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INFLUENCE OF NEGATIVE IONS ON ION-ACOUSTIC SOLITARY WAVES IN A TWO-ELECTRON-TEMPERATURE PLASMA

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Ion-acoustic solitary waves in a drift negative ion plasma have been investigated for two-electron temperatures. Tagare and Reddy studied the effect of higher-order nonlinearity on ion-acoustic waves with cold positive and negative ions for isothermal and non-isothermal electrons using reductive perturbation technique. In this work, we study warm positive and negative ions with two-temperature isothermal electrons using the pseudopotential method. It is found that the concentration of negative ions, drift velocities, mass ratios, equal temperatures of ions (particular case) and presence of two groups of electrons and their ratios modify the profiles of the Sagdeev pseudopotential curves of the solitary waves in the plasma of the first- (ϕ_1) and second-order (ϕ_2) solitary-wave solutions.

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1. Introduction

In non-relativistic and relativistic plasmas, solitary waves and double layers [1 − 5] have been found to be most important in the context of different non-linear phenomena observed both in laboratory and space. Davies, Lust and Schluter [6] were most probably the first to introduce the pseudopotential method. Washimi and Tanuiti [7] were also very remarkable in this field and they derive the Korteweg-de-Vries (K-dV) equation [8] for the study of ion-acoustic solitary waves in a cold plasma. The solitary waves were first experimentally observed by Ikezi et al. [9]. But the experimental results did not obey the theoretically predicted values of amplitude, width and velocity of the solitary waves. For this situation researchers introduced various parameters, e.g. ion-temperature [10], non-isothermality [11], two-temperature electrons [12], inhomogeneity [13] etc. to fit the suitable physical situations in the plasma and for this, some important experimentally verified results were obtained for solitary waves. In ion-acoustic solitary waves, higher-order contribution of non-linear and dispersive terms have been also found to be more effective for removing the difference between the experimental and theoretical results [14 – 16]. Two-temperature-electron plasmas have been observed in strongly interacting electron beam plasma system and in thermonuclear plasma having high-energy tail [17]. In a two-electron-temperatures plasma, Wickens and Allen [18] studied a free expansion into a vacuum of a collisionless quasi-neutral plasma and they showed that this quasi-neutral solution is not single-valued when electron temperature ratio is greater than or equal to 10. In a two distinct Maxwellian electron populations with different temperatures, several authors [19 – 21] have studied the propagation of ion-acoustic solitary waves. Some of them have investigated the effect of ion temperature and also the excitation of solitary waves by high-frequency waves in a three-component plasma having two-temperature isothermal electron components with adiabatic ions. However, negative ions in a plasma have dominant role on the formation of ion-acoustic solitary waves [22]. Drift motion of the ions also plays an important role for the existence of ion-acoustic solitary waves in a plasma [23 – 24]. In the present paper, our intention is to consider the presence of both the drift velocity and negative ions for the study of the propagation of ion-acoustic solitary waves using the pseudo-potential method [25 – 26] which is different from the standard method used by previous investigators. In contrast with the K-dV equation for ion-acoustic waves in a two-electron-temperature plasma, which admits both rarefactive and compressive solitary waves, our K-dV equation is found to admit rarefactive solitary-wave solutions except in a special case of both rarefactive and compressive solitary wave solutions only. Profiles of ion-acoustic solitary waves are shown for the plasma having (He^+, O^-), (He^+, Cl^-) and (H^+, O^-) ions for different negative-ion concentration, drift velocity, mass ratio, ionic temperature and double Maxwellian electron distribution parameters.

The outline of this paper is as follows. Section 2 gives the plasma model and relevant equations for the formation of the problem. In Section 3, conditions for the existence of solitary wave solutions are discussed with the special attention of rarefactive solitary waves. Section 4 is kept for the results and discussions of this problem. Concluding remarks are given in Section 5.

2. Formulation

We consider a collisionless, unmagnetised and isothermal plasma consisting of cold and warm electrons and two-types of ions. Further, we assume that the ions are hot and in the equilibrium state they have constant streaming velocities. Thus the basic set of equations governing the plasma dynamics in unidirectional propagation for positive and negative ions can, therefore, be written in the dimensionless form

$$\frac{\partial n_\alpha}{\partial t} + \frac{\partial}{\partial x}(n_\alpha u_\alpha) = 0\,, \tag{1}$$

$$\frac{\partial u_\alpha}{\partial t} + u_\alpha \frac{\partial u_\alpha}{\partial x} + \frac{\sigma_\alpha}{Q_\alpha n_\alpha}\frac{\partial p_\alpha}{\partial x} = -\frac{Z_\alpha}{Q_\alpha}\frac{\partial \phi}{\partial x}\,, \tag{2}$$

$$\frac{\partial p_\alpha}{\partial t} + u_\alpha \frac{\partial p_\alpha}{\partial x} + 3p_\alpha \frac{\partial u_\alpha}{\partial x} = 0\,, \tag{3}$$

$$\frac{\partial^2 \phi}{\partial x^2} = n_c + n_h - \sum_\alpha Z_\alpha n_\alpha\,, \tag{4}$$

where n_α, u_α, p_α, Q_α, Z_α and σ_α, are the number density, velocity, pressure, mass ratio, charge and temperature of the respective ions. In this case

$$Q_\alpha = \frac{m_j}{m_i}\,, \quad \sigma_\alpha = \frac{T_\alpha}{T_{\text{eff}}}\,, \quad T_{\text{eff}} = \frac{T_c T_h}{\mu T_h + \nu T_c}\,, \quad n_c = \mu \ \exp\left(\frac{\phi}{\mu + \nu\beta}\right),$$
$$n_h = \nu \ \exp\left(\frac{\beta\phi}{\mu + \nu\beta}\right), \quad \beta = \frac{T_c}{T_h} < 1\,, \quad \mu + \nu = 1\,. \tag{5}$$

The subscript $\alpha = i$ is for positive ions and $\alpha = j$ for negative ions. $Z_\alpha = 1$ and $Q_\alpha = 1$ for positive ions (i) and $Z_\alpha = -Z$, $Q_\alpha = Q$ for negative ions (j), μ and ν are the normalized initial cold and hot electron concentrations and $\beta(= T_c/T_h)$ is the cold to hot electron temperature ratio, $\sigma_\alpha(= T_\alpha/T_{\text{eff}})$ is the ion temperature normalized by the effective electron temperature $T_{\text{eff}}(= T_c T_h/(\mu T_h + \nu T_c))$. In the above equations, we have normalized the velocities u_α by the effective ion-acoustic velocity $u_{\text{eff}} = \sqrt{(T_{\text{eff}}/m_\alpha)}$, all densities by the unperturbed equilibrium plasma density n_0, the length by the effective Debye length $\lambda_{\text{eff}} = \sqrt{(T_{\text{eff}}/(4\pi e^2 n_0)}$ whereas the potential ϕ is normalized by (T_{eff}/e), the time t by the reciprocal of ion plasma frequency ω_{pi}^{-1} ($\omega_{pi} = \sqrt{(4\pi n_0 e^2/m_i)}$), pressures p_α are normalized by the ion equilibrium pressure $p_0 = n_0 T_i$, so that the equations appear totally in dimensionless form.

To obtain the solitary wave solutions, we make the dependent variable depend on the single independent variable

$$\eta = x - Vt\,, \tag{6}$$

where V is the velocity of the solitary waves.

We assume that the basic equations are supplemented by the following boundary conditions:

$$u_\alpha \to u_{\alpha 0}, \quad n_\alpha \to n_{\alpha 0}, \quad p_\alpha \to 1, \quad n_c \to \mu, \quad n_h \to \nu, \quad \text{and} \quad \phi \to 0 \quad \text{as} \quad |x| \to \infty\,. \tag{7}$$

The charge neutrality condition of the plasma is

$$\sum_{\alpha} n_{\alpha 0} Z\alpha = 1\,. \tag{8}$$

Using (6), (7) and integrating properly, we get finally from equations (1) to (3)

$$p_\alpha = \frac{(V-u_{\alpha 0})^3}{V-u_\alpha}\,, \tag{9}$$

and

$$n_\alpha = \sqrt{\frac{n_{\alpha 0}^3 Q_\alpha}{6\sigma_\alpha}} \sqrt{\left(K_{1\alpha} - \frac{2Z_\alpha \phi}{Q_\alpha}\right) - \sqrt{\left(K_{1\alpha} - \frac{2Z_\alpha \phi}{Q_\alpha}\right)^2 - K_{2\alpha}^2}}\,, \tag{10}$$

where

$$K_{1\alpha} = (V-u_{\alpha 0})^2 + \frac{3\sigma_\alpha}{Q_\alpha n_{\alpha 0}}\,, \tag{11}$$

and

$$K_{2\alpha}^2 = \frac{12\sigma_\alpha}{Q_\alpha n_{\alpha 0}}(V-u_{\alpha 0})^2\,. \tag{12}$$

The condition for which n_α ($\alpha = i,\, j$) is real is [1]

$$-\frac{Q}{2Z}\left[V - u_{j0} - \sqrt{\frac{3\sigma_j}{Q n_{j0}}}\,\right]^2 < \phi < \frac{1}{2}\left[V - u_{i0} - \sqrt{\frac{3\sigma_i}{n_{i0}}}\,\right]^2. \tag{13}$$

Inequality (13) is the more general form than that of Ghosh et al. [27], and in the absence of negative ions (i.e. when $n_{j0} \to 0$), the condition (13) reduces to the condition of Ghosh et al. [27].

By using (5), (6) and (10), we get from equation (4)

$$\frac{\partial^2 \phi}{\partial \eta^2} = \mu \exp\left(\frac{\phi}{\mu + \nu\beta}\right) + \nu \exp\left(\frac{\beta\phi}{\mu+\nu\beta}\right)$$

$$-\sum_\alpha Z_\alpha \sqrt{\frac{n_{\alpha 0}^3 Q_\alpha}{6\sigma_\alpha}} \sqrt{\left(K_{1\alpha} - \frac{2Z_\alpha \phi}{Q_\alpha}\right) - \sqrt{\left(K_{1\alpha} - \frac{2Z_\alpha \phi}{Q_\alpha}\right)^2 - K_{2\alpha}^2}}\,. \tag{14}$$

Equation (14) can be written in the pseudopotential form

$$\frac{\partial^2 \phi}{\partial \eta^2} = -\frac{\partial \psi}{\partial \phi}\,. \tag{15}$$

After integration of Eq. (15) and using (7), one obtains the "energy law"

$$\frac{1}{2}\left(\frac{\mathrm{d}\phi}{\mathrm{d}\eta}\right)^2 + \psi(\phi) = 0\,, \tag{16}$$

where

$$\begin{aligned}\psi(\phi) &= \mu(\mu+\nu\beta)\left[1-\exp\left(\frac{\phi}{\mu+\nu\beta}\right)\right] + \frac{\nu}{\beta}(\mu+\nu\beta)\left[1-\exp\left(\frac{\beta\phi}{\mu+\nu\beta}\right)\right] \\ &-\frac{1}{6}\sum\sqrt{\frac{n_{\alpha0}Q_\alpha^3}{3\sigma_\alpha}}\left[\left(K_{1\alpha}-\frac{2Z_\alpha\phi}{Q_\alpha}+K_2\alpha\right)^{3/2}\right. \\ &\left.-\left(K_{1\alpha}-\frac{2Z_\alpha\phi}{Q_\alpha}-K_2\alpha\right)^{3/2}+(K_{1\alpha}-K_{2\alpha})^{3/2}-(K_{1\alpha}+K_{2\alpha})^{3/2}\right].\end{aligned} \tag{17}$$

The function $\psi(\phi)$ in Eq. (17) is called the modified Sagdeev potential in a non-relativistic plasma having two-temperature electrons and drifting positive and negative ions. In the absence of negative ion, Eq. (17) exactly reduces to the equation of Ghosh et al. [27] and also reduces to the Sagdeev equation [28] for $\sigma_\alpha \to 0$ and $n_{j0} \to 0$.

3. *Existence of solitary wave solution*

The form of the Sagdeev pseudopotential $\psi(\phi, V)$ in (17) determines whether a soliton-like solution of Eq. (17) will exist or not. For the solitary-wave solution, the Sagdeev potential $\psi(\phi, V)$ in (17) must satisfy the following conditions:

(*i*) $\psi(\phi,V)\Big|_{\phi=0} = \dfrac{\partial\psi(\phi,V)}{\partial\phi}\Big|_{\phi=0} = 0 \quad \text{for all } V.$

(*ii*) $\psi(\phi,V)=0$ for some $\phi=\phi_m$ and V where ϕ_m is some maximum value of ϕ.

(*iii*) $\psi(\phi,V) < 0$ in the interval $0 < |\phi| < |\phi_m|$.

The non-linear disperson relation from condition (*ii*) is the following

$$\begin{aligned}&\mu(\mu+\nu\beta)\left[1-\exp\left(\frac{\phi_m}{\mu+\nu\beta}\right)\right] + \frac{\nu}{\beta}(\mu+\nu\beta)\left[1-\exp\left(\frac{\beta\phi_m}{\mu+\nu\beta}\right)\right] \\ &-\frac{1}{6}\sum_\alpha\sqrt{\frac{n_{\alpha0}^3Q_\alpha^3}{3\sigma_\alpha}}\left[\left(K_{1\alpha}-\frac{2Z_\alpha}{Q_\alpha}\phi_m+K_2\alpha\right)^{3/2}\right. \\ &\left.-\left(K_{1\alpha}-\frac{2Z_\alpha}{Q_\alpha}\phi_m-K_2\alpha\right)^{3/2}+(K_{1\alpha}-K_{2\alpha})^{3/2}-(K_{1\alpha}+K_{2\alpha})^{3/2}\right]=0\,.\end{aligned}$$

In addition, for localized solitary wave solution we require the following condition

$$\left.\frac{\partial^2\psi(\phi,V)}{\partial\phi^2}\right|_{\phi=0} < 0\,, \tag{18}$$

which implies

$$\sum_\alpha \frac{Z_\alpha^2 n_{\alpha 0}^2}{Q_\alpha(V-u_{\alpha 0})^2 n_{\alpha 0} - 3\sigma_\alpha} < 1\,, \tag{19}$$

or

$$\frac{n_{i0}}{(V-u_{i0})^2 - 3\sigma_i/n_{i0}} + \frac{Z^2 n_{j0}}{Q(V-u_{j0})^2 - 3\sigma_j/n_{j0}} < 1\,. \tag{20}$$

This is the condition for the existence of a potential well. After simplifying, we get from (20) the equation for the critical negative ion density (n_{jc}) as

$$\theta_{11} n_{jc}^3 + \theta_{12} n_{jc}^2 + \theta_{13} n_{jc} + \theta_{14} = 0\,, \tag{21}$$

where

$$\theta_{11} = Z^2(QM_j^2 + ZM_i^2)\,,$$

$$\theta_{12} = 2ZQM_j^2 + Z^2M_i^2 - 3Z^2\sigma_i - 3Z^2\sigma_j - ZQM_i^2M_j^2\,,$$

$$\theta_{13} = QM_j^2 + 3Q\sigma_i M_j^2 + 3Z\sigma_j M_i^2 - QM_i^2M_j^2 - 6Z\sigma_j\,, \tag{21a}$$

$$\theta_{14} = 3M_i^2\sigma_j - 3\sigma_j - 9\sigma_i\sigma_j\,,$$

$$M_i = V - u_{i0}\,,$$

$$M_j = V - u_{j0}\,.$$

In the absence of negative ion, the inequality (20) reduces to

$$V > u_{i0} \pm \sqrt{n_{i0} + \frac{3\sigma_i}{n_{i0}}}\,. \tag{22}$$

This is more general form than in Refs. [25, 27, 28]. When $n_{i0} \to 1$ and $\sigma_i \to \sigma$, then $V > u_{i0} \pm \sqrt{1+3\sigma}$, which exactly reduces to the warm-ion condition [25, 27], and for $\sigma = 0$, the Sagdeev's modified cold-ion condition [28] (with drift velocity) $V > u_{i0} \pm 1$ is obtained for single temperature electron plasma. But for non-drifting plasma (for $u_{i0} = 0$), the relation $V > u_{i0} \pm 1$ is reduced to $V > 1$ (taking positive value only), which supports exactly Sagdeev's cold-ion condition [28].

From (16) we have $\mathrm{d}\phi/\mathrm{d}\eta = \pm\sqrt{-2\psi(\phi)}$, but in our present analysis, $\mathrm{d}\phi/\mathrm{d}\eta$ is negative for every $\phi < 0$ (except in a special case when $\phi > 0$ for which compressive solitary waves are observed) so that rarefactive ion-acoustic solitary waves are found for which an extra condition

$$\psi\left[\phi=-\frac{Q}{2Z}\left((V-u_{j0})-\sqrt{\frac{3\sigma_j}{Qn_{j0}}}\right)^2\right]>0$$

is satisfied. This implies

$$\mu\exp\left[\frac{-Q}{2Z(\mu+\nu\beta)}\left(V-u_{j0}-\sqrt{\frac{3\sigma_j}{Qn_{j0}}}\right)^2\right]+\frac{\nu}{\beta}\exp\left[\frac{-\beta Q}{2Z(\mu+\nu\beta)}\left(V-u_{j0}-\sqrt{\frac{3\sigma_j}{Qn_{j0}}}\right)^2\right]$$

$$<\frac{n_{i0}}{\mu+\nu\beta}\left[M_i^2+\frac{\sigma_i}{n_{i0}}-\frac{1}{6}\sqrt{\frac{n_{i0}}{3\sigma_i}}\left(\left\{\left(M_i+\sqrt{\frac{3\sigma_i}{n_{i0}}}\right)^2+\frac{Q}{Z}\left(V-u_{j0}-\sqrt{\frac{3\sigma_j}{Qn_{j0}}}\right)^2\right\}^{3/2}\right.\right.$$

$$\left.\left.-\left\{\left(M_i-\sqrt{\frac{3\sigma_i}{n_{i0}}}\right)^2+\frac{Q}{Z}\left(V-u_{j0}-\sqrt{\frac{3\sigma_j}{Qn_{j0}}}\right)^2\right\}^{3/2}\right)\right]$$

$$+\left(\mu+\frac{\nu}{\beta}\right)+\frac{Qn_{j0}}{\mu+\nu\beta}\left[(V-u_{j0})^2+\frac{\sigma_j}{Qn_{j0}}-8(V-u_{j0})^{3/2}\left(\frac{3\sigma_j}{Qn_{j0}}\right)^{1/4}\right] \tag{23}$$

where $M_i = V - u_{i0}$.

$\psi(\phi)$ is complex for $\phi < (Q/(2Z))\{(V-u_{j0})-\sqrt{3\sigma_j/(Qn_{j0})}\,\}^2$. The condition (23) is the modified form of Ref. [29] for two-temperature-electron plasma with warm positive and negative ions and in the absence of negative ion $[n_{j0}\to 0]$, and with $\beta = 1$ and $\sigma_i = \sigma_j = \sigma = 0$, $Z = 1$, this condition reduces exactly to Sagdeev's condition [28] for cold ion.

To obtain the solitary wave solution of Eq. (15), we expand $\psi(\phi)$ in power series of ϕ and get from (15),

$$\frac{\mathrm{d}^2\phi}{\mathrm{d}\eta^2}=L_1\phi+L_2\phi^2+L_3\phi^3+L_4\phi^4+L_5\phi^5+\cdots, \tag{24}$$

where,

$$L_1=1-\sqrt{\frac{n_{i0}^3}{12\sigma_i p_{i0}}}\left\{\left(V-u_{i0}-\sqrt{\frac{3\sigma_i p_{i0}}{n_{i0}}}\right)^{-1}-\left(V-u_{i0}+\sqrt{\frac{3\sigma_i p_{i0}}{n_{i0}}}\right)^{-1}\right\}$$

$$+\sqrt{\frac{Z^4 n_{j0}^4}{12Q\sigma_j p_{j0}}}\left\{\left(V-u_{j0}-\sqrt{\frac{3\sigma_j p_{j0}}{Qn_{j0}}}\right)^{-1}-\left(V-u_{j0}+\sqrt{\frac{3\sigma_j p_{j0}}{Qn_{j0}}}\right)^{-1}\right\}$$

$$L_2 = \frac{\mu+\nu\beta^2}{2(\mu+\nu\beta)^2} - \frac{1}{4}\sqrt{\frac{n_{i0}^3}{3\sigma_i p_{i0}}}\left\{\left(V-u_{i0}-\sqrt{\frac{3\sigma_i p_{i0}}{n_{i0}}}\right)^{-3} - \left(V-u_{i0}+\sqrt{\frac{3\sigma_i p_{i0}}{n_{i0}}}\right)^{-3}\right\}$$

$$+\frac{Z^3}{4Q^2}\sqrt{\frac{Qn_{j0}^3}{3\sigma_j p_{j0}}}\left\{\left(V-u_{j0}-\sqrt{\frac{3\sigma_j p_{j0}}{Qn_{j0}}}\right)^{-3} - \left(V-u_{j0}+\sqrt{\frac{3\sigma_j p_{j0}}{Qn_{j0}}}\right)^{-3}\right\}$$

$$L_3 = \frac{\mu+\nu\beta^3}{6(\mu+\nu\beta)^3} - \frac{1}{4}\sqrt{\frac{n_{i0}^3}{3\sigma_i p_{i0}}}\left\{\left(V-u_{i0}-\sqrt{\frac{3\sigma_i p_{i0}}{n_{i0}}}\right)^{-5} - \left(V-u_{i0}+\sqrt{\frac{3\sigma_i p_{i0}}{n_{i0}}}\right)^{-5}\right\}$$

$$+\frac{Z^4}{4Q^3}\sqrt{\frac{Qn_{j0}^3}{3\sigma_j p_{j0}}}\left\{\left(V-u_{j0}-\sqrt{\frac{3\sigma_j p_{j0}}{Qn_{j0}}}\right)^{-5} - \left(V-u_{j0}+\sqrt{\frac{3\sigma_j p_{j0}}{Qn_{j0}}}\right)^{-5}\right\}$$

$$L_4 = \frac{\mu+\nu\beta^4}{24(\mu+\nu\beta)^4} - \frac{5}{8}\sqrt{\frac{n_{i0}^3}{12\sigma_i p_{i0}}}\left\{\left(V-u_{i0}-\sqrt{\frac{3\sigma_i p_{i0}}{n_{i0}}}\right)^{-7} - \left(V-u_{i0}+\sqrt{\frac{3\sigma_i p_{i0}}{n_{i0}}}\right)^{-7}\right\}$$

$$-\frac{5}{8}\frac{Z^5}{Q^3}\sqrt{\frac{n_{j0}^3}{12\sigma_j Q p_{j0}}}\left\{\left(V-u_{j0}-\sqrt{\frac{3\sigma_j p_{j0}}{Qn_{j0}}}\right)^{-7} - \left(V-u_{j0}+\sqrt{\frac{3\sigma_j p_{j0}}{Qn_{j0}}}\right)^{-7}\right\}$$

$$L_5 = \frac{\mu+\nu\beta^5}{120(\mu+\nu\beta)^5} - \frac{7}{8}\sqrt{\frac{n_{i0}^3}{12\sigma_i p_{i0}}}\left\{\left(V-u_{i0}-\sqrt{\frac{3\sigma_i p_{i0}}{n_{i0}}}\right)^{-9} - \left(V-u_{i0}+\sqrt{\frac{3\sigma_i p_{i0}}{n_{i0}}}\right)^{-9}\right\}$$

$$-\frac{7}{8}\frac{Z^6}{Q^4}\sqrt{\frac{n_{j0}^3}{12Q\sigma_j p_{j0}}}\left\{\left(V-u_{j0}-\sqrt{\frac{3\sigma_j p_{j0}}{Qn_{j0}}}\right)^{-9} - \left(V-u_{j0}+\sqrt{\frac{3\sigma_j p_{j0}}{Qn_{j0}}}\right)^{-9}\right\} \quad (25)$$

Equation (24) has the first order K-dV soliton solution [30]

$$\phi_1 = \frac{3L_1}{2L_2}\,\mathrm{sech}^2\left(\sqrt{\frac{L_1}{4}}\,\eta\right). \quad (26)$$

For the effect of higher-order non-linearity on the ion-acoustic solitary wave, we take the terms up to ϕ^3 from (15) and get

$$\frac{\mathrm{d}^2\phi}{\mathrm{d}\eta^2} = L_1\phi + L_2\phi^2 + L_3\phi^3\,, \quad (27)$$

where L_1, L_2 and L_3 are given in (25).

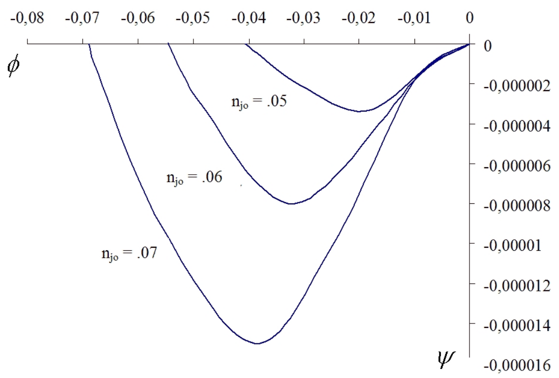
Integrating (27) properly, we finally get the higher-order M-KdV solitary wave solution [30] as

$$\phi_2 = \frac{6L_1}{2L_2 + \sqrt{4L_2^2 - 18L_1L_3}\,\left[2\cosh^2\left(\sqrt{L_1/4}\;\eta\right) - 1\right]} . \tag{28}$$

4. *Results and discussion*

We have investigated the characteristics of ion-acoustic solitary waves for a plasma having drifting positive and negative ions with two-temperature electrons. The profiles of the solitary waves are shown in Figs. 1 to 6 for two equal ion temperatures ($\sigma = \sigma_i = \sigma_j$). In Fig. 1, it is observed that for different negative-ion concentrations ($n_{j0} = 0.05,\ 0.06,\ 0.07$) the rarefactive solitary waves are found for some fixed values of $(V - u_{\alpha 0})$ $[\alpha = i$ and $j]$, σ, Q, β and μ. It is also found that as the negative ion concentration (n_{j0}) increases, the amplitude of the solitary waves increases. When $\mu = 1$, $\nu = 0$, $\beta = 0$ and $\sigma \neq 0$, the profiles of $\psi(\phi)$ follow those of Ref. [31], and when $\mu = 1$, $\nu = 0$, $\beta = 0$ and $\sigma = 0$, the profiles of $\psi(\phi)$ follow those of Ref. [32].

Figure 2 shows the rarefactive solitary waves for the variation of $(V - u_{\alpha 0})$ when σ,Q, n_j, β and μ are fixed. Also it is seen that the velocities including drifts $(V - u_{\alpha 0})$ have dominant role on the formation of solitary waves. When the velocities including drifts are small, the amplitude of the solitary waves becomes large,



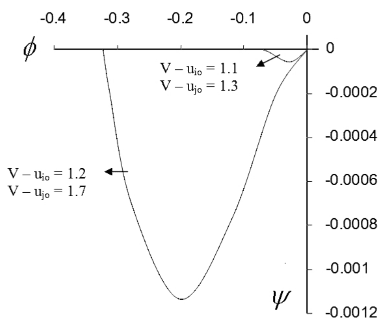


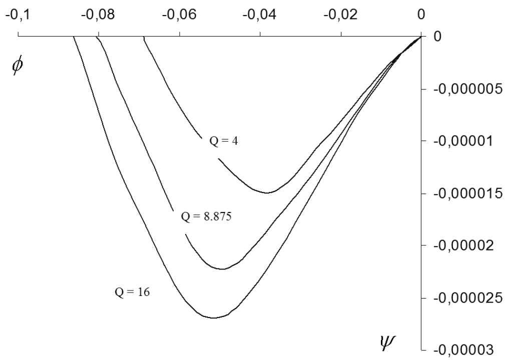
Fig. 1 (left). Solitary-wave profiles (ψ) vs. electrostatic potential (ϕ) for two-temperature electron plasma with the variation of negative ion concentration $[n_{j0}]$ for some fixed values of $Q = 4$, $V - u_{i0} = 1.1$, $V - u_{j0} = 1.3$, $\mu = 0.15$, $\nu = 0.85$, $\beta = 0.025$, $\sigma = 1/30$ and $Z = 1$.

Fig. 2. Solitary-wave profiles (ψ) vs. electrostatic potential (ϕ) for two-temperature electron plasma with the variation of velocities of positive and negative ions including drifts $[V - u_{i0},\ V - u_{j0}]$ for constant values of $Q = 4$, $n_{j0} = 0.05$, $\mu = 0.15$, $\nu = 0.85$, $\beta = 0.025$, $\sigma = 1/30$ and $Z = 1$.

but the solitary waves may not exist in the plasma when the amplitude of the solitary waves is very very small. This happens when the velocities of the ions including drifts are very close to the phase velocity of the wave. When $\mu = 1$, $\nu = 0$, $\beta = 0$ and $\sigma \neq 0$, the profiles of $\psi(\phi)$ follow those of Ref. [31]. In the absence of negative ion [$n_{j0} = 0$ and $V - u_{j0} = 0$] the Sagdeev potential $\psi(\phi)$ follows that of Ref. [27] and for cold ion [$\sigma = 0$] it follows that of Ref. [32].

When the mass ratios [$Q = m_j/m_i$] are increased [$Q = 4,\ 8.875,\ 16$], the rarefactive solitary waves are still formed with higher amplitudes than the previous one for some fixed values of $V - u_{\alpha 0}$, σ, n_{j0}, β and μ. For $\mu = 1$, $\nu = 0$ and $\beta = 0$, the compressive solitary waves are found in the presence of σ [Ref. 31] and in absence of σ [Ref. 32]. Also it is found that the influence of heavier negative ion has a more predominant role than of positive ion. This is shown in Fig. 3.

The effect of temperature (σ) on Sagdeev potential (ψ) profile is the most interesting case. Figure 4 shows the variation of ion temperatures (σ) on Sagdeev potential profiles when $V - u_{\alpha 0}$, n_{j0}, Q, β and μ are fixed. From this figure follows that as the ion-temperature decreases (from $\sigma = 1/15$ to $\sigma = 0$), the amplitude of the rarefactive solitary wave is gradually increasing and it is maximum for the cold-ion case ($\sigma = 0$). This case follows that of Ghosh et al. [27]. When $\mu = 1$, $\nu = 0$, $\beta = 0$ and $\sigma \neq 0$, the Sagdeev potential profiles follow those of Ref. [31] for different respective parameters, and it follows that of Ref. [32] for when $\mu = 1$, $\nu = 0$, $\beta = 0$ and $\sigma = 0$.



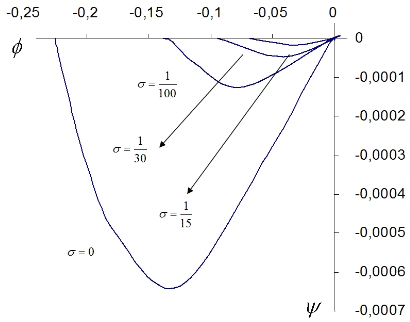


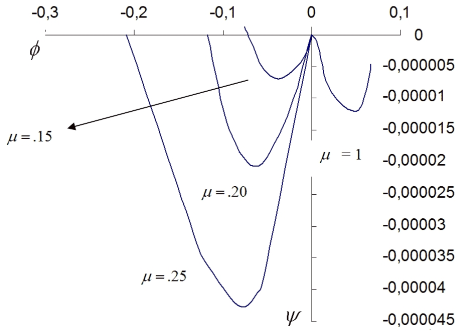
Fig. 3 (left). Solitary-wave profiles (ψ) vs. electrostatic potential (ϕ) for two-temperature electron plasma with the variation of the negative to positive ion masses (Q) for $V - u_{i0} = 1.1$, $V - u_{j0} = 1.3$, $n_{j0} = 0.05$, $\mu = 0.15$, $\nu = 0.85$, $\beta = 0.025$, $\sigma = 1/30$ and $Z = 1$.

Fig. 4. Solitary-wave profiles (ψ) vs. electrostatic potential (ϕ) for two-temperature electron plasma with the variation of ion temperature [σ] for constant values of $Q = 4$, $V - u_{i0} = 1.1$, $V - u_{j0} = 1.3$, $n_{j0} = 0.05$, $\mu = 0.15$, $\nu = 0.85$, $\beta = 0.025$, and $Z = 1$.

Figure 5 shows the effect of cold and hot electron concentration (μ, ν) on Sagdeev potential (ψ) profile. In this case both compressive and rarefactive solitary waves are found. When μ increases, ($\mu = 0.15$,0.20, 0.25, 1) or ν decreases

($\nu = 0.85,\ 0.80,\ 0.75,\ 0$), the amplitude of the wave increases [27] if $V - u_{\alpha 0}$, n_{j0}, Q, σ and β are fixed. For $\mu = 1$ and $\nu = 0$, the interesting case appears for which compressive solitary waves are found instead of rarefactive solitary waves. Rice et al. [33] have also indicated the existence of large amplitude solitary waves for sufficiently large μ. The initial increase in amplitude is slow for low μ, but it starts increasing quite rapidly for large μ.

The effect of cold to hot electron temperature ratio (β) in the presence the of negative ion is shown in Fig. 6. As β increases [$\beta = 0.01,\ 0.025,\ 0.04$], the amplitude of the waves gradually increases. When $\beta = 0$, $\mu = 1$, $\nu = 0$, the profile of $\psi(\phi)$ follows that Ref. [32].



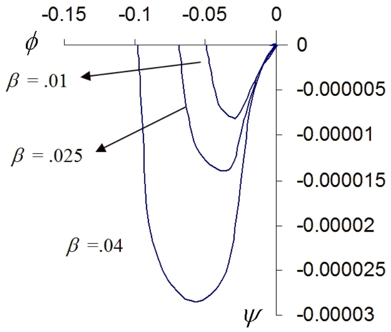


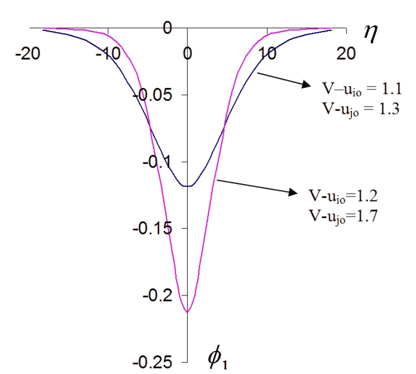
Fig. 5 (left). Solitary-wave profiles (ψ) vs. electrostatic potential (ϕ) for two-temperature electron plasma with the variation of cold and hot electron concentration [μ, ν] for fixed values of $Q = 4$, $V - u_{i0} = 1.1$, $V - u_{j0} = 1.3$, $n_{j0} = 0.05$, $\beta = 0.025$, $\sigma = 1/30$ and $Z = 1$.

Fig. 6 Solitary-wave profiles (ψ) vs. electrostatic potential (ϕ) for two-temperature electron plasma with the variation of cold to hot electron temperature ratio [β] for constant values of $Q = 4$, $V - u_{i0} = 1.1$, $V - u_{j0} = 1.3$, $n_{j0} = 0.05$, $\mu = 0.15$, $\nu = 0.85$, $\sigma = 1/30$ and $Z = 1$.

The effect of the first order (ϕ_1) solitary wave solution and higher-order (ϕ_2) nonlinearity on the propagation of nonlinear ion acoustic waves in a collisionless plasma consisting of positive ions, negative ions and two electron temperature plasma are shown in Figs. 7 to 18 with the variation of the parameters.

In Fig. 7, the profiles of the first order (ϕ_1) solitary wave solutions [ϕ_1 vs. η] for rarefactive solitary waves ($\phi < 0$) are shown with the variation of velocities of positive and negative ions including drifts [$V - u_{i0} = 1.1$, $V - u_{j0} = 1.3$; $V - u_{i0} = 1.2$, $V - u_{j0} = 1.7$] for the fixed values of $Q = 4$, $n_{jo} = 0.01$, $\mu = 0.15$, $\nu = 0.85$, $\beta = 0.025$, $\sigma = 1/30$ and $Z = 1$. When the values of η increase, the values of ϕ_1 [width] decrease. Moreover, the values of ϕ_1 increase when the velocities of positive and negative ions, including drifts, are increased up to a certain a certain value of η, and beyond this value of η the values of ϕ_1 decrease with the variation of velocities of ions including drifts.

Figure 8 shows the profiles of the first-order (ϕ_1) rarefactive solitary wave



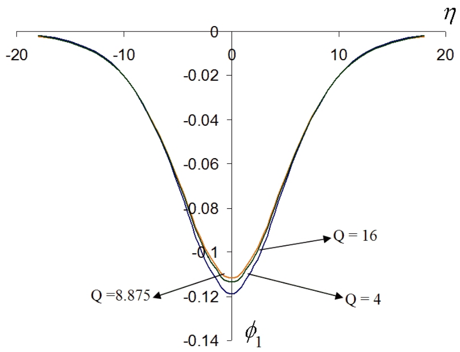


Fig. 7 (left). Profiles of the first-order (ϕ_1) K-dV solitary wave solutions [ϕ_1 vs. η] with the variation of velocities of positive and negative ions including drifts [$V - u_{i0},\ V - u_{j0}$] for fixed values of $Q = 4$, $n_{j0} = 0.01$, $\mu = 0.15$, $\nu = 0.85$, $\beta = 0.025$, $\sigma = 1/30$ and $Z = 1$.

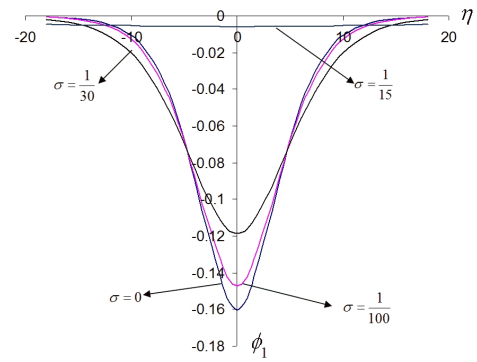
Fig. 8. Profiles of the first-order (ϕ_1) K-dV solitary wave solutions [ϕ_1 vs. η] with the variation of negative to positive ion masses [Q] for $V - u_{i0} = 1.1$, $V - u_{j0} = 1.3$, $n_{j0} = 0.01$, $\mu = 0.15$, $\nu = 0.85$, $\beta = 0.025$, $\sigma = 1/30$ and $Z = 1$.

solutions [ϕ_1 vs. η] with the variation of negative to positive ion mass ratios [$Q =$ 4, 8.875, 16] for the constant values of $V - u_{i0} = 1.1$, $V - u_{j0} = 1.3$, $n_{j0} = 0.01$, $\mu = 0.15$, $\nu = 0.85$, $\beta = 0.025$, $\sigma = 1/30$ and $Z = 1$. The values of ϕ_1 [widths] decrease for different values of Q [$Q = 4$, 16], but for $Q = 8.875$, the value of ϕ_1 increases when $\eta = 0$.

The effect of ionic temperatures (σ) on the first-order (ϕ_1) rarefactive solitary wave solutions [ϕ_1 vs. η] is shown in Fig. 9 for $Q = 4$, $V - u_{i0} = 1.1$, $V - u_{j0} = 1.3$, $n_{j0} = 0.01$, $\mu = 0.15$, $\nu = 0.85$, $\beta = 0.025$ and $Z = 1$. When the ionic temperature (σ) increases ($\sigma = 0,\ 1/100,\ 1/30,\ 1/15$), the values of ϕ_1 decrease up to a certain value of η for the above constant values of the parameters, i.e., the width of the solitary waves decreases with increasing ion temperature (σ) [Ref. 34].

The profiles of the first-order (ϕ_1) rarefactive solitary wave solutions [ϕ_1 vs. η] with the variation of negative ion concentration ($n_{jo} = 0.0,\ 0.05,\ 0.09$) are shown in Fig. 10 for constant values of $Q = 4$, $V - u_{i0} = 1.1$, $V - u_{j0} = 1.3$, $\mu = 0.15$, $\nu = 0.85$, $\beta = 0.025$, $\sigma = 1/30$ and $Z = 1$. The values of ϕ_1 decrease up to a certain value of η when negative ion concentration (n_{j0}) increases and after that ϕ_1 increases for increasing values of η and other respective parameters. Generally, the widths of the rarefactive solitary waves decrease with the increasing values of the negative ion concentration (n_{j0}).

In Fig. 11, the profiles of the first-order (ϕ_1) rarefactive solitary waves solutions [ϕ_1 vs. η] with the variation of cold and hot electron concentration ($\mu,\ \nu$) [$\mu = 0.08,\ \nu = 0.92$; $\mu = 0.15,\ \nu = 0.85$; $\mu = 0.25,\ \nu = 0.75$; $\mu = 0.35,\ \nu = 0.65$] are shown for the constant values of $Q = 4$, $V - u_{i0} = 1.1$, $V - u_{j0} = 1.3$, $n_{j0} = 0.01$, $\beta = 0.025$, $\sigma = 1/30$ and $Z = 1$. As the values of ($\mu,\ \nu$) increase, the values of ϕ_1 also increase for the fixed values of the respective parameters. This shows that, generally, the width increases with increasing μ except for a very small μ ($\mu < 0.1$ for the chosen set of parameters), where with increasing μ it initially decreases for large amplitude solutions and then increases [Ref. 34].



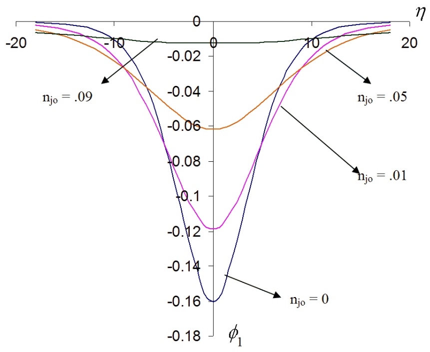
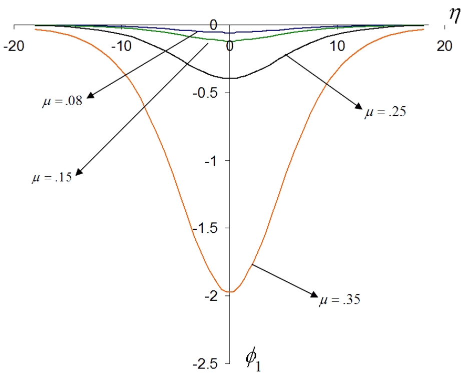


Fig. 9 (left). Profiles of the first-order (ϕ_1) K-dV solitary wave solutions [ϕ_1 vs. η] with the variation of ionic temperature [σ] for constant values of $Q = 4$, $V - u_{i0} = 1.1$, $V - u_{j0} = 1.3$, $n_{j0} = 0.01$, $\mu = 0.15$, $\nu = 0.85$, $\beta = 0.025$ and $Z = 1$.

Fig. 10. Profiles of the first-order (ϕ_1) K-dV solitary wave solutions [ϕ_1 vs. η] with the variation of negative ion concentration [n_{j0}] for $Q = 4$, $V - u_{i0} = 1.1$, $V - u_{j0} = 1.3$, $\mu = 0.15$, $\nu = 0.85$, $\beta = 0.025$, $\sigma = 1/30$ and $Z = 1$.



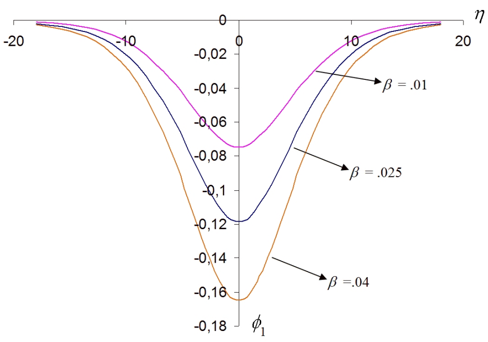


Fig. 11 (left). Profiles of the first-order (ϕ_1) K-dV solitary wave solutions [ϕ_1 vs. η] with the variation of cold and hot electron concentration [μ, ν] for the fixed values of $Q = 4$, $V - u_{i0} = 1.1$, $V - u_{j0} = 1.3$, $n_{j0} = 0.01$, $\beta = 0.025$, $\sigma = 1/30$ and $Z = 1$.

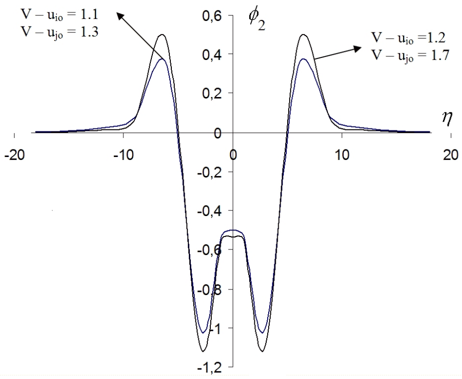
Fig. 12. Profiles of the first-order (ϕ_1) K-dV solitary wave solutions [ϕ_1 vs. η] with the variation of cold to hot electron temperature ratio [β] for $Q = 4$, $V - u_{i0} = 1.1$, $V - u_{j0} = 1.3$, $n_{j0} = 0.01$, $\mu = 0.15$, $\nu = 0.85$, $\sigma = 1/30$ and $Z = 1$.

Figure 12 shows the profiles of the first-order (ϕ_1) rarefactive solitary wave solutions [ϕ_1 vs. η] with the variation of cold to hot electron temperature ratio (β)

[$\beta = 0.01,\ 0.025,\ 0.04$]. The values of ϕ_1 [width of the rarefactive solitary waves] increase [Ref. 34] when β increases for the fixed values of $Q = 4$, $V - u_{i0} = 1.1$, $V - u_{j0} = 1.3$, $n_{j0} = 0.01$, $\mu = 0.15$, $\nu = 0.85$, $\sigma = 1/30$ and $Z = 1$.

In Fig. 13, the profiles of the second order (ϕ_2) rarefactive solitary wave solutions [ϕ_2 vs. η] with the variation of velocities [$V - u_{i0} = 1.1,\ V - u_{j0} = 1.3,\ V - u_{i0} = 1.2,\ V - u_{j0} = 1.7$] of positive and negative ions, including drifts are shown for the constant values of $Q = 4$, $n_{j0} = 0.01$, $\mu = 0.15$, $\nu = 0.85$, $\beta = 0.025$, $\sigma = 1/30$ and $Z = 1$. When the velocities of positive and negative ions, including drifts, are increasing, the profiles of ϕ_2 are larger, but in this case ϕ_2 has a special W-type shape which Tagare [3] found earlier.

Figure 14 shows the second order (ϕ_2) rarefactive solitary wave solutions [ϕ_2 vs. η] with W-type shape for the variation of negative to positive ion mass ratios (Q) [$Q = 4,\ 8.875,\ 16$] for fixed values of $V - u_{i0} = 1.1$, $V - u_{j0} = 1.3$, $n_{j0} = 0.01$, $\mu = 0.15$, $\nu = 0.85$, $\beta = 0.025$, $\sigma = 1/30$ and $Z = 1$. The second order (ϕ_2) solution has a central maximum with a positive value for different values of the mass ratios (Q) which supports the results of Ref. [3].



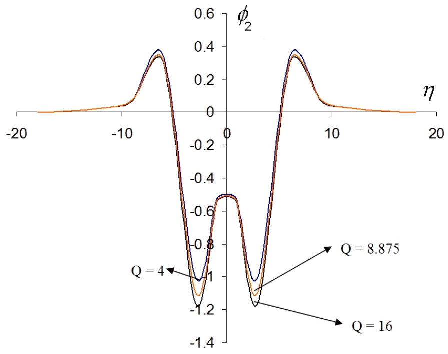
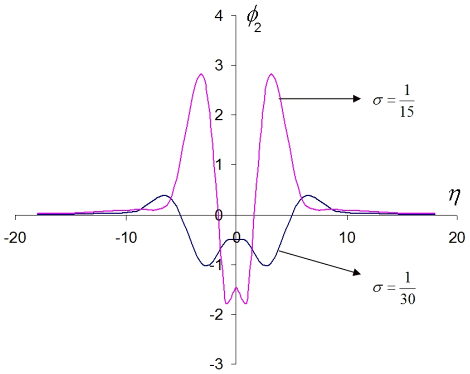


Fig. 13 (left). Profiles of the second-order (ϕ_2) solitary wave solutions [ϕ_2 vs. η] with variation of velocities of positive and negative ions including drifts [$V - u_{i0},\ V - u_{j0}$] for the constant of $Q = 4$, $n_{j0} = 0.01$, $\mu = 0.15$, $\nu = 0.85$, $\beta = 0.025$, $\sigma = 1/30$ and $Z = 1$.

Fig. 14. Profiles of the second-order (ϕ_2) solitary wave solutions [ϕ_2 vs. η] with the variation of negative to positive ion masses [Q] for $V - u_{i0} = 1.1$, $V - u_{j0} = 1.3$, $n_{j0} = 0.01$, $\mu = 0.15$, $\nu = 0.85$, $\beta = 0.025$, $\sigma = 1/30$ and $Z = 1$.

The effect of ionic temperatures (σ) on the second-order (ϕ_2) solitary wave solutions [ϕ_2 vs. η] is shown in Fig. 15. The profiles of the solitary waves are of rarefactive nature with the special W-type shape under the variation of ionic temperatures (σ) [$\sigma = 1/30,\ 1/15$] for fixed values of $Q = 4$, $V - u_{i0} = 1.1$, $V - u_{j0} = 1.3$, $n_{j0} = 0.01$, $\mu = 0.15$, $\nu = 0.85$, $\beta = 0.025$ and $Z = 1$. For $\sigma < 1/30$ (even if $\sigma = 0$, i.e. for cold ion and $\sigma = 1/100$) the higher-order solution (ϕ_2) does not exist for the respective parameters.

In Fig. 16, the second order (ϕ_2) solitary wave solutions [ϕ_2 vs. η] with the variation of negative ion concentration (n_{j0}) $[n_{j0} = 0.01,\ 0.05,\ 0.09]$ is shown for the parameters $Q = 4$, $V - u_{i0} = 1.1$, $V - u_{j0} = 1.3$, $\mu = 0.15$, $\nu = 0.85$, $\beta = 0.025$, $\sigma = 1/30$ and $Z = 1$. It is also of rarefactive nature with W-type shape. As the negative ion concentration (n_{j0}) increases, the values of the second-order (ϕ_2) soliton solution also increase for the constant value of the above parameters. In a collisionless plasma containing negative ions, Tagare et al. [3] showed that higher-order nonlinearity increases the amplitude of the soliton. As the negative ion concentration increases (but still $n_{j0} < n_{jc}$), the second-order one-soliton solution also has W-type shape. The amplitude of the ion-acoustic soliton decreases because of higher-order nonlinearity and this effect becomes more pronounced as the negative ion concentration increases.



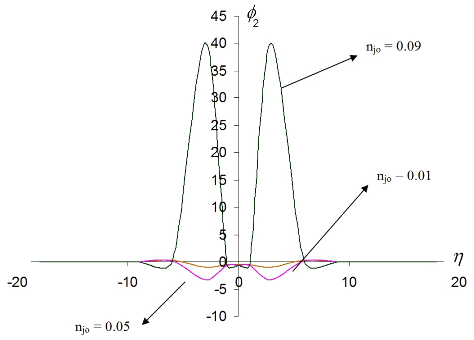
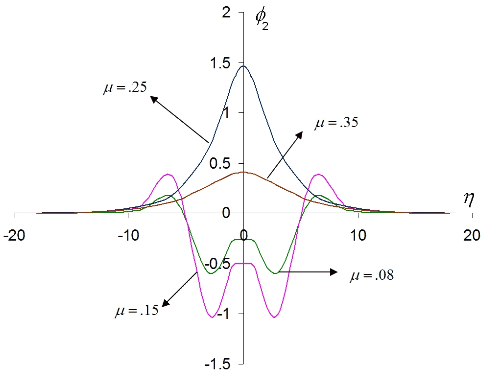


Fig. 15 (left). Profiles of the second-order (ϕ_2) solitary wave solutions $[\phi_2$ vs. $\eta]$ with the variation of ionic temperature $[\sigma]$ for $Q = 4$, $V - u_{i0} = 1.1$, $V - u_{j0} = 1.3$, $n_{j0} = 0.01$, $\mu = 0.15$, $\nu = 0.85$, $\beta = 0.025$ and $Z = 1$.

Fig. 16. Profiles of the second-order (ϕ_2) solitary wave solutions $[\phi_2$ vs. $\eta]$ with the variation of negative ion concentration $[n_{j0}]$ for $Q = 4$, $V - u_{i0} = 1.1$, $V - u_{j0} = 1.3$, $\mu = 0.15$, $\nu = 0.85$, $\beta = 0.025$, $\sigma = 1/30$ and $Z = 1$.

Figure 17 shows the second-order (ϕ_2) solitary wave solutions $[\phi_2$ vs. $\eta]$ with the variation of cold and hot electron concentrations $(\mu,\ \nu)$ $[\mu = 0.08,\ \nu = 0.92;$ $\mu = 0.15,\ \nu = 0.85;$ $\mu = 0.25,\ \nu = 0.75;$ $\mu = 0.35,\ \nu = 0.65]$ for the constant values of $Q = 4$, $V - u_{i0} = 1.1$, $V - u_{j0} = 1.3$, $n_{j0} = 0.01$, $\beta = 0.025$, $\sigma = 1/30$ and $Z = 1$. As the values of cold and hot electron concentration $(\mu,\ \nu)$ increase [provided $\mu \neq 0.25,\ \nu \neq 0.75;\ \mu \neq 0.35,\ \nu \neq 0.65$], the values of the second-order (ϕ_2) solutions also increase with W-type shape and are of rarefactive nature. For $\mu = 0.25,\ \nu = 0.75$ and $\mu = 0.35,\ \nu = 0.65$, with the above other respective parameters, the values of the second-order solitary wave solutions (ϕ_2) are positive everywhere and no negative value of ϕ_2 is obtained, what is an interesting situation.

The second-order (ϕ_2) solitary-wave solutions $[\phi_2$ vs. $\eta]$ with the variation of cold to hot electron temperature ratio (β) are shown in Fig. 18. The profiles of higher-order solution of ϕ_2 are of W-type shape and are of rarefactive nature. When β increases $[\beta = 0.01,\ 0.025,\ 0.04]$, the values of ϕ_2 decrease up to a certain value of η for $\beta = 0.01$ and 0.025 with $Q = 4$, $V - u_{i0} = 1.1$, $V - u_{j0} = 1.3$, $n_{j0} = 0.01$,



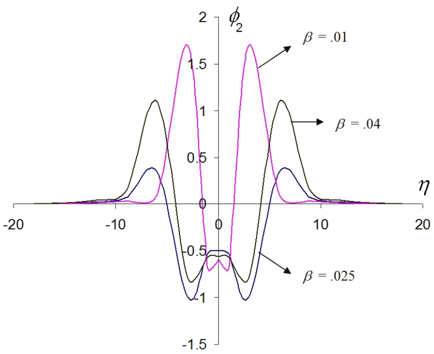


Fig. 17 (left). Profiles of the second-order (ϕ_2) solitary wave solutions [ϕ_2 vs. η] with the variation of cold and hot electron concentration [$\mu,\ \nu$] for the fixed values of $Q = 4$, $V - u_{i0} = 1.1$, $V - u_{j0} = 1.3$, $n_{j0} = 0.01$, $\beta = 0.025$, $\sigma = 1/30$ and $Z = 1$.

Fig. 18. Profiles of the second-order (ϕ_2) solitary wave solutions [ϕ_2 vs. η] with the variation of cold to hot electron temperature ratio [β] for $Q = 4$, $V - u_{i0} = 1.1$, $V - u_{j0} = 1.3$, $n_{j0} = 0.01$, $\mu = 0.15$, $\nu = 0.85$, $\sigma = 1/30$ and $Z = 1$.

$\mu = 0.15$, $\nu = 0.85$, $\sigma = 1/30$ and $Z = 1$, but after that value of η, the value of ϕ_2 is increasing for the same parameters. For $\beta = 0.04$, the values of ϕ_2 decrease for the above parameters.

5. Concluding remarks

We have theoretically investigated the existence of ion-acoustic solitary waves in a drift negative ion-plasma with two electron temperatures having first- (ϕ_1) and second-order (ϕ_2) solitary wave solutions. The inclusion of negative ion reduces considerably the existence domain of rarefactive ion-acoustic solitary waves in a two-electron-plasma in the study using the Sagdeev pseudopotential method. It is known that the negative ion-plasma can be produced in laboratory [35] and also exists in space [36]. In astrophysical plasma, drifting ions are observed during solar bursts and pulsar radiation etc. Thus our present study would be relevant for both the astrophysical as well as laboratory plasma. It should also to be mentioned that the low-temperature electrons (μ) as well as the high-temperature electrons (ν) occur in double plasma machines [37], hot-cathode discharges [38], thermonuclear plasma [39], space-plasma [36] and in laser-irradiated plasma [21]. Nakamura et al. [35] and others [9, 40] experimentally observed the ion-acoustic solitons and shocks. But the experiment on soliton and shocks in a negative-ion plasma having both low- and high-temperature electrons ($\mu,\ \nu$) has not yet been reported. The Sagdeev potential $\psi(\phi)$ from relation (17) is convergent for small values of ϕ given by inequality (13), from which a range of validity is obtained so that the convergence in the expansion of $\psi(\phi)$ is assured. We are now making a plan for studying ion-acoustic solitary waves in two-temperature electron plasma with positive and negative ion drifts and electron drifts.

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UTJECAJ NEGATIVNIH IONA NA IONSKO-ZVUČNE SOLITARNE VALOVE U PLAZMI S DVIJE TEMPERATURE ELEKTRONA

Istražujemo ionsko-zvučne solitarne valove u protječnoj negativnoj plazmi s dvije temperature elektrona. Tagare i Reddy su proučavali učinak nelinearnosti višeg reda na ionsko-zvučne valove s hladnim pozitivnim i negativnim ionima primjenom reduktivne metode smetnje. U ovom se radu proučavaju vrući pozitivni i negativni ioni metodom pseudopotencijala. Nalazimo da koncentracija negativnih iona, brzine protjecanja, omjeri masa, poseban slučaj jednakih temperatura iona i prisutnost dvije grupe elektrona i njihov omjer mijenjaju profile krivulja Sagdeevog pseudopotencijala solitarnih valova u plazmi za rješenja prvog (ϕ_1) i drugog (ϕ_2) reda.