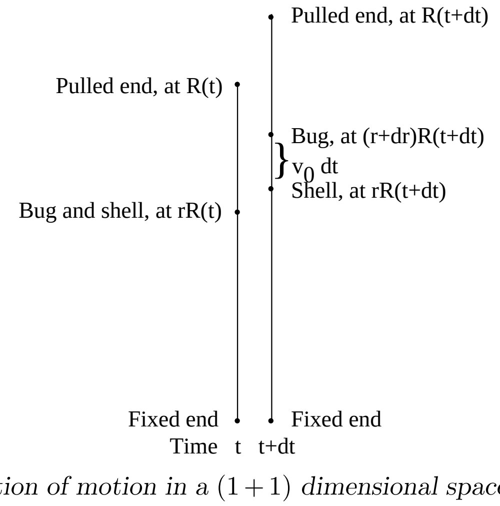


Fig. 1. Illustration of motion in a $(1+1)$ dimensional space – a bug on a stretching string.

3. *Three simple models of cosmic expansion*

3.1. *Constant rate of expansion*

The simplest model is when we assume

$$\dot{R}(t) = constant = \dot{R}_0 \,, \quad \text{resulting in} \quad R(t) = \dot{R}_0 t \,. \tag{4}$$

From Eq. (3),

$$r_1 = \int_{t_1}^{t_0} \frac{c\mathrm{d}t}{\dot{R}_0 t} = \frac{c}{\dot{R}_0} \ln \frac{t_0}{t_1} \,, \tag{5}$$

and we can find the time of emission of the signal

$$t_1 = t_0 \mathrm{e}^{-r_1 \dot{R}_0 / c}. \tag{6}$$

So, at the time of emission of the signal, the velocity and the distance of the object were

$$r_1 \dot{R}_0 = constant \,, \qquad \text{and} \qquad r_1 R(t_1) = v_0 \, t_0^{-v_0/c}. \tag{7}$$

The Hubble parameter $\dot{R}(t)/R(t)$ is not a constant as follows from Eqs. (4).

3.2. *Accelerated rate of expansion*

We assume that the rate of cosmic expansion is a linear function of the cosmic time t,

$$\dot{R}(t) = \dot{R}_0 + \ddot{R}_0 t \,, \qquad \text{hence} \qquad R(t) = \dot{R}_0 t + \frac{1}{2} \ddot{R}_0 t^2, \tag{8}$$

where $\dot{R}_0 = \dot{R}(t=0)$ is the initial velocity of the object (right after the Big Bang) and $\ddot{R}_0$ is a constant acceleration. Using Eq. (3), we have

$$r_1 = \int_{t_1}^{t_0} \frac{c\mathrm{d}t}{\dot{R}_0 t + \frac{1}{2}\ddot{R}_0 t^2} = \frac{c}{\dot{R}_0} \ln \frac{\dot{R}_0/t_1 + \ddot{R}_0/2}{\dot{R}_0/t_0 + \ddot{R}_0/2} \,. \tag{9}$$

Introducing the parameters $\alpha_0 = r_1 \ddot{R}_0 t_0 / c$ and $\beta_0 = r_1 \dot{R}_0 / c$, the solution of Eq. (9) can be written

$$\frac{t_1}{t_0} = \left[\mathrm{e}^{\beta_0} \left(1 + \frac{\alpha_0}{2\beta_0} \right) - \frac{\alpha_0}{2\beta_0} \right]^{-1} . \tag{10}$$

Assuming $\alpha_0 = 0$, the solution in Eq. (6) is recovered.

Using the solution (10), the velocity and the distance of the object at the time of emission t_1 are

$$\dot{R}(t_1) = \dot{R}_0 + \ddot{R}_0 t_1\,, \qquad \text{and} \qquad R(t_1) = \dot{R}t_1 + \frac{1}{2}\ddot{R}_0 t_1^2\,, \tag{11}$$

respectively.

Again, the Hubble parameter $\dot{R}(t)/R(t)$ is not a constant as follows from Eqs. (8).

3.3. Hubble law valid all time

We assume now

$$\frac{\dot{R}(t)}{R(t)} = H(t) = H_0\,, \quad \text{or} \quad \frac{\mathrm{d}R(t)}{R(t)} = H_0 \mathrm{d}t \tag{12}$$

at all times. Integration from the variable time t to the present age of the Universe $t_0 = 1/H_0$ yields

$$\ln R(t)\Big|_t^{t_0} = H_0(t_0 - t) = 1 - t/t_0\,, \tag{13}$$

leading to

$$R(t) = R(t_0)\mathrm{e}^{-(1-t/t_0)}\,, \quad \text{and} \quad \dot{R}(t) = \frac{1}{t_0}R(t_0)\mathrm{e}^{-(1-t/t_0)}. \tag{14}$$

The light signal from the galaxy is observed at the moment t_0 (the present time) and it was emitted at a previous moment $t_1 < t_0$. From the time t_1, the light signal travelled toward us (at the position $r = 0$), in the direction opposite to the direction of sight, lowering its r value (the light signal has not a fixed value of the relative position r). Using Eq. (3), we have

$$r_1 = \frac{-c}{R(t_0)} \int_{t_1}^{t_0} \mathrm{e}^{1-t/t_0}\,, \tag{15}$$

$$r_1 = \frac{c}{R(t_0)}\left(1 - \mathrm{e}^{1-t/t_0}\right) \tag{16}$$

and

$$t_1 = t_0\left[1 - \ln\left(1 + \frac{r_1 R(t_0)}{ct_0}\right)\right]. \tag{17}$$

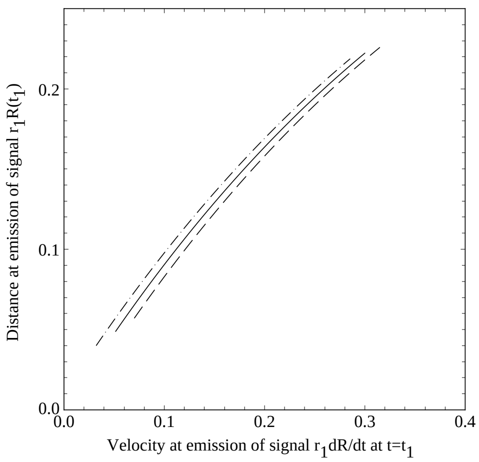
Using this expression for the time of emission of the light signal and choosing t_0 and $r_1 R(t_0)$ as parameters, we can calculate the time of emission of the light signal t_1. With this value of t_1, using Eqs. (14), we can calculate the distance and velocity of the object,

$$r_1 R(t_1) = r_1 R(t_0) \mathrm{e}^{-(1-t_1/t_0)}, \quad \text{and} \quad r_1 \dot{R}(t_1) = \frac{1}{t_0} r_1 R(t_0) \mathrm{e}^{-(1-t_1/t_0)}. \tag{18}$$

In this case, the Hubble parameter $\dot{R}(t)/R(t)$ is constant as was assumed in Eqs. (12).

4. Results

To illustrate the results of the previous section, we made numerical calculations assuming realistic values of the parameters. Figures 2 and 3 show the results for the distance vs. velocity and the relative time t_1/t_0 vs. velocity for the cosmic expansion at constant rate and accelerated rates, all at the moment of emission of the light signal, for various values of the input parameters. The effect of the variability of the time delay is clearly seen. The faster objects reach larger distances in the same times, so the time delay of signals carrying data on their velocity and position take a longer time to reach us. As they are observed on Earth at the same time, the data we receive from faster objects refer to earlier time because it took a longer time the light signals to reach us. These results are very similar to our results of the nonrelativistic and relativistic calculations (in Euclidean space), Ref. (9).



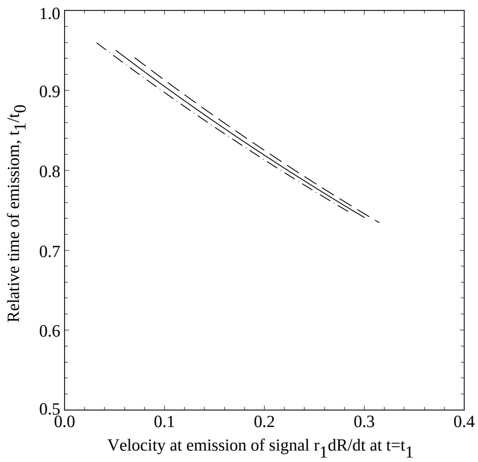
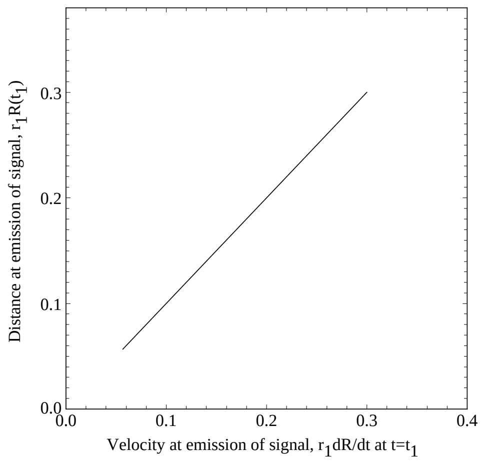


Fig. 2 (left). Plot of distance vs. velocity for cosmic objects at the time of emission of the light signal t_1 for accelerated (dashed line) and deccelerated (dash-dot line) expansion, and constant rate of expansion (full line). The acceleration is taken via the parameter α_0, +0.02, -0.02 and zero (for constant rate of expansion).

Fig. 3. Plot of the relative time of emission, t_1/t_0 vs. velocity for cosmic objects at the time of emission of the light signal t_1 for accelerated and constant-rate cosmic expansion (the same lines as in Fig. 2).

Figures 4 and 5 show the results for the case $\dot{R}(t)/R(t) = constant$ at all time (the Hubble law). One obtains a linear distance-to-velocity plot, as is assumed. The distances covered by the galaxies increase exponentially with time, what appears to be unrealistic. The dependence of the relative time t_1/t_0 on velocity is similar to the previous cases.



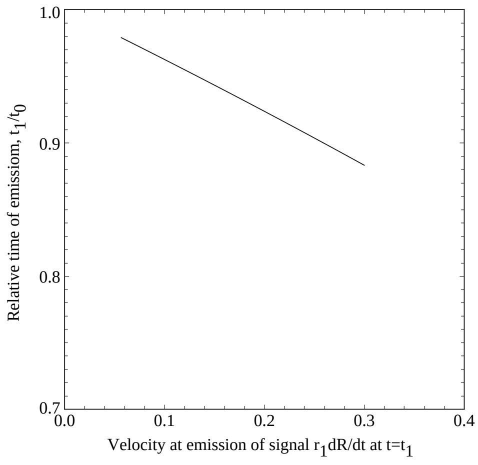


Fig. 4 (left). Plot of distance vs. velocity for cosmic objects at the time of emission of the light signal t_1 for expansion according to the Hubble's law. The gradient of the straight line is simply the age of the Universe, $t_0 = 13.7$ Ga.

Fig. 5. Plot of the relative time of emission, t_1/t_0 vs. velocity for cosmic objects at the time of emission of the light signal t_1 for cosmic expansion following the Hubble's law.

5. Conclusions

A simplified description of the theory of cosmic expansion, based on the FRW metric, is presented. The Friedmann's idea of the solution of Einstein's equations of GRT assuming the cosmic objects to be fixed in a homogeneous and isotropic space at positions that are described by the parameter of relative position r, and that the whole Universe changes by the space expansion described by the cosmic scale function $R(t)$ is very attractive. It is applied in many analyses of cosmic evolution. We present results for the distance-to-velocity relations and relative time-to-velocity relations for three idealized cosmic scale functions. The distances reached by the accelerated and decelerated expansion are smaller and larger, respectively. With the assumption of the Hubble's law, a linear distance-to-velocity relation is obtained, as assumed, while the distances increase exponentially. It is hoped that students will find this work informative and interesting to study.

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GIBANJE DALEKIH TIJELA U SVEMIRU

Dajemo kratak pregled istraživanja o građi Svemira i njegovom širenju u svrhu nastave. Kako danas znamo, Svemir je neshvatljivo velik, s ogromnim brojem galaksija, a svaka galaksija sadrži ogroman broj zvijezda. Galaksije se udaljuju od nas s vrlo velikim brzinama koje su usporedive brzini svjetlosti. Razvijene su mnoge teorije radi objašnjenja mnoštva podataka dobivenih u astronomskim opažanjima. Izlažemo jednostavnu teoriju širenja Svemira i opisujemo ishode računa za tri modela za funkciju svemirske ljestvice $R(t)$: za stalnu brzinu i ubrzano (usporeno) širenje, te za stalan Hubbleov parametar.