Printed ISSN 1330–0008
Online ISSN 1333–9125
CD ISSN 1333–8390
CODEN FIZAE4

SINGLE-POINT OBSERVATION OF RAPIDLY MOVING OBJECTS

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Received 25 November 2007; Accepted 28 December 2007
Online xx February 2008

We study the kinematical problem of observation of motion of objects that move at velocities comparable to the velocity of signals which are used to determine the velocity and distance of the objects. The main purpose of this work is to demonstrate the importance of time delay between the emission of signals and their reception at the observation point. The simplest cases are considered, i.e., rectilinear motion and assuming the observational point at the path of the moving object. Sound and light signals are considered to transmit the information on the motion. The results relating to the motion at constant velocity and for the acceleration due to a constant force are presented.

PACS numbers: 45.50.-j UDC 531.12, 531.113
Keywords: observation of motion, single observation point, constant velocity, accelerated motion

1. Introduction

In our everyday life, we often experience sounds from rapidly moving vehicles (cars, trains, etc.). Many people have a very good ability to asses the distance and velocity of such objects by listening the intensity and pitch of the sound produced by the vehicles. Of course, viewing the vehicles gives additional (often higher-quality) information. Mostly, the time delay of the sound signals travelling from the objects to the listener is not taken into account. One case may be taken as an exception, the observation of airplanes in flight. We often see airplanes in the sky and observe that the sound of their engines arives from points far behind their visual appearance.

University textbooks and other books treating the description of motion (kinematics) generally do not take into account these delays [1]. The local times are assumed (or the "absolute" time).

The problem of taking into account the time it takes the signals to reach the observer from rapidly moving objects has been generally solved [2]. Different approaches are known. In aviation the application of radars allows quite precise determination of positions and velocities of flying objects using the electromagnetic waves. Since the waves are trasmitted with very small delays, the problem of delay of signals is of minor importance.

In special theory of relativity, the Lorentz transformations relate space-time coordinates of inertial systems. In each system, position (three space coordinates) and time are associated with each point, assuming that the local clocks (which determine the local time) are synchronized in each system, separately. This concept is essential for understanding the Lorentz transformations and their consequences. We mention here only one of them, the Lorentz contraction: bodies moving at speeds comparable to the velocity of light are shorter in the direction of motion. This result of the Lorentz transformations is correct only if the positions of points on the body are determined at the same local time in the system of observation in which the body moves. If this condition is not satisfied, the dimensions of the body may take vastly different values.

R. Penrose in "The apparent shape of a relativistically moving sphere" [3] and J. Terrell in "Invisibility of the Lorentz contraction" [4] discovered unusual effets if the observation of moving bodies is made, say, by taking a photograph from an observation point. The different results are due to the determination of positions of points on the moving body not being at the same local time in the system of observation but at different local times because light rays from points on the body take different times to reach the observation point (the camera). The results are unusual, a sphere is seen as a sphere (not an elipsoid), it is rotated, etc. These results show that in single-point observations of rapidly moving bodies time delays must be taken into account.

The main purpose of the present work is to study the cases when, during the transmission of signals used for observation, the objects move a non-negligible distance. The simplest cases are considered to explain the importance of the signal delays.

2. *Kinematics*

The signals received from the moving objects are used to determine the distance and velocity of the objects. For example, the intensity of signals may be used for the determination of the distance, while the frequency of the signals (as modified by the Doppler effect) may be used to determine the velocity. Other methods are possible.

Regarding the motion, we assume the simplest cases: rectilinear motion, the observation point at the path of the object (the initial point), and motion at constant velocity or accelerated (decelerated) motion due to a constant force. At the moment of passage of the object by the observation point, the time is set to zero and the "initial" velocity is determined. After the elapse of a predetermined time interval, t_{obs} (the observation time), the signal from the object is recorded and analysed to derive the distance and velocity of the object at the time of emission of the signal, t_{em}.

2.1. Non-relativistic case

We consider the detection of sound waves at the observation point after the passage of the object. As an example, one can imagine an airplane flying over observers at low altitude during an air show, or a small submarine moving under the sea surface and past a skin diver. Very simple relations are obtained for the case of constant velocity of the object, $v = v_0$,

$$t_{\mathrm{obs}} = t_{\mathrm{em}} + \frac{s}{V_{\mathrm{sound}}}\,, \tag{1}$$

where s is the distance of the object at the moment of emission of the signal, t_{em} and V_{sound} is the speed of sound. Since $s = v_0 t_{\mathrm{em}}$,

$$t_{\mathrm{em}} = \frac{t_{\mathrm{obs}}}{1 + v_0/V_{\mathrm{sound}}} \quad \text{and} \quad s = \frac{v_0 t_{\mathrm{obs}}}{1 + v_0/V_{\mathrm{sound}}}\,. \tag{2}$$

If the motion under a constant force, F, is considered, and no change of the mass of the object is assumed, a constant acceleration (deceleration) is obtained. At the moment t_{em}, the distance of the object is

$$s = v_0 t_{\mathrm{em}} + \frac{1}{2} a t_{\mathrm{em}}^2\,, \tag{3}$$

and its velocity

$$v = v_0 + a t_{\mathrm{em}}\,, \tag{4}$$

where $a = F/m = const.$ is the acceleration and m the mass of the object. Using Eqs. (1) and (3), one obtains the quadratic equation,

$$t_{\mathrm{em}}^2 + t_{\mathrm{em}} \frac{2V_{\mathrm{sound}}}{a}(1 + \frac{v_0}{V_{\mathrm{sound}}}) - \frac{2V}{a} t_{\mathrm{obs}} = 0\,. \tag{5}$$

This equation has two roots. Since we consider the observation of the signals after the passage of the object, only the positive value of t_{em} is the solution, i.e.,

$$t_{\mathrm{em}} = \frac{V_{\mathrm{sound}}}{a} \left(\sqrt{\left(1 + \frac{v_0}{V_{\mathrm{sound}}}\right)^2 + \frac{2a t_{\mathrm{obs}}}{V_{\mathrm{sound}}}} - \left(1 + \frac{v_0}{V_{\mathrm{sound}}}\right) \right)\,. \tag{6}$$

The position and the velocity, as determined at the observation point at the moment t_{obs}, are obtained from Eqs. (3), (4) and (6).

2.2. Relativistic case

The velocities of objects comparable to the velocity of light are considered. Regarding the motion, the same assumptions are used as given at the beginning of Section 2.1, but light signals are assumed to transmit the data on the position and velocity of objects. As an example, we may take the observation of motion of a bunch of high-speed ions in an accelerating tube, which are excited and emit light.

For the constant velocity of the observed objects, one obtains the results which closely correspond to the non-relativistic case, the only difference being replacement of V_{sound} by the velocity of light, c,

$$t_{\mathrm{em}} = \frac{t_{\mathrm{obs}}}{1+v_0/c} \quad \text{and} \quad s = \frac{v_0 t_{\mathrm{obs}}}{1+v_0/c}, \tag{7}$$

Uniform acceleration is not a realistic assumption because of the change of mass due to relativistic effects. Therefore, a constant force, F, is assumed. For the rectilinear motion, we have

$$F = \frac{\mathrm{d}p}{\mathrm{d}t}, \tag{8}$$

and

$$p = \frac{mv}{\sqrt{1-v^2/c^2}}\,, \tag{9}$$

where m is the rest mass of the object. Simple integration of Eq. (9) yields

$$p = \frac{mv_0}{\sqrt{1-v_0^2/c^2}} + Ft = p_0 + Ft\,, \tag{10}$$

where v_0 is the initial velocity of the object and $p_0 = mv_0/\sqrt{1-v_0^2/c^2}$.

To calculate the distance s of the object at the time t_{em}, we express the velocity in the following way

$$v = c\frac{(p_0+Ft)/(mc)}{\sqrt{1+(p_0+Ft)^2/(mc)^2}}\,. \tag{11}$$

Hence, the distance is given by

$$s = \int\limits_{t=0}^{t_{\mathrm{em}}} v\mathrm{d}t = c\int\limits_{t=0}^{t_{\mathrm{em}}} \frac{(p_0+Ft)/(mc)}{\sqrt{1+(p_0+Ft)^2/(mc)^2}}\,. \tag{12}$$

Integrating of this expression is simple because $\int x\mathrm{d}x/\sqrt{1+x^2} = \sqrt{1+x^2}$. So one obtains

$$s = \frac{mc^2}{F}\left(\sqrt{1+(p_0+Ft_{\mathrm{em}})^2/(mc)^2} - \sqrt{1+p_0^2/(mc)^2}\right)\,. \tag{13}$$

The time of observation is given by

$$t_{\rm obs} = t_{\rm em} + \frac{s}{c}\,.$$

This expression, with s given by Eq. (13), yields

$$t_{\rm em} = \frac{t_{\rm obs}}{2}\;\frac{Ft_{\rm obs}/(mc) + 2\gamma_0}{p_0/(mc) + Ft_{\rm obs}/(mc) + \gamma_0}\,, \tag{14}$$

since $\sqrt{1 + p_0^2/(mc)^2} = 1/\sqrt{1 - v_0^2/c^2} = \gamma_0$.

Equations (13) and (11), with $t_{\rm em}$ given by Eq. (14) represent the results for s and v, respectively, at the moment $t_{\rm em}$ as observed at the later moment $t_{\rm obs}$ at the observation point.

3. Results

To illustrate the results presented above, the values of $t_{\rm em}$, and of s and v at the moment $t_{\rm em}$ have been calculated using parameters given in the captions of Figs. 1 and 2.

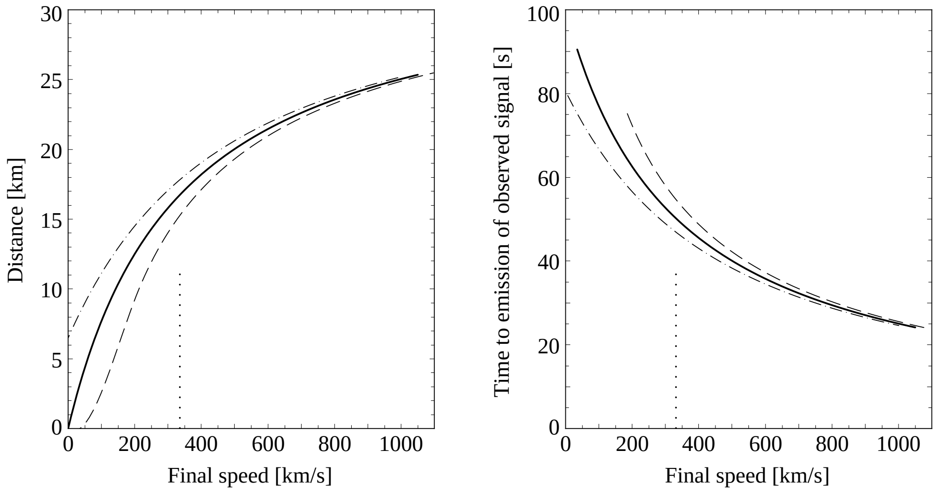

Fig. 1. Use of sound signals for the determination of the distance s, velocity v and time of emission of signals $t_{\rm em}$, assuming a predetermined observation time $t_{\rm obs}$ = 100 s, the signal velocity $V_{\rm sound}$ = 334 m/s and a fixed observation time $t_{\rm obs}$ = 100 s. At left, the distance s is plotted against v, both at the time $t_{\rm em}$, as determined from the sound signals received at the observation time $t_{\rm obs}$. At right, the time of emission of the observed sound signal, $t_{\rm em}$, is plotted against the velocity at the same instant. Full lines show the results for motion at constant velocities, while the dashed and dash-dot-dash lines show the results for accelerated ($a = F/m = 2$ m/s^2) and decelerated motion ($a = F/m = -2$ m/s^2), respectively. The dotted line indicates the speed of sound.

Figure 1 shows the non-relativistic results for observations using sound signals. At left, the calculated values of distance s are displayed against the velocity v at the time t_{em}, as observed at the predetermined observation time $t_{\mathrm{obs}} = 100$ s for various initial velocities v_0. For $v = v_0 = const.$, the object travels at a constant velocity and its distance incresases linearly with the local time. But due to the increasing delay of signals from larger distances for larger values of v, larger fractions of t_{obs} are taken by the signals to reach the observer, as shown in Fig. 1 (right). So the distances s do not increase linearly with v. These relations are valid also at supersonic velocities. It may be noted, while approaching the observational point at a supersonic velocity, no sound is detected for $t < 0$, around $t = 0$ the supersonic boom is heard followed by sound signals emitted prior to the moment $t = 0$ in reverse order. In the case of accelerated motion, we find that the values of s for various values of v, both determined at the time t_{em}, are smaller and for decelerated motion are, respectively, larger than those of motion at $v = v_0 = const.$

Figure 2 shows the results for relativistic velocities and for $t_{\mathrm{obs}} = 1$ μs. Although the velocities are about a million times larger, the results are very similar to the

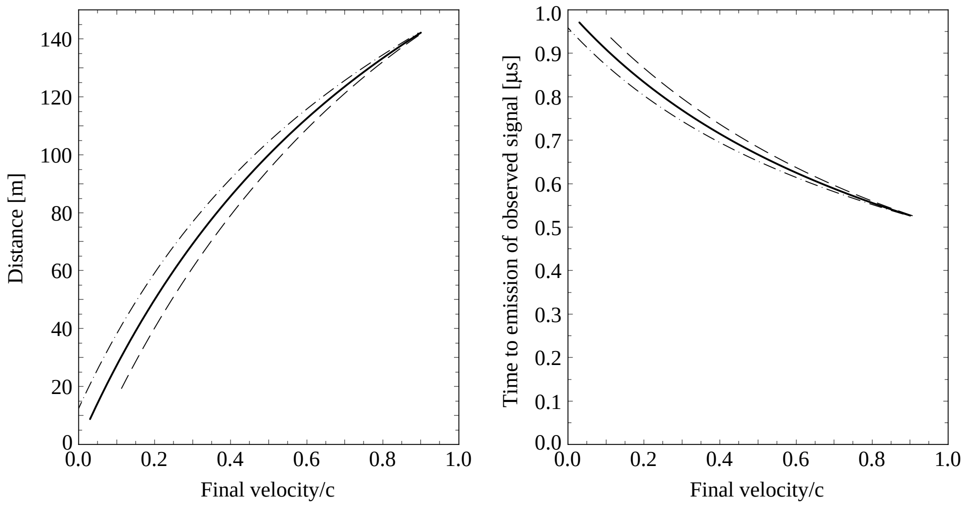

Fig. 2. Use of light signals for the determination of the distance s, velocity v and time of emission of signals t_{em}, assuming a predetermined observation time $t_{\mathrm{obs}} = 1$ μs, the signal velocity (velocity of light) $c = 300$ m/μs and a fixed observation time $t_{\mathrm{obs}} = 1$ μs. At left, the distance s is plotted against v, both at the time t_{em}, as determined from the light signals received at the observation time t_{obs}. At right, the time of emission of the observed light signal, t_{em}, is plotted against the velocity at the same instant. Full lines show the results for motion at constant velocities, while the dashed and dash-dot-dash lines show the results for accelerated and decelerated motion, respectively, due to a constant force-to-mass ratio ($F/m = 25$ m/(μs)^2) and ($F/m = -25$ m/(μs)^2), respectively.

non-relativistic case. The basic difference is that the velocity of light cannot be exceeded.

4. *Conclusions*

The observation of rapidly moving objects from a single point has been considered, assuming rectilinear motion at constant velocities and under the action of a constant force. A constant time between the passage of the object by the observational point at the signal detection, t_{obs}, is assumed. For increasing initial velocities, a larger fraction of t_{obs} is taken for the signal to reach the observer, so the time of motion to the emission point decreases. Therefore, the observed distances are shorter than those calculated using the local time equal to t_{obs}. Similar results are obtained for non-relaticistic velocities and sound signals and for relativistic velocities and light signals.

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OPAŽANJE GIBANJA BRZIH TIJELA IZ JEDNE TOČKE

Razmatramo kinematički problem opažanja gibanja tijela čije su brzine usporedive s brzinama signala koji se rabe za određivanje udaljenosti i brzine tih tijela. Osnovni je cilj ovog rada pokazivanje važnosti vremenskog kašnjenja od trenutka emisije do prijama signala u točki opažanja. Razmatraju se najjednostavniji slučajevi, tj. gibanje duž pravca i položaj opažača na putu promatranog tijela. Pretpostavlja se prijenos podataka o gibanju zvučnim i svjetlosnim signalima. Prikazuju se ishodi računa za gibanje stalnom brzinom i pod djelovanjem stalne sile.