Printed ISSN 1330–0008
Online ISSN 1333–9125
CD ISSN 1333–8390
CODEN FIZAE4

MAGNETIC-FIELD INDUCED HOPPING CONDUCTION OF A PSEUDO-GAP INTERACTING SYSTEM

DEBNARAYAN JANA

Department of Physics, University College of Science and Technology, 92 A. P. C. Road, Kolkata 700 009, W.B., India E-mail address: djphy@caluniv.ac.in

Received 17 June 2009; Revised manuscript received 20 November 2009
Accepted 7 July 2010 Online 19 July 2010

We present a simple scaling form of Mott conductivity formula in case of amorphous material in presence of an external magnetic field in an arbitrary spatial dimension d. The model of the interacting system is chosen as the variable range hopping (VRH) one with density of states having a soft gap at the Fermi energy. We indicate the relation between the conductivity exponents (both in weak and strong magnetic field) with that without the magnetic field. The exponents related to the variation of activation energy with temperature have also been computed. We also indicate the lower as well as the upper bound of the conductivity exponents. Besides, a connecting formula between the exponents (non-interacting, interacting with weak magnetic field and interacting with strong magnetic field) shows that only two of them are independent. Previously-known results can be obtained from this generalized form of the conductivity. Finally, a comparison of the exponents has been done with the strong electric field case.

PACS numbers: 72.80.Ng, 73.43.Qt UDC 537.633, 537.312, 538.935
Keywords: variable range hopping (VRH) conduction, pseudo-gap amorphous material

1. Introduction

It is well known that the wave functions for non-interacting electrons in a typical regular solid are extended Bloch states. But, in the presence of disorder and impurities, some of these states become localized. In the limit of strong disorder, the wave function turns out to be extremely distorted and is exponentially localized. When the electronic states at the Fermi energy are localized, the material is an insulator, i. e., there is no conductivity at $T = 0$. But for any finite conductivity at $T = 0$, the material is termed as a metallic one. The detailed analysis of this metal-insulator transition [1, 2] and metal-nonmetal transition [3] as a function of strength has been reviewed in the literature for various situations. The configurationally averaged d.c.

conductivity with states near the appropriate energy at $T = 0$ for these localized states is zero. In other words, in the thermodynamic limit, these localized states do not carry any current. Hence, the conduction involving localized states can only take place by means of transitions of electrons from full states to neighboring empty states with the help of phonons. The extended states, which can carry current at $T = 0$ (i.e. for finite conductivity), can be distinguished from localized one by a quantity known as generalized inverse participation ratio (GIPR). For localized states, GIPR vanishes, while for extended ones GIPR is non-zero when the system size (L) is taken to infinity. This GIPR has been recently used in 2D disordered system in a magnetic field to shed light on the nature of the energy eigenstates in the band [4].

Amorphous materials have attracted considerable attention recently with regard to their electrical properties [5−8]. To understand the behavior of electrical conduction of such a material, a mechanism originally due to Mott [9] was developed. The atoms in amorphous material are distributed at random and the electrons associated with them have distribution of energies. In fact, a band of localized states appears in an amorphous semiconductor. An atom might have an empty state whose energy is slightly higher than that of the occupied state of a chosen given atom. Typically, the activation energy required for electrons to hop to an empty state is very small and hence the hopping conduction [15] takes place. The hopping distance varies because of the random arrangement of the atoms having suitable energy scales and hence the name is given as variable range hopping (VRH). This hopping process differs from the usual electrical conduction in normal metal. Here, in the hopping process, phonons assist the transport while in typical band theory, the transport is impeded by the destruction in periodicity caused by the lattice vibration.

This VRH model has been applied in various branches of condensed matter physics to explore the behavior of electrical conduction at low temperature. In recent years, VRH model has been invoked in metal insulator transition [16, 17], insulating amorphous alloys [18], thin film transistors [19], inorganic compounds [20] and in mesoscopic carbon networks [21]. The VRH model has also been applied to describe the electrical conduction in the DNA double helix [22]. A variant of VRH model known as quasi 1D VRH model [23] has been applied to charge transport in the disordered regime of HCl-doped PAN-ES samples. VRH model has also been proposed in complex systems [24] in the light of percolation theory. The famous Mott's 1/4 law in three dimension was obtained in the system having a constant density of states at the Fermi surface. A soft gap is obtained when the electron-electron Coulomb interaction is taken into account, and is more realistic than a constant density of states (DOS) [9]. In particular, a detailed analysis taking into account the Coulomb interaction [25] between the charged traps in an amorphous semiconductor showed the reduction of the DOS [26] at the Fermi surface by a factor of 2 below its values without the interaction. The exponent 1/2 was obtained in a dilute interacting impurity model in 3 dimensions known as Efros-Shklovskii (ES) limit [27, 28]. The change of exponent from 1/4 to 1/2 is due to the Coulomb gap [29] in VRH. In fact, a better treatment was developed by Larkin and Khemelnitskii [30] for a large localization length to solve the electrostatic problem of Coulomb interaction between the electrons hopping to a new site and hole produced there. Recently, a kinetic Monte-Carlo method has been invoked [31] to test the validity of ES mechanism and other related scaling relations. Considerable discussions regarding the value of the exponent are going on in the literature [32]. Recently, a unifying theory for dc transport has been developed for two-dimensional and quasi-1D interacting electronic systems [33, 34].

Hopping conduction has been observed in high-temperature superconductor (HTSC) to identify the nature of electrical conduction [35 − 37]. More recently, the study on 55 MeV Li^{+3} ion beam irradiation on Bi-2212 system reveals that the variation of normal state resistivity with temperature shows several crossovers [38] of the VRH exponents. These crossovers occurred in a narrow window of temperature and the data point out the existence of a soft gap near the Fermi energy. The long-range interaction in vortex of superconducting thin films is seen to modify the Mott's 1/3 law [39].

Metal-insulator transition below T_c in vacuum-deposited amorphous MoO_3-TeO_2 films [40] has been explored via the Mott's 1/4 law. In the semiconducting regime, a crossover from Mott VRH to single polaron hopping (SPH) has been observed which is absent in the bulk phase [41]. A crossover from Mott to Efros-Shklovskii (ES) model has been noticed [42] in low-doped manganites $La_{0.875}Sr_{0.125-x}Ca_xMnO_3$ ($0 \leq x \leq 0.125$). This x-dependent crossover occurs in the paramagnetic phase. The physical origin of such a transition lies in the change of DOS at the Fermi surface.

The semiconducting nature of the antiferromagnetic material $CaMn_{1-x}Cu_xO_3$ ($0 \leq x \leq 0.20$) has been analyzed in the framework of ES model and the authors have shown the fitting of experimental data with the exponent 1/2 in greater accuracy [43] than with the Mott's 1/4 exponent.

VRH conduction has also been observed [44] in the bulk samples of single-walled carbon nanotubes. In particular, the electrical conduction study in the network of single-wall carbon nanotubes (SWCNT) reveals that the transport process is 2D VRH [45]. Moreover, the electrical field dependence conductivity in this system [45] at strong electrical field was confirmed by the theoretical prediction by us [46]. The doping of boron (B) in carbon networks indicates significant changes [47, 48] in the structural, chemical and optical properties. Recently, 3D VRH conduction has been observed in B-doped multi-wall carbon nanotubes (MWCNT) [49] in the temperature range between 5 K to 290 K. A crossover has been noticed from Mott to ES for $T < 55$ K in amorphous conducting carbon films [50] with different doping levels of boron.

The conduction measurement [51] of n-type CdSe nanocrystal film showed the importance of Coulomb interaction in the temperature regime 10 K $< T <$ 120 K in which 1/2 exponent was obtained. A magnetic-field induced crossover [52] from Mott VRH to a weakly insulating (power law divergence) is seen in n-CdSe samples. A theoretical study of spin-orbit assisted VRH in a strong magnetic field reveals [53] a sharp increase of DOS in the hopping region.

A rigorous calculation of the transverse conductivity [54] of finite sample of amorphous materials in a strong magnetic field shows a transition from Mott to ES regime at some critical temperature. A magneto-resistance (MR) study [55] with insulating $Al_{70}Pd_{22.5}Re_{7.5}$ quasi-crystal reveals a crossover from Mott to ES regime. With this motivation, we would like to compute the conductivity exponent in a magnetic field with the more generalized density of states via a scaling approach.

In this paper, we compute the conductivity exponent in the case of a soft gap under an arbitrary (high and low) uniform static magnetic field in an arbitrary spatial dimension d. The experimental data as discussed in earlier paragraphs could lead to crossover to different exponents under the appropriate conditions. This situation motivated us to derive a set of more general exponents and their interrelationship. The paper is organized as follows. In the next section, we give a very brief introduction to the celebrated VRH model. In Section 3, we discuss the results of this model for interacting system without the magnetic field. In Section 4, we compute the exponents in the case of weak as well as strong magnetic field and discuss their bounds. The comparison of various exponents in different cases is discussed in Section 6. Finally, we give our conclusions in Section 7.

2. *The variable range hopping (VRH) model*

In one sentence, the variable range model is a typical one associated with the low-temperature conduction of localized states in amorphous material. In fact, it is this low-temperature conduction which is of interest to semiconductor device industry because of their attempt to replace the single-crystal device by glass (amorphous) layers. Thus, in this conduction process, a nontrivial competition of inelastic scattering with thermal activation and long-range Coulomb interaction takes place. Before Mott, Miller and Abrahams [10] studied the conduction process through disordered systems via random resistor network known as RRN model [11] in the literature. This model (RRN) is based on the idea on percolation used extensively in studying the geometrical phase transition in statistical mechanics. The variable range model (VRH) defined in the literature [5, 12] for strongly localized system without the external magnetic field has been justified in the light of percolation theory [13, 14]. For the sake of completeness, we here mention some salient features of the model. In the Mott-VRH version, the non-interacting electrons in the presence of phonons (that provide the necessary inelastic scattering for transition) hop from one state to another one having different energy.

We consider two localized states, one filled at or slightly below the Fermi energy E_F, and the other empty above E_F; their energy and spatial separations are W and R, respectively. The hopping transition rate p is given by

$$p = \nu_0 \exp\big(- 2\alpha R - \beta W(R)\big) . \tag{1}$$

The first factor is the quantum mechanical tunnelling probability and is just the overlap between the two localized states decaying with the same characteristic localization length α^{-1}. In typical amorphous semiconductors, the localization length α^{-1} is much smaller than the distance between the impurities. In fact, we will notice later that this helps strongly in the process of hopping conduction.The second factor arises because of the fact that the different localized states must have different energy. This difference in energy $W(R)$ is supported by the phonon scattering and governed by the Boltzmann weight at temperature T. Note that $W(R)$ essentially depends on the separation between the localized states. This mechanism is effective only at low temperature. It is to be noted that the first factor $\exp(-2\alpha R)$ favors short hops. With increase of R, $W(R)$ decreases while the exponential weight increases. Hence, the second factor, the energy activation, favors long hops. In other words, with larger hops, one can reduce the activation energy. Hence, the conductivity is estimated by optimizing the competition between these two terms. At high enough temperature, the variable-range hopping distance R becomes equal to the nearest neighbor distance. The so called attempt frequency ν_0 essentially depends on the electron-phonon coupling and phonon density of states but independent of R and W as defined above. However, in Miller and Abrahams model [10], ν_0 depends on R as well as W.

The above equation (1) can be understood in the light of percolation theory [13] as follows. The conduction of the electrons from site i to site j depends on the transition rate τ_{ij} and whether the site i is occupied along with site j unoccupied and vice versa. Hence, the conduction can be simply written as

$$\sigma = \tau_{ij} f(E_i)\big(1 - f(E_j)\big)\,, \tag{2}$$

where $f(E_i)$ is the Fermi-Dirac function of electrons at site energy E_i. Now, τ_{ij} crucially depends on the localized states and density of phonons at that particular temperature T. Therefore, the transition rate can be written as

$$\tau_{ij} = \nu_0 \exp\big(-2\alpha R_{ij}\big)\rho\big(\Delta_{ij}\big)\,. \tag{3}$$

Since phonons do obey Bose-Einstein distribution, the density of phonons is $\rho\big(\Delta_{ij}\big) = 1/\big(\exp(\beta\Delta_{ij}) - 1\big)$. The mismatch energy $\Delta_{ij} = (E_i - E_j)$ is required for hopping from site i to site j. Combination of these two equations (2) and (3) justify the equation (1) and the requirement of low temperature for occurring of this process.

The transition probability is maximized by the optimization of the exponent $\mathcal{P} = 2\alpha R + \beta W(R)$. For a constant density of states in an arbitrary dimension d [7], it was found that the conductivity, which is related to the transition probability, varies with temperature as

$$\sigma(T) = \sigma_0 \exp\Big(-A\,T^{-1/(d+1)}\Big)\,. \tag{4}$$

The exponents in two and three dimensions have been verified experimentally [15]. All these results were obtained for a *flat* density of states and neglecting the electron-electron interaction.

Let me point out an important feature associated with the above well-known derivation. The hopping distance has been cleverly used in two senses of the above discussion. In one sense, it represents the distance between two localized states, while in other sense, it implies the radius of the sphere within which the hopping conduction takes place. Thus, one has to be very careful to take into account the appropriate weights of the various conduction within the sphere. If one considers the states very close to the sphere, then one may obtain a different power law apart from the usual 1/4 in 3 dimensions. Moreover, the DOS near the Fermi level turns out to be quite high when compared to the standard value of $10^{28}\mathrm{cm}^{-3}(\mathrm{eV})^{-1}$ without metallic conduction. For a good discussion of these points, the readers are referred to Refs. [56, 57]. This does not indicate at all that the underlying qualitative concept is wrong. Till date, Mott's 1/4 law is one of the inspirations to all experimentalists to study the electrical conduction of amorphous materials at low temperature, although it may not be the best fit throughout the whole range of temperature. That is why we are looking for a more generalized description in which the values of the exponents can change with the variation of the density of states.

3. *Results of VRH for pseudo-gap system without the magnetic field*

In this section, we would like to generalize the non-interacting result to interaction one following the single-particle DOS as

$$N(E) \sim |E - E_F|^\nu \,. \tag{5}$$

The non-negative parameter ν actually determines the way the DOS vanishes at the Fermi surface. This particular nature of the density of states arises while studying the localized electrons interacting via Coulomb interaction [27] at low temperature. A bound for the exponent ν was also obtained as [29]

$$\nu \geq d - 1 \tag{6}$$

for an arbitrary dimension d. This argument is based on the following two important points as indicated by Efros and Shklovskii [27, 29]. Firstly, all excitation energies are positive and secondly, the Coulomb interaction energy E_{ij}^{int} at separation R_{ij} scales with the density in an arbitrary d dimension as

$$E_{ij}^{\mathrm{int}} = -e^2 \left[\bar{N}(E_j - E_i)\right]^{1/2}, \tag{7}$$

where $\bar{N}$ is the average density. These two facts immediately point out that the interaction energy cannot have a larger magnitude than $(E_j - E_i)$. With the above density of states, the unitarity condition yields [46] that within the energy intervals $E_F \pm W(R)$ in an arbitrary dimension d, the activation energy $W(R)$ scales as

$$\int \mathrm{d}^d R \int\limits_{E_F}^{E_F+W(R)} N(E)\, dE = 1\,, \qquad W(R) \sim R^{-d/(\nu+1)}\,. \tag{8}$$

Incorporating the above density of states in the Fermi-Dirac distribution function, Eq. (1) takes the form

$$\sigma \sim 2e^2 R^2 T^{\nu} \nu_0 \exp\big(- 2\alpha R - \beta W \big)\,. \tag{9}$$

Hence, the exponent $\mathcal{P}$ in the hopping transition rate in Eq. (1), $p = \nu_0 \exp(-\mathcal{P})$, becomes

$$\mathcal{P} = 2\alpha R + \beta b R^{-d/(\nu+1)}\,. \tag{10}$$

Note that b is a constant independent of R, W and T. Maximizing the hopping transition rate with respect to R, we obtain [46, 61]

$$\sigma(T) = \sigma_0 T^{B(\nu,d)} \exp\big(- AT^{-\phi}\big), \tag{11}$$

where the exponent ϕ is given by

$$\phi_{\rm int}(d,\nu) = \frac{\nu+1}{d+\nu+1}\,, \quad B(\nu,d) = \frac{\nu^2+d\nu-\nu-2}{d+\nu+1}\,, \tag{12}$$

which matches with an earlier result [58]. Even for a power-law type of density of states ($N(E) \propto E^{\nu}$), the same exponent $\phi(d,\nu)$ was obtained recently [33]. For $d = 3$, the exponent reduces to the result $\phi_{\rm int}(3,\nu) = (\nu+1)/(4+\nu)$ derived earlier by Pollak [59] and Hamiltonian [60].

It is also clear from the above derivation that the condition of applying VRH model is

$$W > k_B T, \qquad \alpha R > 1\,. \tag{13}$$

The relation in Eq. (11) can also be written as

$$\rho(T) = \rho_0 \exp\left(\frac{E_a}{k_B T}\right), \tag{14}$$

where $\rho_0 = T^{-B(\nu,d)}$ and $E_a = k_B T \big(T_0/T\big)^p$. This is useful in analyzing experimental data. The weak temperature dependence of the ρ_0 (pre-factor of the resistivity) for the Mott and ES regime is shown in Table 1.

TABLE 1. Temperature dependence of the pre-factor of conductivity.

Dimension	ν	$B(\nu,d)$
$d=1$	0 (Mott)	-1
$d=2$	0 (Mott)	$-\frac{2}{3}$
$d=3$	0 (Mott)	$-\frac{1}{2}$
$d=1$	2 (ES)	$\frac{1}{4}$
$d=2$	2 (ES)	$\frac{2}{5}$
$d=3$	2 (ES)	1

The scaling approach adopted for pseudo-gap system has been further generalized to compute the a.c. conductivity [61] of the system in an arbitrary spatial dimension d.

4. VRH conduction in a magnetic field

Metal insulator transition can also be produced by applying magnetic field to a delta layer [62]. This delta layer in the semiconductor is produced by a dopant sheet having few angstroms thickness buried typically few tens of nanometers below the surface of the thin film. A typical delta layer consists of donor atoms which are randomly distributed in space. Therefore, the above VRH model can again be applied to study the electrical conduction of such a low-dimensional disordered system in an external magnetic field. In two dimensions, for non-interacting case, Pepper [63] showed that the value of the exponent is 1/2. Subsequently, in a Si : Sb layer, it was confirmed experimentally [64]. Hopping conduction in lightly doped semiconductor in both weak and strong magnetic fields for constant density of states at the Fermi level was computed [65]. Equation (4) in the strong magnetic field limit takes the form

$$\sigma(T) = \sigma_0 \exp\big(- AT^{-1/d}\big). \tag{15}$$

5. Generalization to arbitrary dimension

In this section, we would like to generalize the Mott conductivity in a magnetic field taking into account the density of states (DOS) which has a soft gap at the Fermi energy. We assume the DOS as

$$N(E) \sim |E - E_F|^{\nu}\,. \tag{16}$$

The bound [29] on the exponent ν was obtained as

$$\nu \geq d - 1\,, \tag{17}$$

in an arbitrary dimension d. The magnetic length scale $l_c = \sqrt{\hbar c/(eB)}$ automatically arises from the solution of the Schroedinger equation for a particle in a magnetic field. This leads to a typical wave function in the asymptotic limit $\exp\big(- R^2/(4l_c^2)\big)$. Thus, instead of exponential localization, the magnetic field introduces a Gaussian one. Following the calculation done in 3 dimensions with the above density of states [15], the unitarity condition within the energy intervals $E_F \pm W(R)$ in an arbitrary dimension d indicates that

$$\int \mathrm{d}^d R \int\limits_{E_F}^{E_F+W(R)} N(E)\; dE = 1, \qquad C_d R^{2(d-1)} \frac{W^{\nu+1}}{\nu+1} = 1\,. \tag{18}$$

Note that the magnetic field affects only $(d-1)$ dimension. It is clear from Eq. (18) that $W(R)$ is essentially the average energy spacing among states spatially located within a hyper-sphere of radius R. This gives

$$W(R) \sim R^{-2(d-1)/(\nu+1)} \tag{19}$$

The conductivity exponent depends on whether the system is exposed to strong or weak magnetic field. Typically, for heavily doped and highly compensated sample like n-Ge, 8 T field is regarded as weak magnetic field. In a weak magnetic field, the magnetic length l_c becomes large enough so that the amplitude of the zero point motion in the lowest Landau level becomes appreciable. If the state radius r_a without the magnetic field is smaller than this magnetic length l_c, then the magnetic potential energy term becomes smaller compared to the Coulomb energy within the distance r_a. Thus, in this case, the modification over the $B = 0$ case is simple to calculate. The average hopping distance according to Eq. (1) for the generalized density of states varies with temperature as

$$R(T) \sim T^{-(\nu+1)/(d+\nu+1)}. \tag{20}$$

Considering the average energy integral in a weak magnetic field [8], the above average hopping distance is modified to

$$R(T) \sim T^{-d(\nu+1)/(d+\nu+1)}. \tag{21}$$

This implies immediately, the Mott conductivity will vary with temperature as

$$\sigma(T) = \sigma_0 \exp\big(-AT^{-\phi_1}\big), \qquad \phi_1 = \frac{d(\nu+1)}{d+\nu+1}\,. \tag{22}$$

The above analysis fails in a strong magnetic field. In a strong magnetic field, the exponential factor in Eq. (1) changes. Hence, the optimization exponent $\mathcal{P}$ in the hopping transition rate in Eq. (1), $p = \nu_0 \exp(-\mathcal{P})$ becomes

$$\mathcal{P} = \frac{R^2}{2l_c^2} + \frac{\beta b}{R^{2(d-1)/(\nu+1)}}\,. \tag{23}$$

Here b is a constant independent of R, W and T. Maximizing the hopping transition rate with respect to R, we obtain

$$\sigma(T) = \sigma_0 \exp\big(-AT^{-\phi_2}\big), \qquad \phi_2 = \frac{\nu+1}{d+\nu}\,. \tag{24}$$

For three spatial dimensions, $\phi_2 = (\nu+1)/(\nu+3)$ which matches exactly the result derived earlier by Tokumoto et al. [66]. A simple physical argument in support of the value of the exponent ϕ_2 can be given as follows. We generalize the arguments given for the non-interacting case at strong electric field [5] in three dimensions. In a strong magnetic field, the diamagnetic term dominates over the hopping energy giving rise to a simple inequality

$$\frac{eBR^2}{12\hbar c} \geq \beta W\,. \tag{25}$$

In this limit, the conductivity is governed by $\exp(-R^2/l_c^2)$, which in turn fixes the conductivity exponent in the conductivity expression. It is interesting to note that the exponents ϕ_1 and ϕ_2 remain the same even for DOS $N(E) \propto E^\nu$ or $N(E) \propto (E-E_F)^\nu$ with no restriction on ν.

5.1. Inter-relationship between the exponents and their bounds

In fact, the zero magnetic field [46, 61, 58] and strong magnetic field exponents can be combined in a single compact form as

$$\phi_\gamma = \frac{\nu + 1}{d + \nu + \gamma}\,, \tag{26}$$

where $\gamma = 0$ and $\gamma = 1$ correspond to the strong magnetic field and zero magnetic field case, respectively. If we denote the zero magnetic field exponent as ϕ_0, then the relation between the three exponents is given as

$$(\nu + 1)(d\phi_2 - \phi_1) = d\phi_2\phi_0\,. \tag{27}$$

The exponents ϕ_1, ϕ_2 and ϕ_0 essentially depend on the density of states close to the Fermi surface and the spatial dimension. Equation (27) also implies that out of these three exponents ϕ_1 ,ϕ_2 and ϕ_0, any two of them are independent. For the non-interacting case, Eq. (27) reduces to

$$d\phi_2 - \phi_1 = d\phi_2\phi_0\,. \tag{28}$$

For a special case of Coulomb gap in three spatial dimensions (where $\nu = 2$), Eq. (27) becomes

$$3\phi_2 - \phi_1 = \phi_2\phi_0\,. \tag{29}$$

A quick look to the value of ϕ_2 indicates that

$$\frac{d}{2d - 1} < \phi_2 < 1\,. \tag{30}$$

This follows simply because of the strict bound on the value of ν, as shown in Eq. (6). Therefore, we have been able to get both the lower as well as the upper bound of the Mott conductivity exponent in strong magnetic field. For two dimensions, the above inequality indicates that

$$\frac{2}{3} \le \phi_2 < 1\,, \tag{31}$$

while for 3 dimensions

$$\frac{3}{5} \le \phi_2 < 1\,. \tag{32}$$

It is interesting to note that this type of inequality is also satisfied in the case of an interacting system without the external magnetic field [61]. Of course, in that case, the lower bound was different ($\frac{1}{2}$) instead of $\frac{2}{3}$ or $\frac{3}{5}$. Also, the variation of this ϕ_2 exponent with ν is different in comparison to ϕ_3.

The local activation ϵ_a is an effective way [67] of computing the exponents and hence, identifying the Mott and ES processes. The activation energy [8] at a given temperature can be computed for the above three cases. If ρ is the resistivity, then the activation energy ϵ_a is defined as

$$\epsilon_a = \frac{\mathrm{d}(\ln \rho)}{\mathrm{d}(k_B T)^{-1}} . \tag{33}$$

We find that the activation energy ϵ_a varies with temperature T as

$$\epsilon_a \sim T^{\alpha_1}, \qquad \epsilon_a \sim T^{\alpha_2}, \qquad \epsilon_a \sim T^{\alpha_3} , \tag{34}$$

where α_1, α_2 and α_3 are, respectively, the exponents for the strong magnetic field, zero magnetic field and weak magnetic field. The relation between the exponents α_1, α_2 and α_3 is given by

$$(d + \nu + 1)(\alpha_2 - \alpha_3) = \alpha_1 (d + \nu)(\nu + 1) . \tag{35}$$

The above equation implies that only *two* of the exponents are independent. The bounds obeyed by these exponents are

$$\frac{d-1}{2d-1} \le \alpha_1 < 1, \qquad \frac{1}{2} \le \alpha_2 < 1, \qquad \left(1 - \frac{d}{2}\right) \le \alpha_3 < 1 . \tag{36}$$

6. *Comparison of the exponents*

We present below a comparison of the exponents in various cases in Table 2, Table 3 and Table 4.

TABLE 2. Conductivity exponent in the non-interacting case ($\nu = 0$).

Dimension	$B = 0$	Small B	Large B
$d = 2$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{2}$
$d = 3$	$\frac{1}{4}$	$\frac{3}{4}$	$\frac{1}{3}$

TABLE 3. Conductivity exponent in the interacting case ($\nu = 2$).

Dimension	$B = 0$	Small B	Large B
$d = 2$	$\frac{3}{5}$	$\frac{6}{5}$	$\frac{3}{4}$
$d = 3$	$\frac{1}{2}$	$\frac{3}{2}$	$\frac{3}{5}$

TABLE 4. Conductivity exponent in the interacting case ($\nu = 1$).

Dimension	$B = 0$	Small B	Large B
$d = 1$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{2}{3}$

For the non-interacting case, we take explicitly $\nu = 2$ and $\nu = 1$ values for the comparison of the exponents.

It is evident from the above tables that in the presence of an external uniform static magnetic field, the value of the Mott conductivity exponent is higher than that in zero magnetic field in *all* dimensions. The physical arguments given for the three dimensions [8] can also be generalized to an arbitrary dimension d. In a weak magnetic field, it is the phase space restriction in comparison to the zero magnetic field which signals the larger value of the exponent in the magnetic field case. It is also noticed that the weak field exponents are higher than the large B exponents in *all* dimensions. But the application of a strong external magnetic squeezes or shrinks the electronic wave function and thus the typical overlap between the electronic wave functions decreases in comparison to the zero magnetic field case. This fact in turn implies an exponentially higher resistivity and a higher conductivity exponent in comparison to the zero field case.

We also note that the exponents in the interacting cases have higher values in comparison to the non-interacting case in all spatial dimension. A simple physical argument presented in the electric field case [61] can also be repeated here. From Table 4, we notice that for the one-dimensional case, with DOS in the Coulomb gap varying linearly with energy, that the exponents are same for small as well as strong magnetic field.

For different values of the transverse magnetic field from 0.4 T to 1.5 T, Tokumoto et al. [68] found a range of exponents between 0.25 to 0.7 for n-InSb sample with the doping concentration 4.3×10^{14} cm^{-3}. However, surprisingly, majority of the samples obeyed the exponent 1/2 in three dimensions for the $B = 0$ case. This can be understood from the theory developed by Shklovskii [69, 70] for non-resonant scattering processes in the hopping conduction in a strong magnetic field. In such a situation, the asymptotic form of the wave function changes to exponential from the Gaussian one. Because of this exponential nature of the wave function, the exponent 0.5 is same as that without the magnetic field. However, it should be remembered that the magnetic field must be transverse to the direction of tunnelling of electrons. In Table 5 we show a comparison of some experimental exponents with theoretical values.

TABLE 5. Comparison of conductivity exponents.

Dimension	$B = 0$	Small B	Large B	Experimental values
$d = 3$	0.50	1.50	0.60	(0.25 to 0.7), 0.5 for n-InSb
$d = 3$	0.25	0.75	0.33	(0.25 and 0.33), $Al_{70}Pd_{22.5}Re_{7.5}$
$d = 3$	0.25	0.75	0.33	0.25, carbon nanotube, field upto 0.7 T
$d = 3$	0.50	1.50	0.60	0.5, n-Ge, field upto 8 T
$d = 3$	0.50	1.50	0.60	0.5, $In_xGa_{1-x}As$, at 10.5 T and 0.8 T
$d = 2$	0.33	0.66	0.50	0.33, SWCNT with a nickel impurity
$d = 2$	0.33	0.66	0.50	0.33, InAs/GaAs, field upto 35 T

7. *Conclusions and perspectives*

In summary, we have obtained the generalized form of the conductivity in an external magnetic field in an arbitrary dimension d with a density of states having a soft gap. In the non-interacting case, it is noticed that in *all* spatial dimensions both for weak as well as strong magnetic field, the (Mott) conductivity exponent is higher than that without the magnetic field. This conclusion is also valid for the interacting cases. We have also calculated the activation energy as a function of temperature. The lower and upper bounds of the exponents have also been estimated.

Acknowledgements

This work is supported by DST-FIST, Govt. of India, New Delhi. I have benefitted discussions with Dr. Upendranath Nandy of Scottish Church College, Kolkata. We would also like to thank two anonymous referees for many critical comments and suggestions to improve the quality of the paper.

References

[1] P. A. Lee and T. V. Ramakrishnan, Rev. Mod. Phys. **57** (1985) 287.

[2] N. F. Mott, *Metal Insulator Transitions*, 2nd ed., Taylor Francis, London (1990).

[3] P. P. Edwards, T. V. Ramakrishnan and C. N. R. Rao, J. Phys. Chem. **99** (1995) 5228.

[4] D. Jana, Indian J. Phys. A **74** (2000) 547.

[5] N. F. Mott and E. A. Davies, *Electronic Processes in Non-Crystalline Materials*, 2nd ed., Oxford University Press (1979).

[6] S. R. Ellot, *Physics of Amorphous Materials*, London, New York, Longman (1984).

[7] R. Zallen, *The Physics of Amorphous Solids*, Wiley-Interscience Publication (1998).

[8] B. I. Shklovskii and A. L. Efros, *Electronic Properties of Doped Semiconductor*, Springer, Berlin (1984).

[9] N. F. Mott, J. Non-Cryst. Solids **1** (1968) 1; Phil. Mag. **19** (1969) 835.

[10] A. Miller and E. Abrahams, Phys. Rev. **120** (1960) 745.

[11] M. J. Hirsch, Comments on Cond. Mat. Phys. **13** (1987) 249.

[12] N. F. Mott, Adv. Phys. **50** (2001) 865.

[13] V. Ambegaokar, B. I. Halperin and J. S. Langer, Phys. Rev. B **4** (1971) 2612.

[14] S. R. Kirkpatrick, Rev. Mod. Phys. **45** (1973) 574.

[15] M. Pollak and B. I. Shklovskii, *Hopping Conduction in Semiconductors*, North-Holland (1990).

[16] T. G. Castner, Phys. Rev. B **61** (2000) 16596.

[17] T. E. Whall, Contemp. Phys. **33** (1992) 369.

[18] A. Y. Rogatchev and U. Misutani, Phys. Rev. B **61** (2000) 15550.

[19] J. K. Schon, C. Kloc and B. Batlogg, Phys. Rev. B **63** (2001) 5304.

[20] S. Ebisu, T. Sogabe, M. Hayashi and S. Nagata, J. Phys. Chem. Solids **61** (2000) 869.

[21] V. A. Samuilov, J. Galibert, V.K. Ksenevich, V. J. Goldman, M. Rafailovich, J. Sokolov, I. A. Bashmarov and V. A. Dorosinets, Physica B **294** (2001) 319.

[22] Z. G. Yu and X. Song, Phys. Rev. Lett. **86** (2001) 6018.

[23] Z. H. Wang, C. Li, E. M. Scherr, A. G. McComick and A. J. Epstein, Phys. Rev. Lett. **66** (1991) 1475.

[24] Asok K. Sen and Somenath Bhattacharya, in *Continuum Models and Discrete Systems*, eds. D. Bergman and E. Inam, Kluwer Academic Publishers, Dordecht (2004), pp. 367-373.

[25] M. Pollak, Discuss. Faraday Soc. **50** (1971) 13.

[26] G. Srinivasan, Phys. Rev. B **4** (1971) 2581.

[27] A. L. Efros and B. I. Shklovskii, J. Phys. C **8** (1975) L49.

[28] J. H. Davies, P. A. Lee and T. M. Rice, Phys. Rev. B **29** (1984) 4240.

[29] A. L. Efros, J. Phys. C **9** (1976) 2021.

[30] A. I. Larkin and D. E. Khmelnitskii, Sov. Phys. JETP **56** (1982) 647.

[31] D. N. Tsigankov and A. L. Efros, Phys. Rev. Lett. **88** (2002) 176602.

[32] N. F. Mott, Contemp. Phys. **26** (1985) 203.

[33] M. M. Fogler, S. Teber and B. I. Shklovskii, Phys. Rev. B **69** (2004) 035413.

[34] S. Teber, Euro. J. Phys. B **49** (2006) 289.

[35] M. F. Cromnie, A. Y. Liu, M. L. Cohen and A. Zettl, Phys. Rev. B **41** (1990) 2526.

[36] D. Mandrus, L. Forro, C. Kendziora and L. Mihaly, Phys. Rev. B **44** (1991) 2418.

[37] A. M'Michirgui, M. Zouaoui, F. Ben Azzouz, B. Yangui and M. Ben Salem, Solid State Commun. **125** (2003) 71.

[38] A. Sarkar, S. Chattopadhyay, R. Kumar and U. De, Nucl. Instr. and Meth. in Phys. Res. B **230** (2005) 284.

[39] M. P. A. Fisher, T. A. Tokuyasu and A. P. Young, Phys. Rev. Lett. **66** (1991) 2931.

[40] S. Mukherjee, H. Sakata and B. K. Chaudhuri, Chin. J. Phys. **43** (2005) 736.

[41] S. Mukherjee, S. Karmakar, H. Sakata and B. K. Chaudhuri, J. Appl. Phys. **97** (2005) 123707.

[42] E. Bose, S. Karmakar, B. K. Chaudhuri, S. Pal, C. Martin, S. Hebert and A. Maignam, J. Phys.: Condens. Matter **19** (2007) 266218.

[43] M. Kar, S. M. Borah and S. Ravi, Mat. Sci. Eng. B **129** (2006) 54.

[44] Y. Yosida, Tech. Rep. ISSP Ser A, 3496, 13P (1999).

[45] M. Jaiswal, W. Wang, K. A. S. Fernando, Y. -P. Sun and R. Menon, J. Phys.: Condens. Matter **19** (2007) 446006.

[46] D. Jana and J. Fort, Physica B **344** (2004) 62.

[47] O. Stephane, P. M. Ajayan, C. Coliex, Ph. Redlich, J. M. Lambert et. al., Science **266** (1994) 1683.

[48] D. Jana, L.-C. Chen, C. W. Chen, S. Chattopadhyay and K.-H. Chen, Carbon **45** (2007) 1482.

[49] S. Bandow, S. Numao and S. Iijima, J. Phys. Chem. C **111** (2007) 11763.

[50] P. N. Vishwakarma and S. V. Subramanyam, Phil. Mag. **87** (2007) 811; J. Appl. Phys. **100** (2006) 113702.

[51] Dong Yu, Congjun Yang, Brian L. Wehrenberg and Philippe Guyot-Sionnet, Phys. Rev. Lett. **92** (2004) 216802.

[52] Jonathan R. Friedman, Y. Zhang, P. Dai and M. P. Sarachik, Phys. Rev. B **53** (1996) 9528.

[53] Y. Meir and B. L. Altshuler, Europhys. Lett. **37** (1997) 127.

[54] Er'el Granot, Phys. Rev. B **55** (1997) 15828.

[55] H. -C. Fang, Tzung-I Su and Shui-Tien Lin, J. Non-Cryst. Solids, **334/335** (2004) 417.

[56] D. K. Paul and S. S. Mitra, Phys. Rev. Lett. **31** (1973) 1000.

[57] J. M. Marshall and C. Main, J. Physics: Condensed Matter **20** (2008) 285210.

[58] A. J. Zabrodskii, Sov. Phys. Semiconduct. **11** (1977) 345.

[59] M. Pollak, J. Non-Cryst. Solids **11** (1972) 1.

[60] E. M. Hamiltonian, Phil. Mag. **26** (1972) 1043.

[61] D. Jana and J. Fort, in *Frontiers in Condensed Matter Research*, ed. John V. Chang, Nova Science, New York (2006), pp. 111-128.

[62] T. E. Whall, Contemp. Phys. **33** (1992) 369.

[63] M. Pepper, Phil. Mag. **37** (1978) 83; Contemp. Phys. **26** (1985) 257.

[64] N. L. Mattey, T. E. Whall, R. G. Biswas, R. A. A. Kubiak and M. J. Kearney, Phil. Mag. B **66** (1992) 379.

[65] B. I. Shklovskii, Sov. Phys. Semiconduct. **6** (1972) 1053.

[66] H. Tokumoto, R. Mansfield and M. J. Lea, Phil. Mag. **46** (1980) 93.

[67] T. G. Castner, in *Hopping Transport in Solids*, ed. M. Pollak and B. Shklovskii, Elsevier, Amsterdam (1991).

[68] H. Tokumoto, R. Mansfield and M. J. Lea, Solid State Commun. **35** (1980) 961.

[69] B. I. Shklovskii, Sov. Phys. JETP Lett. **36** (1982) 287.

[70] B. I. Shklovskii and B. Z. Spivak, *Scatering and Intereference Phenomena in Variable Range Hopping Conduction*, in *Hopping Conduction in Semiconductor*, ed. M. Pollak and B. I. Shklovskii, North-Holland, Amsterdam (1990).

PRESKOČNA VODLJIVOST UZROKOVANA MAGNETSKIM POLJEM U SUSTAVU S PSEUDO-PROCIJEPOM

Opisuje se jednostavan izraz za sumjeravanje za Mottov izraz za vodljivost u amorfnim tvarima u vanjskom magnetskom polju uz proizvoljnu prostornu dimenziju d. Kao model odabrano je međudjelovanje preskakivanjem promjenljivog dosega u sustavu s mekim procijepom na Fermijevoj energiji. Pokazujemo odnos eksponenata vodljivosti bez magnetskog polja s onima u slabom i jakom magnetskom polju. Izračunali smo također eksponente u svezi s promjenama aktivacijske energije s temperaturom. Pokazuju se donje i gornje granice eksponenata vodljivosti. Relacija među eksponentima (bez međudjelovanja, te s međudjelovanjem u slabom i u jakom magnetskom polju) pokazuje da su samo dva eksponenta nezavisna. Od ranije poznati podaci mogu se izvesti iz novog izraza za vodljivost. Na kraju se daje usporedba eksponenata s onima za slučaj snažnog električnog polja.