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PLANETARY ORBITS IN SOLAR AND EXTRASOLAR SYSTEMS

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The analysis of orbital parameters of planets and main planetary satellites of the solar system, published by the present authors, suggests that the Sun's system could be a "prototype" for the distribution of orbits in extrasolar planetary systems. Owing to the recent endeavours in detecting exoplanets, it became possible and suitable to check this assumption. Particularly useful in this work are the multiple extrasolar system with at least four planets. Unfortunately, there are only four stars satisfying this requirement. At the present time eleven stars with three planets have also been observed, which may also be taken into account in reaching reasonable assertions. Quantization of orbits in the solar system by orbital number, the integer n, and quantization of the product nv_n (v_n is the orbital velocity) by the spacing number, integer k, is also found in extrasolar planets. It is expected that new discoveries will support the present findings.

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1. Introduction

In our previous articles [1a, b, c, d], the square law for orbits has been deduced by the analysis of orbital parameters of Sun's planets and main satellites of Jupiter, Saturn and Uranus. The Sun's planets are classified in two subsystems: terrestrial and Jovian. Therefore, there are five subsystems in the solar system, for which the orbital distributions follow the square law in the form

$$r_n = r_1 n^2 \,. \tag{1}$$

The values of n are consecutive integer numbers in a definite range and r_1 is the radius of the orbit with $n = 1$, dependent on the subsystem. The existing orbits, as an example, for terrestrial planets are distributed from $n = 3$ for Mercury and ending with $n = 8$ for Ceres. Similar results are obtained for other systems, as will be shown later in relevant graphs and tables. In the terrestrial system of planets, the Earth's moon, by hypothesis, had its primordial orbit $n = 7$ between Mars ($n = 6$) and Ceres ($n = 8$). Thus, the Moon is considered to be a planet, which was captured by the Earth [1e]. This hypothesis is supported by the analysis of masses, volumes and periods of all terrestrial planets. Why the Moon at orbit $n = 7$ migrated through the orbit of Mars to become a satellite of the Earth at $n = 5$ is not clear, as well as the problem of its chemical constitution. If the Moon was born at orbit $n = 7$, then it would be expected to contain a significant amount of water, like both Mars and Ceres. However, the absence of volatile elements and water in Moon's materials brought by astronauts suggests another origin: a giant impact of a body as large as Mars with the Earth. But it is hard to accept that such a cataclysmic process could have resulted in the Earth's satellite with parameters compatible with that of the present Moon and with those of all terrestrial planets. Therefore, the problem of the origin of the Moon remains open. Here we treat the Moon as a small planet of the terrestrial group of planets.

Physical basis for the square law (1) is a quantization of the specific angular momentum of planets. Details are presented in Ref. [1]. Equation (1) in extended form [1c,d] is given by

$$r_n = \frac{1}{v_0^2}\, GM \frac{n^2}{k^2}\,. \tag{2}$$

Other relevant relations are:

$$\text{specific angular momentum} \qquad \frac{J_n}{m_n} = \frac{1}{v_0}\, GM \frac{n}{k}\,, \tag{3}$$

$$\text{period} \qquad T_n = 2\pi \frac{1}{v_0^3}\, GM \frac{n^3}{k^3}\,, \tag{4}$$

$$\text{velocity} \qquad v_n = v_0 \frac{k}{n}\,, \tag{5}$$

where n is the orbital number, G is the universal gravitational constant, M is the mass of the central body, v_0 is the velocity constant for all subsystems in the solar system (close to 24 kms^{-1}), and k is spacing number that depends on the system and defines the packing of orbits. In these formulae, circular orbits are assumed with radii equal to the semi-major axes of actual orbits.

For a definite value of k, Eqs. 2 to 5 are simply:

$$r_n^{1/2} \sim n\,, \tag{2$'$}$$

$$J_n/m_n \sim n\,, \tag{3$'$}$$

$$T_n^{1/3} \sim n\,, \tag{4$'$}$$

$$nv_n = const. \tag{5$'$}$$

It is important to point out that in a given system nv_n is constant. In the solar system, there are five subsystems, but each with its own value of nv_n. These values are determined by the number k. Jovian planets and satellites of Uranus have $k = 1$ and almost equal values nv_n. For terrestrial planets, $k = 6$ and similarly for other systems (see Table 1). It means that physical laws are equal in planar gravitational systems regardless the mass m_n of orbiting bodies and the mass M of the central

TABLE 1. Solar and extrasolar planetary systems with at least four planets. The masses are expressed in terms of mass of Jupiter (M_J) or Earth (M_E). T is the period of rotation, a the semimajor axis, n the arbital number, nv_n the product of the orbital number and velocity and k is the spacing number.

System		Mass	T (days)	a (AU)	n	$\boldsymbol{nv_n}$ (km/s)	$\boldsymbol{k}$
HD 160691	c	0.0332 M_J	9.638	0.09094	1	102.6	
	d	0.5219 M_J	310.55	0.921	3	96.8	
	b	1.676 M_J	643.25	1.5	4	101.5	
	e	1.814 M_J	4205.8	5.235	7	94.8	
						$\boldsymbol{\langle nv_n\rangle=99\pm3}$	**4**
55 Cnc	(e)	0.024 M_J	2.81705	0.038	1	(146.8)	
	b	0.824 M_J	14.65162	0.115	2	170.8	
	c	0.169 M_J	44.3446	0.24	3	176.6	
	f	0.144 M_J	260	0.781	5	163.4	
	d	3.835 M_J	5218	5.77	14	168.4	
						$\boldsymbol{\langle nv_n\rangle=170\pm4}$	**7**
GI 581	e	0.006104M_J	3.14942	0.03	3	310.9	
	b	0.0492 M_J	5.34874	0.041	4	332.3	
	c	0.01686 M_J	12.9292	0.07	5	294.5	
	d	0.02231 M_J	66.8	0.22	9	322.5	
						$\boldsymbol{\langle nv_n\rangle=315\pm10}$	**13**
Ter.pl.	Me	0.0056 M_E	87.96	0.387	3	143.6	
	V	0.815 M_E	224.70	0.723	4	140.0	
	E	1 M_E	365.26	1	5	148.9	
	Ma	0.107 M_E	686.98	1.524	6	144.8	
	Moon?	0.012 M_E	1089	2.07	7	144.8	
	Ce	0.00016 M_E	1680	2.77	8	143.5	
						$\boldsymbol{\langle nv_n\rangle=144\pm2}$	**6**
Jov.pl.	J	318 M_E	4333	5.203	2	26.1	
	S	95 M_E	10759	9.54	3	28.9	
	U	14.5 M_E	30685	19.18	4	27.2	
	N	17.2 M_E	60188	30.06	5	27.2	
	Pl	0.002 M_E	90700	39.44	6	28.4	
						$\boldsymbol{\langle nv_n\rangle=28\pm1}$	**1**
HD 10180	b(?)	1.35 M_E	1.17768	0.02225	1	205.542 (?)	
	c	13.10 M_E	5.75979	0.0641	2	242.147	
	d	11.75 M_E	16.3579	0.1286	3	256.586	
	e	25.1 M_E	49.745	0.2699	4	236.108	
	f	23.9 M_E	122.76	0.4929	5	218.409	
	g	21.4 M_E	601.2	1.422	9	231.588	
	h	64.4 M_E	2222	3.4	14	233.058	
						$\boldsymbol{\langle nv_n\rangle=236\pm9}$	**10**

body, provided that m_n are much smaller than M. Consequently, it is expected that the stars with their own planets must also follow the same physical laws. A premonition of that statement was given in Ref. [1c], but at the time of publication, only three extrasolar systems, each with three planets, had been detected: PSR B1267+12, PSR1828 11 and ν Andromedae. Obviously, insufficient observational data could not present a convincible proof. However, nowadays there are 11 detected multiple extrasolar system with thee planets (Table 2), two systems with four planets, one system with five planets and one system (the most recently discovered, Ref. [5] update December 2010) with seven planets. The systems with at least four planets (Table 1) are the best to confirm the square law for the distribution of orbits. This will be discussed in the next section.

2. *Analysis of observational data*

Orbital radii of terrestrial and Jovian planets and of main satellites of Jupiter, Saturn and Uranus are distributed according the square law (1). The circular orbits are assumed with radii equal to semi-major axes. The fundamental physical reason is the quantization of the specific angular momentum, which in the used approximation is given by

$$J_n/m_n = (GMr_1)^{1/2}n.$$

For elliptical orbits, this relation is given by [4]

$$J_n/m_n = [G(M+m_n)a_n(1-e_n^2)]^{1/2},$$

where a_n is the semi-major axis and e_n the eccentricity related to the n-th orbit (subcript n is added by the present authors). For $m_n \ll M$ and small values of e_n, the approximation of circular orbits is very good.

Using the model defined by Eqs. (1 − 5), or Eqs. (2′ − 5′) for a given $k = const.$, the numbers n of all orbits in a system are easily determined by the following simple calculation. The values of $T_n^{1/3}$ are each divided by one number from a choice of small integer numbers (see Eq. (4′)) with the aim to obtain a constant quotient for all orbits.

For example: for the star 61 Vir, the periods of the planets b, c and d (Ref. 5 and Table 2) are: T(days) = 4.215, 38.021 and 123.01. Consequently, $T^{1/3} = 1.615$, 3.363, 4.973. One simply obtains $1.615/1 = 1.615$, $3.363/2 = 1,682$ and $4.973/3 = 1.658$, so the orbital numbers are $n = 1$, 2 and 3. The resulting approximate set of orbital periods is $T_n^{1/3} = 1.65n$, $n = 1$, 2, 3.

The conclusion is that this star has three planets in successive orbits with $n = 1$, 2, 3. There maybe other planets with higher n that have not been detected yet.

However, another set of possible numbers n is obtained taking $1.615/2 = 0.808$, $3.363/4 = 0.841$ and $4.973/6 = 0.829$, so the orbital numbers could be $n = 2$, 4 and 6. The resulting set of orbits is then $T_n^{1/3} = 0.83n$. Other orbits would then be

TABLE 2. Extrasolar planetary systems with three planets.

System		Mass (M_J)	T (days)	a (AU)	n	$\boldsymbol{nv_n}$ (km/s)	$\boldsymbol{k}$
61 Vir	b	0.016	4.215	0.05020	1	129.6	
	c	0.0573	38.021	0.2175	2	124.4	
	d	0.072	123.01	0.476	3	126.3	
						$\boldsymbol{\langle nv_n \rangle = 127 \pm 3}$	**5**
Ups And	b	0.69	4.617136	0.059	1	139.0	
	c	1.92	241.33	0.832	4	150.0	
	d	4.13	1278.1	2.51	7	149.6	
						$\boldsymbol{\langle nv_n \rangle = 146 \pm 6}$	**6**
HD 69830	b	0.033	8.667	0.0785	2	197.1	
	c	0.038	31.56	0.186	3	192.4	
	d	0.058	197	0.63	6	208.8	
						$\boldsymbol{\langle nv_n \rangle = 200 \pm 9}$	**8**
GLIESE 876	d	0.02	1.93785	0.021	2	233.6	
	c	0.83	30.258	0.132	5	237.3	
	b	2.64	61.067	0.211	6	225.5	
						$\boldsymbol{\langle nv_n \rangle = 232 \pm 6}$	**10**
HD 40307	b	0.0132	4.345	0.047	3	355.8	
	c	0.0216	9.62	0.081	4	366.4	
	d	0.0288	20.46	0.134	5	356.3	
						$\boldsymbol{\langle nv_n \rangle = 360 \pm 6}$	**15**
PSR 1257+1	b	7e-05	25.2620	0.19	5	409.1	
	c	0.013	66.5419	0.36	7	412.0	
	d	0.012	98.2114	0.46	8	407.6	
						$\boldsymbol{\langle nv_n \rangle = 409 \pm 3}$	**17**
HD 181433	b	0.238	9.3743	0.08	1	92.8	
	c	0.64	962	1.76	5	99.5	
	d	0.54	2172	3.0	6	90.2	
						$\boldsymbol{\langle nv_n \rangle = 94 \pm 5}$	**4**
HD 74156	b	1.88	51.65	0.294	1	61.9	
	d	0.396	336.6	1	2	65.3	
	c	8.03	2476	3.85	4	67.7	
						$\boldsymbol{\langle nv_n \rangle = 65 \pm 3}$	**3**
HD 37124	b	0.64	154.46	0.529	2	74.5	
	d	0.624	843.6	1.64	3	64.5	
	c	0.683	2295	3.19	4	60.5	
						$\boldsymbol{\langle nv_n \rangle = 68 \pm 7}$	**3**
HIP 14810	b	3.88	6.67386	0.0692	1	112.9	
	c	1.28	147.73	0.545	3	121.7	
	d	0.57	962	1.89	5	108.0	
						$\boldsymbol{\langle nv_n \rangle = 114 \pm 7}$	**5**
47 Uma	b	2.53	1078	2.1	3	63.6	
	c	0.54	2391	3.6	4	65.5	
	d	1.64	14002	11.6	7	63.1	
						$\boldsymbol{\langle nv_n \rangle = 64 \pm 2}$	**3**

possible. For $n = 1$, the period would be $T_1 = 0.57$ days. In Ref. [5], there are no stars with planets with such a small period (only the star WASP 19 has the planet b with the period equal to 0.789 day [5]).

Similar calculations may be performed with semi-major axes. For the planets b, c and d of the star 61 Vir, the semi-major axes (in AU) are 0.0502, 0.2175 and 0.476 (Table 2). Then the values of $a^{1/2}$ are 0.224, 0.466 and 0.690. It follows that 0.224:1= 0.224, 0.466:2= 0.233 and 0.690:3= 0.231. As expected from the previous considerations, the orbital numbers are $n = 1$, 2, and 3 and $a_n^{1/2} = 0.23n$. As above, another possible set of numbers for known orbits is $n = 2$, 4, 6, then $a_n^{1/2} = 0.115n$, which for $n = 1$ gives $a_1 = 0.013$ AU. Note, there are no stars with such a small planetary orbit (only star GJ 1214 has planet b with the semi-major axis $a = 0.014$ AU [5]). The number of missing planets would be three, at $n = 1$, 3 and 5. Which set of numbers is real has to be confirmed by additional observational data. This ambiguity is at present unavoidable.

Another example is the star 47 Uma with three detected planets, with assigned orbital numbers $n = 3$, 4, and 7 (see Table 2). Square root of a_n of the planets b, c and d are 1.45, 1.90 and 3.41 $(\mathrm{AU})^{1/2}$. Following Eq. (2′), one obtains: $1.45/3 = 0.48$, $1.90/4 = 0.48$ and $3.41/7 = 0.49$. Consequently, occupied orbits are $n = 3$, 4 and 7 and orbits 1, 2, 5 and 6 have not been detected, or may even be nonexisting. Again, a definitive conclusion may be obtained only by new observational data.

In the examples above, the systems with three planets are discussed in which the method of the determination of orbital numbers in a planetary system is given.

In the following analysis, the systems with at least four planets are examined. These are the Sun's terrestrial and Jovian planets, and the planets of stars HD160691, 55 Cnc, GI 581 and HD 10180.

Table 1 shows the parameters of the planets: the masses, periods, semi-major axes, calculated orbital numbers n, and the products nv_n with average values and errors, and for each system the spacing number k (last column).

Orbital velocity is calculated using the simple formula $v_n = 2\pi a_n/T_n$ AU/day, in which it is assumed that a_n could be taken as the radius of circular orbits. If a_n in this formula is in units AU and T_n in days, then $v_n = 1.0879\ 10^4 a_n/T_n\ \mathrm{km\,s^{-1}}$. It was shown that nv_n is nearly constant for a particular system according to Eq. (5), but depends on the value of the integer k [1c].

The dependence of $T^{1/3}$ on n is illustrated in Fig. 1.

Figure 1 are also shows the data for the satellites of Jupiter, Saturn and Uranus, also given in Table 3. This is done according to our statement that in all planar systems, the existing bodies rotate about the central large body in orbits according to the same physical law, and in particul satisfy the quantization of the specific angular momentum. It means that Jupiter with its satellites may be considered as a small planetary system. That similarly holds for other systems. For example, planetary system of the star Cnc 55 "has some basic structural attributes found in our solar system" [6]. It is also pointed out that the HD 10180 planetary system shows the regular pattern of planets' orbits, as is also seen in the solar system [7].

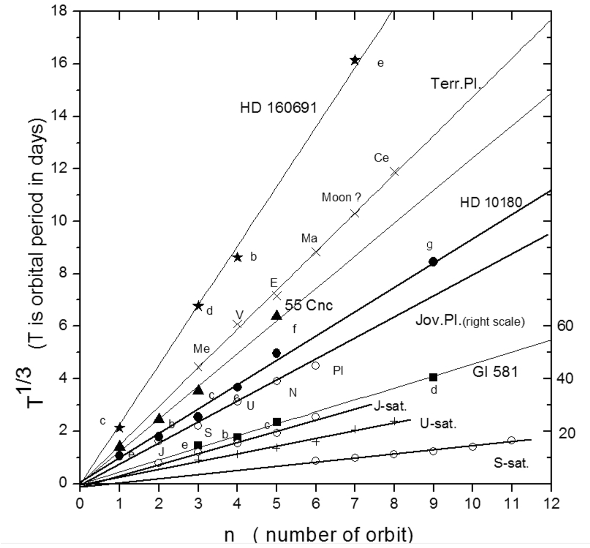

Fig. 1. Third roots of periods divided by chosen small integers n give the straight lines for all bodies in each planetary system. The ns are the orbital numbers (see Eq. (5′)). Systems with at least four bodies are shown.

TABLE 3. Systems of satellites of Jupiter, Saturn and Uranus.

System		T (days)	a (AU)	n	nv_n (km/s)	k
Jupiter	Am.	0.498	0.00121	2	52.65	
	Io	1.569	0.00282	3	52.01	
	Eu.	3.551	0.00449	4	54.98	
	Gan.	7.155	0.00715	5	54.42	
	Call.	16.689	0.0126	6	49.23	
					$\langle nv_n\rangle$=**52.7±1.6**	**2**
Saturn	Jan.	0.693	0.00101	6	95.11	
	Mim.	0.942	0.00124	7	100.24	
	Enc.	1.370	0.00159	8	101.07	
	Teth.	1.888	0.00197	9	102.16	
	Dione	2.737	0.00252	10	100.27	
	Rhea	4.518	0.00352	11	93.31	
					$\langle nv_n\rangle$=**98.69±3.0**	**4**
Uranus	Ariel	2.520	0.00128	5	27.55	
	Umb.	4.144	0.00178	6	28.04	
	Tit.	8.706	0.00291	7	25.49	
	Ober.	13.463	0.0039	8	25.21	
	Mir.	1.414	0.000865	4	26.62	
	Puck	0.672	0.000575	3	24.62	
					$\langle nv_n\rangle$=**26.26±1.2**	**1**

The dependence of nv_n on k is shown in Fig. 2. Straight line obtained by linear regression is

$$nv_n = (23.64 \pm 0.32)k + 3.76 \pm 0.87 \quad \mathrm{km\,s^{-1}} \tag{6}$$

Since $nv_n = kv_0$

$$\mathbf{v_0 = (23.64 \pm 0.32) \quad km\,s^{-1}}. \tag{7}$$

Note that nv_n for the planet 55 Cnc, e is considerably smaller than those of the other planets with $n = 2$, 3, 5 and 14. The reason for that remains unknown, and in Table 1 both e and nv_1 are given in parentheses and are not included in mean value $\langle nv_n \rangle$.

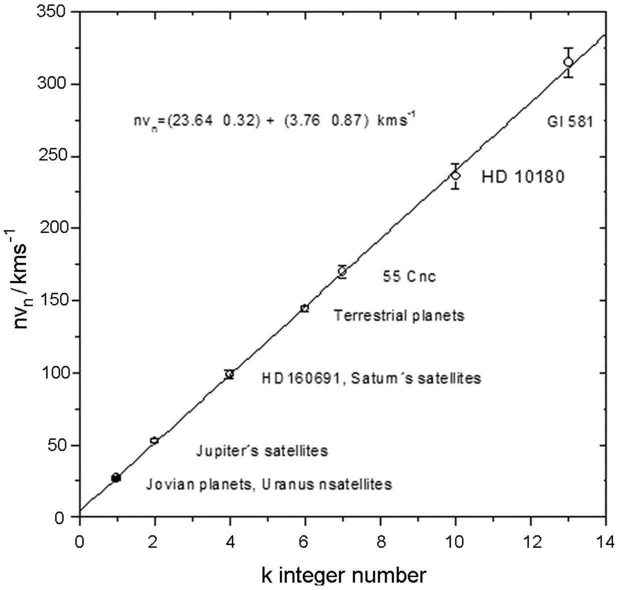

Fig. 2. Products nv_n of orbital number n with orbital velocities v_n for all systems represented in Fig. 1 are shown in steps defined by the spacing number k.

The number k defines the spacing of orbits in a system. It is interesting to point out that Jovian planets and satellites of Uranus both have $k = 1$. It means that orbital velocities decrease with n equaly in both systems. Thus, for the $n = 5$ planet Neptune and Uranian satellite Ariel have the same orbital velocities. This is wonderful having in mind that the planar systems of Sun and Uranus are mutually nearly orthogonal. It is also impressive that planets of the star HD 160691 have $k = 4$ as the satellites of Saturn. Deviation of v_0 in Eq. (6) is less than 2%, owing to the analysis of four or more planets per system.

However, 11 systems with only 3 planets per system have greater dissipation of nv_n, as can be seen in Table 2, where maximum errors are included. Nevertheless, the mean nv_n values may be distributed so that they are close to a straight line defined by Eq. (6), as is illustrated in Fig. 3.

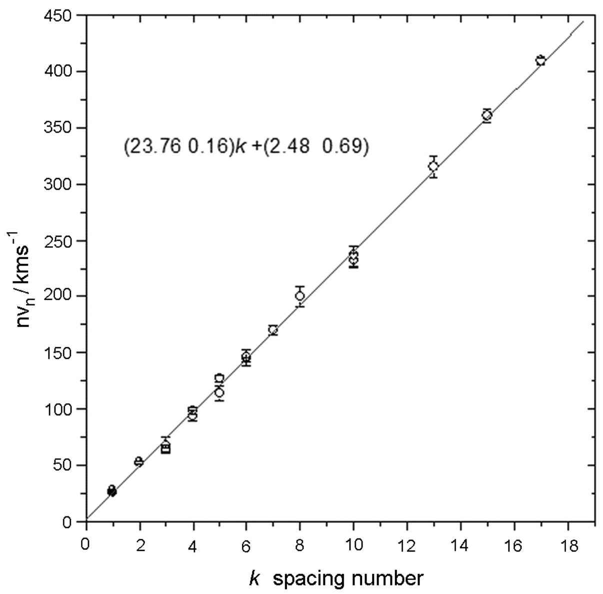

Fig. 3. Dependence of nv_n on k for all systems shown in Fig. 2 and for all systems with three planets.

The straight line determined by all considered systems listed in Tables 1−3 is

$$nv_n = (23.76 \pm 0.16)k + (2.48 \pm 0.69) \;\; \mathrm{km\,s^{-1}}.$$

Therefore, $v_0 = (23.8 \pm 0.2)$ km s^{-1}. In spite of the satisfactory description of the dependence of nv_n on k, the systems with 3 planets cannot safely confirm the quantization of nv_n with the above step of v_0. Additional stars with more planets should be decisive for a final conclusion. Hopefully, advanced technique of observations of extrasolar planetary systems will help in reaching a definite solution.

3. Conclusion

We applied our model of quantization of orbits in the solar system on newly discovered extrasolar planetary systems. We confirmed that the square law for the distribution of orbits deduced from observational orbital parameters in solar system (Eq. (1) and/or Eq. (2)) also holds for the extrasolar planetary systems. Both integers, the orbital number n and the spacing number k, can easily be determined from observed periods and semi-major axes of planets in systems considered. All this may be useful in the classification of orbits. Thus third root of the period divided by some small integer number n needs to be nearly constant for all planets in the system. Then, n is the number of orbit. We emphasize that the square law of orbits defines only the architecture of the planetary system, but details can only be determined by using observational parameters of some real objects belonging to the system considered. When the possible set of orbits is defined on the basis of occupied orbits, then one can anticipate which empty orbits could contain unobserved planets.

For example, the 55 Cnc planets e, b, c, f, and d are, according to our analysis, located at orbits 1, 2, 3, 5 and 14. Eight orbits at $n = 4$, 6,..13 are empty. The first thought is that at orbit 4 could be a small yet undetected planet. Moreover, the authors in Ref. [6] presume that in the gap between periods of 260 days to 13 yr several planets could exist and probably maintain dynamical stability.

In the HD 10180 planetary system, the first five orbits are occupied by the planets b, c, d, e, and f. The last planet h with relatively large mass is at the orbit 14. But at orbit 9 there is the planet g. Similarity with 55 Cnc system is impressive.

The procedure outlined above has also been applied to the origin of the Moon. Namely, in the terrestrial planets, the orbit 7 is empty and is located between Mars and Ceres. We have put forward a hypothesis that the Moon originated at that orbit and later on migrated to be captured by the Earth [1e]. The argument for such an assertion is that definite mass and volume of the Moon are expected when compared with the same quantities of all terrestrial planets.

We hope that determination of possible orbits according to square law could be a guide in the search for extrasolar planetary systems.

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STAZE PLANETA U SUNČEVOM I IZVANSUNČEVIM SUSTAVIMA

Analiza parametra putanja planeta i glavnih planetarnih satelita u sunčevom sustavu, objavljena u našim prijašnjim radovima, upućuje na to da bi sunčev sustav mogao biti prototip i za planetarne sustave zvijezda sličnih Suncu. Zahvaljujući novijim rezultatima u detekciji izvan sunčevih planeta (exoplaneta) omogućena je provjera ove pretpostavke.U tu svrhu su najpogodniji sustavi sa četiri i više planeta, ali nažalost takvih sustava je otkriveno samo nekoliko. Veći broj sustava ima samo tri planete, ali i njhova analiza daje potporu gornjoj pretpostavci iako sa manjom vjerodostojnošću. Kvantizacija putanja u sunčevom sustavu s cijelim brojem n, i kvantizacija prostornosti (pakiranja) putanja sa cijelim brojem k, vodi na relaciju $nv_n = kv_0$, gdje je v_n brzina planete na putanji, a v_0 je konstantna brzina za sve sustave. Ove veličine mogu se odrediti i u izvansunčevim sustavima. Očekujemo da će nova otkrića exoplaneta potvrditi naša dosadašnja saznanja.