Printed ISSN 1330–0008
Online ISSN 1333–9125
CD ISSN 1333–8390
CODEN FIZAE4

STUDY OF LANDAU DAMPING OF ION-ACOUSTIC WAVES IN DOPE PLASMA

S. N. PAUL[a,b], CH. DAS[c], B. PAUL[b], S. K. BHATTACHARYA[d] and B. CHAKRABORTY[a]

[a]*Centre for Plasma Studies, Faculty of Science, Jadavpur University, Calcutta-700032, India E-mail address: sailenpaul@rediffmail.com*

[b]*Centre for Science Education and Research, Shyamali Housing Estate, Salt Lake, Calcutta-700064, India*

[c]*Department of Physics, Uluberia College, West Bengal, India*

[d]*Department of Physics, Meghnad Saha Institute of Technology, East Kolkata Township, Calcutta-700107, India E-mail address: sujay2k2@rediffmail.com*

Received 18 August 2005; Accepted 4 September 2007
Online 8 February 2008

Landau damping of ion-acoustic waves is theoretically studied in a dope plasma of a light inert gas like helium in the presence of plasma of a heavy inert gas like argon, considering both fluid model and kinetic model. Damping rates of the ion-acoustic waves in dope plasma are different for the two models which may be experimentally investigated by imposing some conditions. Other cases of Landau damping in Vlasov plasma are also considered. The study is intended to be in support of the experimental investigation of Landau damping free from the influence of other non-collisional as well as collisional damping.

PACS numbers: 52.40 Dg, 52.40 Db, 52.25 Vy UDC 531.327, 537.52
Keywords: Landau damping, ion-acoustic wave, Vlasov plasma

1. Introduction

Linear and nonlinear propagation of electrostatic and electromagnetic waves has been theoretically and experimentally investigated by many authors considering different kinds of plasma [1−8]. In a partially ionized plasma, some atoms remain neutral, but their valence electrons are weakly bound to their respective nuclei. The nonlinear distortion of orbits of bound electrons contributes to the optical properties of atoms excited by strong electromagnetic radiation [9]. The electrons of neutral atoms are considered as harmonically bound to their respective nuclei. In the presence of the applied electric wave field, the charged constituents of a plasma experience wave-induced displacements. The stationary orbits of bound electrons are also distorted by the wave field. In laser produced plasma, consisting of multiply-ionized ions and highly charged heavy ions, atoms capture many electrons in their high-lying loosely bound orbitals. When the number of bound electrons becomes large, the line spectra may become very complex. The classical study of particle dynamics of bound electrons explains the nonlinear effects.

Assuming the existence of bound electrons in the background of free electrons in plasma, Chakraborty et al. [10], and other authors [11,12] theoretically studied the nonlinearly induced precessional rotation, magnetic moment field etc. However, they did not consider the collective effects of bound electrons, free electrons and free ions for their studies on the propagation of waves in plasma. Chakraborty et al. [13] have first considered the collective effects of the plasma species for study of the propagation of transverse waves in a Vlasov plasma consisting of a mixture of free electrons, weakly bound electrons, free static ions and static ions associated with the bound electrons, maintaining macroscopic charge neutralization. The dependence on temperature of the phase velocity, group velocity and Thomson scattering cross section have been investigated. They have discussed the Lagrangian and the Hamiltonian of a compressible plasma having bound electrons in the fluid mixture approximation. Chakraborty et al. [14] have investigated the collective behaviour of a plasma when the plasma is considered to be as mixture of compressible fluids of free electrons, free ions, weakly bound electrons and neutral atoms. They have developed a classical theory for understanding the dynamical behaviour of such a plasma. It is found that the Poynting theorem for energy conservation and the Maxwell stress elements have some new terms. The particle dynamical analysis and the energy exchange aspects of free electrons and bound electrons have also been considered. Kinetic and kinematical aspects of this model and other models have been discussed briefly. Recently, Paul et al. [15] have evaluated (i) the zero-harmonic magnetic moment (ii) the damping factor and cut-off frequency and (iii) the collisional energy absorption for an elliptically polarized wave, assuming a plasma to be fluid-like, compressible mixture of populations of weakly bound electrons, free electrons and ions. The characteristics of propagation of small-amplitude waves in such a cold plasma have also been studied. They have discussed the relative roles of bound electrons, the characteristic frequency of free electrons, the characteristic frequency of bound electrons, the resonance condition and the cut-off frequency.

In this paper, we have developed the theory for a modification of Landau damping and dispersion of ion acoustic waves in a plasma having a population of bound electrons in addition to a population of free electrons and ions for neutralization. Since, the phase velocity of ion-acoustic waves is independent of the plasma density, it is convenient to perform experiments with ion waves in partially ionized plasma. Damping of waves in the absence of collisions is found to be more interesting than that in the presence of collisions. In fact, damping in a non-collisional plasma was first considered by Landau [16], and this damping is known as Landau damping of waves. Researchers have tackled the problem of Landau damping in plasma in various ways and still this subject is being studied by many researchers [16 – 20]. Ott and Sudan [21,22] considered the effects of Landau damping on the ion acoustic solitary waves in plasma. In a relativistic plasma, nonlinear propagation of waves in the presence of Landau damping has been studied by Roychowdhury et al. [23] and others. Alexeff et al. [24] measured Landau damping length of ion acoustic waves in a dope plasma of light inert gas without the help of an ambient magnetic field along the direction of wave propagation. To eliminate the influence of collissional and other losses of damping, these authors first produced a plasma of a heavy inert gas like argon and xenon, at wave phase velocity greater than the average ion thermal velocity. Landau damping of ion acoustic wave is not possible in this plasma. Actually, the direct measurement of Landau damping by a wave at phase velocity equal to ion thermal velocity is not possible because this noncollissional damping can not be separated from collissional and other losses. So, Alexeff et al. [24] introduced a small amount of a dope (helium or neon) in plasma. As the dope ions are much lighter, they move at higher velocities and so the waves are Landau damped. If the concentration of dope is increased considerably, this Landau damping will not occur because the phase velocity of the wave then exceeds the ion thermal velocity of light gas.

In our present investigation, we have studied Landau damping in a dope plasma considering the motion for both the electrons and ions. The damping rates in both cases have been numerically estimated and graphically discussed. It is found that the damping rate in the fluid model is lower than when electrons are represented by the Boltzmann distribution function.

2. *Formulation*

To study Landau damping for the ion acoustic waves, we consider a collissionless, partially ionized, unmagnetized dope plasma with two inert gases, one heavy (say argon) and one light (say helium). We assume that the dynamics of electrons and ions are represented by fluid motion and distribution function, respectively. All electrons are considered to belong to the non-thermal fluid and bound loosely to their inner cores of positive charges. The distribution functions of time, velocity components and position coordinates for ions of a heavy inert gas and a light inert gas are:

$$f_{\mathrm{ih}}(r, v, t) \quad \text{and} \quad f_{\mathrm{il}}(r, v, t)\,. \tag{1}$$

The applied electric field for longitudinal wave is

$$E(z, t) = E_0 \cos\theta, \qquad \theta = kz - \omega t \tag{2}$$

where, k and ω are the wave number and frequency of the wave. The normalized equilibrium velocity distribution functions of ions of the heavy and light gases are the Maxwellians

$$f_{\mathrm{ih}}^{0}(v^2) = \left(\frac{M_{\mathrm{h}}}{2\pi K_{\mathrm{B}} T_{\mathrm{ih}}}\right)^{1/2} \exp\left(\frac{-M_{\mathrm{h}} v^2}{2K_{\mathrm{B}} T_{\mathrm{ih}}}\right), \tag{3}$$

$$f_{\mathrm{il}}^{0}(v^2) = \left(\frac{M_{\mathrm{l}}}{2\pi K_{\mathrm{B}} T_{\mathrm{il}}}\right)^{1/2} \exp\left(\frac{-M_{\mathrm{l}} v^2}{2K_{\mathrm{B}} T_{\mathrm{il}}}\right), \tag{4}$$

where,M_{h} and M_{l} are the masses, T_{ih} and T_{il} are the temperatures of heavy and light ions. K_{B} is the Boltzmann constant.

The perturbed state distribution functions are expressed as

$$f_{\mathrm{ih}}(r,v,t) = N_{\mathrm{ih}} f_{\mathrm{ih}}^{0}(v^2) + f_{\mathrm{ih}}(z,v,t)\,. \tag{5}$$

$$f_{\mathrm{il}}(r,v,t) = N_{\mathrm{il}} f_{\mathrm{il}}^{0}(v^2) + f_{\mathrm{il}}(z,v,t)\,. \tag{6}$$

where

$$|f_{\mathrm{ih}}(r,v,t)| >> |f_{\mathrm{ih}}(z,v,t)|, \text{ etc.}, \tag{7}$$

and N_{ih} and N_{il} are number densities in the equilibrium state.

The infinitesimally small perturbation in the distribution functions are

$$f_{\mathrm{ih}}(z,v,t) \quad \text{and} \quad f_{\mathrm{il}}(z,v,t)\,. \tag{8}$$

In the case of electron, the perturbed number densities can be written as

$$n_{\mathrm{eh}} = N_{\mathrm{eh}} + n_{\mathrm{eh}}^{(1)}, \tag{9}$$

$$n_{\mathrm{el}} = N_{\mathrm{el}} + n_{\mathrm{el}}^{(1)}, \tag{10}$$

where

$$\mid N_{\mathrm{eh}} \mid \gg \mid n_{\mathrm{eh}}^{(1)}, \qquad N_{\mathrm{el}} \mid \gg \mid n_{\mathrm{el}}^{(1)} \mid \tag{11}$$

N_{eh} and N_{el} are number densities in the equilibrium state and small perturbations number densities are $n_{\mathrm{eh}}^{(1)}$ and $n_{\mathrm{el}}^{(1)}$.

The guiding equations for the four species plasma are

- for ions of argon plasma

$$\frac{\partial f_{\mathrm{ih}}}{\partial t} + v\frac{\partial}{\partial z} f_{\mathrm{ih}} + N_{\mathrm{ih}} \frac{q_{\mathrm{ih}}}{M_{\mathrm{h}}} E \frac{\partial f_{\mathrm{ih}}^{0}}{\partial v} = 0\,, \tag{12}$$

- for ions of helium plasma

$$\frac{\partial f_{\mathrm{il}}}{\partial t} + v\frac{\partial}{\partial z} f_{\mathrm{il}} + N_{\mathrm{il}}\frac{q_{\mathrm{il}}}{M_{\mathrm{l}}} E\frac{\partial f_{\mathrm{il}}^0}{\partial v} = 0\,, \tag{13}$$

- for electrons of argon plasma

$$\ddot{\xi}_{\mathrm{eh}} = -\frac{q_{\mathrm{eh}}}{m} E - \omega_{\mathrm{oh}}^2 \xi_{\mathrm{eh}} - \nu\dot{\xi}_{\mathrm{eh}}\,, \tag{14}$$

- for electrons of helium plasma

$$\ddot{\xi}_{\mathrm{el}} = -\frac{q_{\mathrm{el}}}{m} E - \omega_{\mathrm{ol}}^2 \xi_{\mathrm{el}} - \nu\dot{\xi}_{\mathrm{el}}\,, \tag{15}$$

where q_{ih}, q_{il}, q_{eh}, q_{el} are the per particle charge of the four species. ξ_{el} and ξ_{eh} are wave field induced displacement of the bound electrons of the light element and heavy element, respectively, and the frequencies of their oscillations are ω_{ol} and ω_{oh}, respectively, m is the mass of an electron and ν is the collisional frequency.

Now, the Gauss's divergence law gives

$$\boldsymbol{\nabla}\cdot\boldsymbol{E} = 4\pi(\rho_{\mathrm{ih}} + \rho_{\mathrm{il}} + \rho_{\mathrm{eh}} + \rho_{\mathrm{el}}) \tag{16a}$$

or,

$$\frac{\partial E}{\partial z} = 4\pi\left[\int (q_{\mathrm{ih}} f_{\mathrm{ih}} + q_{\mathrm{il}} f_{\mathrm{il}})\mathrm{d}v + q_{\mathrm{eh}} n_{\mathrm{eh}}^{(1)} + q_{\mathrm{el}} n_{\mathrm{el}}^{(1)}\right] \tag{16b}$$

where, ρ_{ih}, $\rho_{\mathrm{il}}, \rho_{\mathrm{eh}}, \rho_{\mathrm{el}}$ are the charge densities of the four species in the plasma.

The prefield macroscopic charge neutrality ensures that

$$q_{\mathrm{il}} N_{\mathrm{il}} + q_{\mathrm{ih}} N_{\mathrm{ih}} + q_{\mathrm{el}} N_{\mathrm{el}} + q_{\mathrm{eh}} N_{\mathrm{eh}} = 0\,. \tag{17}$$

Time derivatives of ξ_{l} and ξ_{h} give the field induced average velocities v_{l} and v_{h}. So

$$\dot{\xi}_{\mathrm{il}} = \frac{1}{N_{\mathrm{il}}}\int v f_{\mathrm{il}}\mathrm{d}v, \quad \dot{\xi}_{\mathrm{ih}} = \frac{1}{N_{\mathrm{ih}}}\int v f_{\mathrm{ih}}\mathrm{d}v\,. \tag{18}$$

Using Eqs. (2) and (18) in Eq. (12) we find that

$$f_{\mathrm{ih}}(z,v,t) = -\frac{\mathrm{i}q_{\mathrm{ih}} N_{\mathrm{ih}}}{M_{\mathrm{h}}} E\frac{f_{\mathrm{ih}}^{01}(v^2)}{kv-\omega}\,, \tag{19}$$

$$f_{\mathrm{il}}(z,v,t) = -\frac{\mathrm{i}q_{\mathrm{il}} N_{\mathrm{il}}}{M_{\mathrm{l}}} E\frac{f_{\mathrm{il}}^{01}(v^2)}{kv-\omega}\,, \tag{20}$$

where

$$f_{\mathrm{ih}}^{01} = \mathrm{d}f_{\mathrm{ih}}^0/\mathrm{d}v, \ \text{etc.}$$

The solutions of Eqs (14) and (15) are

$$\xi_{\rm eh} = \frac{q_{\rm eh}E/m}{(\omega^2 - \omega_{\rm oh}^2 + {\rm i}\nu\omega)}\,, \tag{21}$$

$$\xi_{\rm el} = \frac{q_{\rm el}E/m}{(\omega^2 - \omega_{\rm ol}^2 + {\rm i}\nu\omega)}\,. \tag{22}$$

Using the continuity equations

$$\frac{\partial(n_{\rm el})}{\partial t} + \boldsymbol{\nabla}\cdot(n_{\rm el}\boldsymbol{v}_{\rm el}) = 0\,, \tag{23}$$

one obtains

$$n_{\rm el}^{(1)} = -\frac{{\rm i}N_{\rm el}q_{\rm el}kE}{m(\omega^2 - \omega_{\rm ol}^2 + {\rm i}\nu\omega)}\,. \tag{24}$$

Similarly,

$$n_{\rm eh}^{(1)} = -\frac{{\rm i}N_{\rm eh}q_{\rm eh}kE}{m(\omega^2 - \omega_{\rm oh}^2 + {\rm i}\nu\omega)}\,. \tag{25}$$

Then Eq. (16) becomes the wave evolution equation

$$k=\omega_{\rm ih}^2\int\frac{f_{\rm ih}^{01}(v^2){\rm d}v}{(kv-\omega)}+\omega_{\rm il}^2\int v\frac{f_{\rm il}^{01}(v^2){\rm d}v}{(kv-\omega)}+\frac{\omega_{\rm eh}^2k}{(\omega^2-\omega_{\rm oh}^2)+{\rm i}\nu\omega}+\frac{\omega_{\rm el}^2k}{(\omega^2-\omega_{\rm ol}^2)+{\rm i}\nu\omega}\,, \tag{26}$$

where $\omega_{\rm ih}^2 = 4\pi q_{\rm ih}^2 N_{\rm ih}/M_{\rm h}$, etc.

Now, carrying out the contour integration over the singularity, we obtain

$$k = \omega_{\rm ih}^2 P\int\frac{f_{\rm ih}^{01}(v^2){\rm d}v}{(kv-\omega)} \pm \frac{\omega_{\rm ih}^2}{k}\pi{\rm i}f_{\rm ih}^{01}(v_p^2) + \omega_{\rm il}^2 P\int\frac{f_{\rm il}^{01}(v^2){\rm d}v}{(kv-\omega)}$$

$$\pm\frac{\omega_{\rm il}^2}{k}\pi{\rm i}f_{\rm il}^{01}(v_p^2) + \frac{\omega_{\rm eh}^2k}{(\omega^2-\omega_{\rm oh}^2)+i\nu\omega} + \frac{\omega_{\rm el}^2k}{(\omega^2-\omega_{\rm ol}^2)+{\rm i}\nu\omega}\,, \tag{27}$$

where P stands for the principal value of the singular integral and $v_p(=\omega/k)$ is the phase velocity.

For ions of the heavy gas $v < v_p$, the residue physically vanishes and the factor $(kv-\omega)^{-1}$ is expanded in positive integral powers of kv/ω. For ions of the light gas, since $v \le \omega/k$, in the singular integro differential equation, the principal value of the integral of $f_{\rm il}(z, v, t)$ containing $(kv-\omega)^{-1}$ is determined after expanding this factor in positive integral powers of kv/ω. Also the residue of this integral is evaluated at the wave phase velocity v_p. Then

$$k = -\frac{\omega_{\rm ih}^2}{\omega}\left(-\frac{k}{\omega} - \frac{3k^2C_{\rm ih}^2}{\omega^3}\right) - \frac{\omega_{\rm il}^2}{\omega}\left(-\frac{k}{\omega} - \frac{3k^2C_{\rm il}^2}{\omega^3}\right) \pm \pi {\rm i}\frac{\omega_{\rm il}^2}{k} f_{\rm il}^{01}(v_p^2)$$

$$+\frac{\omega_{\rm eh}^2 k}{(\omega^2-\omega_{\rm oh}^2)+{\rm i}\nu\omega} + \frac{\omega_{\rm el}^2 k}{(\omega^2-\omega_{\rm ol}^2)+{\rm i}\nu\omega}\,, \tag{28}$$

where $C_{\rm ih} = K_{\rm B}T_{\rm ih}/M_{\rm h},\ C_{\rm il} = K_{\rm B}T_{\rm il}/M_{\rm l}$

For finding the damping rate, we write $\omega = \omega + {\rm i}\gamma$ where $\gamma << \omega$. Then from Eq. (28) one obtains the damping rate

$$\frac{\gamma}{\omega} = \frac{QP_1 + P_2}{(XP_1 - YQ + Z)\omega}\,, \tag{29}$$

where

$$Q = \pi\frac{\omega_{\rm il}^2 v_p}{k^2 C_{\rm il}^3\sqrt{2\pi}}\exp\left(-\frac{v_p^2}{2C_{\rm il}^2}\right)$$

and

$$P_1 = \left[(\omega^2-\omega_{\rm ol}^2)^2(\omega^2-\omega_{\rm oh}^2)^2 + \omega^2\nu^2\left\{2\omega^4 - 2\omega^2(\omega_{\rm oh}^2+\omega_{\rm ol}^2) - (\omega_{\rm oh}^4+\omega_{\rm ol}^4)\right\} + \omega^4\nu^4\right]\,,$$

$$P_2 = \omega^3\nu^3(\omega_{\rm eh}^2+\omega_{\rm el}^2) - \left\{(\omega^2-\omega_{\rm oh}^2)^2\omega_{\rm el}^2 + (\omega^2-\omega_{\rm ol}^2)^2\omega_{\rm eh}^2\right\}\omega\nu\,,$$

$$X = \left[\frac{2}{\omega^3}(\omega_{\rm ih}^2+\omega_{\rm il}^2) + \frac{12k^2}{\omega^5}(\omega_{\rm ih}^2C_{\rm ih}^2 + \omega_{\rm il}^2C_{\rm il}^2)\right]\,,$$

$$Y = [-2(\omega^2-\omega_{\rm ol}^2)(\omega^2-\omega_{\rm oh}^2)(\omega_{\rm oh}^2-\omega_{\rm ol}^2)\nu + 2(\omega_{\rm oh}^2+\omega_{\rm ol}^2)\nu^3\omega^2$$

$$+4\left\{(\omega^2-\omega_{\rm ol}^2)^2 + (\omega^2-\omega_{\rm oh}^2)^2\right\}\omega^2\nu + 8\omega^4\nu^3]\,,$$

$$Z = \omega\omega_{\rm eh}^2\left[2(\omega^2-\omega_{\rm ol}^2)(\omega^2-\omega_{\rm ol}^2-\nu^2)+6\omega^2\nu^2\right] + \omega\omega_{\rm el}^2\left[2(\omega^2-\omega_{\rm oh}^2)(\omega^2-\omega_{\rm oh}^2-\nu^2)+6\omega^2\nu^2\right]\,.$$

Case I : Neglecting collision terms

$$\frac{\gamma}{\omega} = \sqrt{\frac{\pi}{8}}\frac{\omega_{\rm il}^2\omega^2 v_p}{k^2C_{\rm il}^3}\frac{\exp\left(-\dfrac{v_p^2}{2C_{\rm il}^2}\right)}{(\omega_{\rm ih}^2+\omega_{\rm il}^2)\left[1+\dfrac{\omega^4}{(\omega_{\rm ih}^2+\omega_{\rm il}^2)}\left\{\dfrac{\omega_{\rm eh}^2}{(\omega^2-\omega_{\rm oh}^2)^2} + \dfrac{\omega_{\rm el}^2}{(\omega^2-\omega_{\rm ol}^2)^2}\right\}\right]}\,. \tag{30}$$

Case II : If $\omega >> \omega_{\rm oh}, \omega_{\rm ol}$, and neglecting collision term we have

$$\frac{\gamma}{\omega} = \sqrt{\frac{\pi}{8}} \frac{\omega_{\rm il}^2 \omega^2 v_p}{k^2 C_{\rm il}^3} \frac{\exp\left(-\dfrac{v_p^2}{2C_{\rm il}^2}\right)}{(\omega_{\rm ih}^2 + \omega_{\rm il}^2)\left[1 + \dfrac{\omega_{\rm eh}^2 + \omega_{\rm el}^2}{(\omega_{\rm ih}^2 + \omega_{\rm il}^2)}\right]} \,. \tag{31}$$

For finding the Landau damping distance, we replace k by $k+{\rm i}/d$. Then we have

$$d = \frac{\sqrt{2\pi}\omega_{\rm il}^2 v_p \exp\left(-\dfrac{v_p^2}{2C_{\rm il}^2}\right)}{k^3 C_{\rm il}^3 \left[\dfrac{\omega_{\rm ih}^2 + \omega_{\rm il}^2}{\omega^2} + \dfrac{\omega_{\rm eh}^2(\omega^2 - \omega_{\rm oh}^2)}{(\omega^2 - \omega_{\rm oh}^2)^2 + \nu^2\omega^2} + \dfrac{\omega_{\rm el}^2(\omega^2 - \omega_{\rm ol}^2)}{(\omega^2 - \omega_{\rm ol}^2)^2 + \nu^2\omega^2}\right]} \,. \tag{32}$$

Case III : Neglecting collision terms, since $\omega > \omega_{\rm oh}, \omega_{\rm ol}$, we obtain

$$d = \frac{\sqrt{2\pi}\omega_{\rm il}^2 v_p \exp\left(-\dfrac{v_p^2}{2C_{\rm il}^2}\right)}{k^3 C_{\rm il}^3 \left[\dfrac{\omega_{\rm ih}^2 + \omega_{\rm il}^2}{\omega^2} + \dfrac{\omega_{\rm eh}^2 + \omega_{\rm el}^2}{\omega^2}\right]} \,. \tag{33}$$

If we now consider that both electrons and ions follow the Boltzmann-Vlasov equation for the distribution functions instead of the fluid model for the electron motion, then the four distribution functions of time, velocity components, position coordinates for electrons and ions of the heavy inert gas and the light inert gas will be

$$f_{\rm ih}(r,v,t), \;\; f_{\rm il}(r,v,t), \;\; f_{\rm eh}(r,v,t) \;\; \text{and} \;\; f_{\rm el}(r,v,t) \,. \tag{34}$$

The normalized equilibrium velocity distribution functions of electrons are the Maxwellians

$$f_{\rm eh}^0(v^2) = \left(\frac{m}{2\pi K_B T_{\rm eh}}\right)^{1/2} \exp\left(\frac{mv^2}{2K_B T_{\rm eh}}\right), \text{ etc.} \tag{35}$$

The perturbed state of distribution functions for electrons are expressed as

$$f_{\rm eh}(r,v,t) = N_{\rm eh} f_{\rm eh}^0(v^2) + f_{\rm eh}(z,v,t) \,, \tag{36}$$

etc., where

$$|f_{\rm eh}(r,v,t)| >> |f_{\rm eh}(z,v,t)|, \text{ etc.} \tag{37}$$

The number densities in the equilibrium state and the per particle charge of the four species are $N_{\rm eh}, N_{\rm el}, N_{\rm ih}, N_{\rm il}, q_{\rm eh}, q_{\rm el}, q_{\rm ih}, q_{\rm il}$.

The infinitesimally small perturbations in the distribution functions are

$$f_{\rm eh}(z,v,t),\;\; f_{\rm el}(z,v,t),\;\; f_{\rm ih}(z,v,t) \;\;\text{and}\;\; f_{\rm il}(z,v,t)\,. \tag{38}$$

In the linearized approximation, the Boltzmann-Vlasov equations for the four species of plasma are: Eq. (12) for ions of argon plasma and Eq. (13) for the ions of helium plasma. For the electrons of the above plasma, Eqs. (14) and (15) will be replaced by

$$\frac{\partial f_{\rm eh}}{\partial t} + v\frac{\partial}{\partial z}f_{\rm eh} + N_{\rm eh}\left(\frac{q_{\rm eh}}{m}E + \omega_{\rm oh}^2\xi_{\rm h}\right)\frac{\partial f_{\rm eh}^0}{\partial v} = 0\,, \tag{39}$$

(for electrons of argon plasma)

and

$$\frac{\partial f_{\rm el}}{\partial t} + v\frac{\partial}{\partial z}f_{\rm el} + N_{\rm el}\left(\frac{q_{\rm el}}{m}E + \omega_{\rm ol}^2\xi_{\rm l}\right)\frac{\partial f_{\rm el}^0}{\partial v} = 0\,, \tag{40}$$

(for electrons of helium plasma)

Moreover, Gauss's divergence law gives

$$\frac{\partial E}{\partial z} = 4\pi\int(q_{\rm ih}f_{\rm ih} + q_{\rm il}f_{\rm il} + q_{\rm eh}f_{\rm eh} + q_{\rm el}f_{\rm el}){\rm d}v\,. \tag{41}$$

The time derivatives of $\xi_{\rm l}$ and $\xi_{\rm h}$ are given by the field induced average velocities $v_{\rm l}$ and $v_{\rm h}$

$$\dot{\xi}_{\rm l} = \frac{1}{N_{\rm el}}\int\limits_v vf_{\rm el}{\rm d}v,\;\; \dot{\xi}_{\rm h} = \frac{1}{N_{\rm eh}}\int\limits_v vf_{\rm eh}{\rm d}v\,. \tag{42}$$

Using Eqs. (2) and (42) in Eq. (39), we obtain

$$f_{\rm eh}(z,v,t) = -{\rm i}N_{\rm eh}\left(\frac{q_{\rm eh}}{m}E + \frac{{\rm i}\omega_{\rm eh}^2}{\omega N_{\rm eh}}\int\limits_v vf_{\rm eh}{\rm d}v\right)\frac{f_{\rm eh}^{01}(v^2)}{kv-\omega}\,, \tag{43}$$

where $f_{\rm eh}^{01} = {\rm d}f_{\rm eh}^0/{\rm d}v$, etc.

Hence, the integral $\int\limits_v vf_{\rm eh}{\rm d}v$ is given by

$$\int\limits_v vf_{\rm eh}{\rm d}v = \frac{-{\rm i}N_{\rm eh}\dfrac{q_{\rm eh}}{m}\dfrac{\omega}{k}E\displaystyle\int\limits_v\frac{f_{\rm eh}^{01}(v^2){\rm d}v}{kv-\omega}}{1-\dfrac{\omega_{\rm oh}^2}{k}\displaystyle\int\limits_v\frac{f_{\rm eh}^{01}{\rm d}v}{kv-\omega}}\,. \tag{44}$$

Using this integral in Eq. (43) gives

$$f_{\rm eh}(z,v,t) = -\frac{\dfrac{{\rm i}q_{\rm eh}}{m} N_{\rm eh} E \dfrac{f^{01}_{\rm eh}(v^2)}{kv-\omega}}{1 - \dfrac{\omega^2_{\rm oh}}{k} \displaystyle\int_v \dfrac{f^{01}_{\rm eh}(v^2){\rm d}v}{kv-\omega}} \,. \tag{45}$$

Similarly, proceeding for $f_{\rm el}$ we obtain

$$f_{\rm el}(z,v,t) = -\frac{\dfrac{{\rm i}q_{\rm el}}{m} N_{\rm el} E \dfrac{f^{01}_{\rm el}(v^2)}{kv-\omega}}{1 - \dfrac{\omega^2_{\rm ol}}{k} \displaystyle\int_v \dfrac{f^{01}_{\rm el}(v^2){\rm d}v}{kv-\omega}} \,. \tag{46}$$

Eqs. (45) and (46) represent $f_{\rm ih}$ and $f_{\rm il}$, respectively.

Then Eq. (41) becomes the wave evolution equation

$$k = \omega^2_{\rm ih} \int \frac{f^{01}_{\rm ih}(v^2){\rm d}v}{(kv-\omega)} + \omega^2_{\rm il} \int \frac{f^{01}_{\rm il}(v^2){\rm d}v}{(kv-\omega)}$$

$$+ \frac{\omega^2_{\rm eh} \displaystyle\int \dfrac{f^{01}_{\rm eh}(v^2){\rm d}v}{kv-\omega}}{1 - \dfrac{\omega^2_{\rm oh}}{k} \displaystyle\int_v \dfrac{f^{01}_{\rm eh}(v^2){\rm d}v}{kv-\omega}} + \frac{\omega^2_{\rm el} \displaystyle\int \dfrac{f^{01}_{\rm el}(v^2){\rm d}v}{kv-\omega}}{1 - \dfrac{\omega^2_{\rm ol}}{k} \displaystyle\int_v \dfrac{f^{01}_{\rm el}(v^2){\rm d}v}{kv-\omega}} \,. \tag{47}$$

Carrying out the contour integral over the singularity, we obtain

$$k = \omega^2_{\rm ih} P \int \frac{f^{01}_{\rm ih}(v^2){\rm d}v}{(kv-\omega)} \pm \frac{\omega^2_{\rm ih}}{k} \pi {\rm i} f^{01}_{\rm ih}(v_p^2) + \omega^2_{\rm il} P \int \frac{f^{01}_{\rm il}(v^2){\rm d}v}{(kv-\omega)}$$

$$\pm \frac{\omega^2_{\rm il}}{k} \pi {\rm i} f^{01}_{\rm il}(v_p^2) + \frac{\omega^2_{\rm eh} P \displaystyle\int_v \dfrac{f^{01}_{\rm eh}(v^2){\rm d}v}{kv-\omega} \pm \pi {\rm i} \dfrac{\omega^2_{\rm eh}}{k} f^{01}_{\rm eh}(v_p^2)}{1 - \dfrac{\omega^2_{\rm oh}}{k} P \displaystyle\int_v \dfrac{f^{01}_{\rm eh}(v^2){\rm d}v}{kv-\omega} \pm \pi {\rm i} \dfrac{\omega^2_{\rm eh}}{k} f^{01}_{\rm eh}(v_p^2)}$$

$$+ \frac{\omega^2_{\rm el} P \displaystyle\int_v \dfrac{f^{01}_{\rm el}(v^2){\rm d}v}{kv-\omega} \pm \pi {\rm i} \dfrac{\omega^2_{\rm el}}{k} f^{01}_{\rm el}(v_p^2)}{1 - \dfrac{\omega^2_{\rm ol}}{k} P \displaystyle\int_v \dfrac{f^{01}_{\rm el}(v^2){\rm d}v}{kv-\omega} \pm \pi {\rm i} \dfrac{\omega^2_{\rm el}}{k} f^{01}_{\rm el}(v_p^2)} \,. \tag{48}$$

If the wave is introduced in the plasma of the heavy inert gas at the wave phase velocity v_p greater than its ion thermal velocity C_i, the Landau damping of ion acoustic waves is not possible. So, a trace of the dope plasma of a light inert gas is introduced, the ion thermal velocity of which is more than the thermal velocity of the ions of the heavy gas. The dope ions are Landau damped in this mixture of two plasmas. For electrons of argon and helium $C_{\rm el} > v_p, C_{\rm il} > v_p$, so physically, $v > v_p$, and $v \neq 0$ is not physically possible. For ions of the heavy gas $v < v_p$, the residue physically vanishes and the factor $(kv-\omega)^{-1}$ is expanded in positive integral powers of kv/ω. For ions of the light gas, since $v \leq \omega/k$, in the singular integro-differential equation, the principal value of the integral of $f_{\rm il}(z,v,t)$ containing $(kv-\omega)^{-1}$ is determined after expanding this factor in positive integral powers of kv/ω. Also the residue of this integral is evaluated at the wave phase velocity v_p. Then Eq. (48) becomes

$$k = -\frac{\omega_{\rm ih}^2}{\omega}\left(-\frac{k}{\omega} - \frac{3k^2C_{\rm ih}^2}{\omega^3}\right) - \frac{\omega_{\rm il}^2}{\omega}\left(-\frac{k}{\omega} - \frac{3k^2C_{\rm il}^2}{\omega^3}\right) \pm \pi {\rm i}\frac{\omega_{\rm il}^2}{k} f_{\rm il}^{01}(v_p^2)$$

$$+\frac{\dfrac{\omega_{\rm eh}^2}{k}\displaystyle\int\limits_{v\neq 0} \frac{1}{v}(1+\frac{\omega}{kv}+\frac{\omega^2}{k^2v^2})f_{\rm eh}^{01}(v^2){\rm d}v}{1-\dfrac{\omega_{\rm oh}^2}{k^2}\displaystyle\int\limits_{v\neq 0} \frac{1}{v}(1+\frac{\omega}{kv}+\frac{\omega^2}{k^2v^2})f_{\rm eh}^{01}(v^2){\rm d}v}$$

$$+\frac{\dfrac{\omega_{\rm el}^2}{k}\displaystyle\int\limits_{v\neq 0} \frac{1}{v}(1+\frac{\omega}{kv}+\frac{\omega^2}{k^2v^2})f_{\rm el}^{01}(v^2){\rm d}v}{1-\dfrac{\omega_{\rm ol}^2}{k^2}\displaystyle\int\limits_{v\neq 0} \frac{1}{v}(1+\frac{\omega}{kv}+\frac{\omega^2}{k^2v^2})f_{\rm el}^{01}(v^2){\rm d}v}\,. \tag{49}$$

So, the damping rate for the ion-acoustic wave in a dope plasma when the distribution functions of both the electrons and ions are represented by Boltzmann-Vlasov equations, become

$$\frac{\gamma}{\omega} = \frac{Q}{\omega[X+L-R]}\,, \tag{50}$$

where

$$L = \frac{2\omega\omega_{\rm eh}^2\left(\dfrac{1}{C_{\rm eh}^2}+\dfrac{2\omega^2}{C_{e_1h}^2}\right)\left(1+\dfrac{1}{C_{\rm eh}^2}\right)}{k^2C_{e_1h}^2\left(1+\dfrac{\omega^2}{C_{\rm eh}^2}+\dfrac{\omega^4}{C_{e_1h}^2}\right)} + \frac{2\omega_{\rm eh}^2}{k^2C_{e_1h}^2\left(1+\dfrac{\omega^2}{C_{\rm eh}^2}+\dfrac{\omega^4}{C_{e_1h}^2}\right)}$$

$$R = \frac{2\omega_{\rm eh}^2}{\omega^3\left(1+\dfrac{\omega_{\rm ol}^2}{k^2C_{e_1l}^2}\right)\left[1+\dfrac{\omega^2\omega_{\rm ol}^2}{k^4C_{e_1l}^4\left(1+\dfrac{\omega_{\rm ol}^2}{k^2C_{e_1l}^2}\right)}\right]}$$

$$+\frac{2\omega\omega_{\rm ol}^2\omega_{\rm el}^2/\left(1+\dfrac{\omega_{\rm ol}^2}{k^2C_{e_1l}^2}\right)}{k^4C_{e_1l}^4\left(1+\dfrac{\omega_{\rm ol}^2}{k^2C_{e_1l}^2}\right)\left[1+\dfrac{\omega^2\omega_{\rm ol}^2}{k^4C_{e_1l}^4\left(1+\dfrac{\omega_{\rm ol}^2}{k^2C_{e_1l}^2}\right)}\right]\left\{1+\dfrac{\omega^2\omega_{\rm ol}^2}{k^4C_{e_1l}^4\left(1+\dfrac{\omega_{\rm ol}^2}{k^2C_{e_1l}^2}\right)}\right\}}$$

and

$$\frac{1}{C_{c_1l}^2} = \int_{-\infty}^{-v_p} + \int_{-v_p}^{\infty} \frac{1}{v^2} f_{\rm el}^{01}(v^2){\rm d}v,$$

$$\frac{1}{C_{c_1h}^2} = \int_{-\infty}^{-v_p} + \int_{-v_p}^{\infty} \frac{1}{v^2} f_{\rm eh}^{01}(v^2){\rm d}v\,.$$

Since $\omega_e^2/C_e^2 = \omega_i^2/C_s^2$, where $C_s^2 = K_{\rm B}T_e/M$, and M is the nuclear mass, Eq. (50) becomes

$$\frac{\gamma}{\omega} = \sqrt{\frac{\pi}{8}}\frac{\omega_{\rm il}^2\omega^2 v_p}{k^2C_{\rm il}^3}\frac{\exp\left(-\dfrac{v_p^2}{2C_{\rm il}^2}\right)}{(\omega_{\rm ih}^2+\omega_{\rm il}^2)}\,. \tag{51}$$

For finding the Landau damping at the distance d, we obtain

$$\frac{6k}{\omega^4 d}(C_{\rm ih}^2\omega_{\rm ih}^2 + C_{\rm il}^2\omega_{\rm il}^2) \pm \sqrt{\frac{\pi}{2}}\frac{\omega_{\rm il}^2 v_p}{k^2C_{\rm il}^3}\exp\left(-\frac{v_p^2}{2C_{\rm il}^2}\right)$$

$$= \frac{2kC_{e_1h}^2\omega_{\rm ih}^2C_{\rm eh}^2}{C_{sh}^2 d(k^4C_{\rm eh}^2C_{e_1h}^2 + k^2\omega_{\rm oh}^2C_{e_1h}^2 + \omega^2\omega_{\rm oh}^2)}$$

$$+\frac{2kC_{e_1l}^2\omega_{\rm il}^2C_{s_1}^2}{C_{s1}^2 d(k^4C_{\rm el}^2C_{e_1l}^2 + k^2\omega_{\rm el}^2C_{e_1l}^2 + \omega^2\omega_{\rm ol}^2)}$$

$$-\frac{\omega_{\rm ih}^2C_{\rm eh}^2\omega^2(4k^3C_{e_1h}^2 + 2k\omega_{\rm oh}^2)}{C_{sh}^2(A^2+B^2)d}$$

$$-\frac{\omega_{\rm il}^2C_{\rm el}^2\omega^2(4k^3C_{sh}^2 + 2k\omega_{\rm oh}^2)}{C_{sl}^2(C^2+D^2)d}\,, \tag{52}$$

$$A = k^4 C_{\mathrm{eh}}^2 C_{e_1 h}^2 + k^2 \omega_{\mathrm{oh}}^2 C_{e_1 h}^2 + \omega^2 \omega_{\mathrm{oh}}^2,$$

$$B = \left(\frac{2k}{d}\right) \left\{ 2k^2 C_{\mathrm{eh}}^2 C_{e_1 h}^2 + \omega_{\mathrm{oh}}^2 C_{e_1 h}^2 \right\},$$

$$C = k^4 C_{\mathrm{el}}^2 C_{e_1 l}^2 + k^2 \omega^2 C_{e_1 l}^2 + \omega^2 \omega_{\mathrm{ol}}^2,$$

$$D = \left(\frac{2k}{d}\right) \left\{ 2k^2 C_{\mathrm{el}}^2 C_{e_1 l}^2 + \omega_{\mathrm{ol}}^2 C_{e_1 l}^2 \right\} .$$

3. Results and discussion

From the expressions of Eqs. (29) and (51) we see that the growth rates for the Landau damping of ion acoustic waves are different when we consider the distribution function of both electrons and ions, and also when electron are in fluid motion and ion motion are represented by distribution function. To find the growth rates we consider a model plasma for which the plasma parameters are: $\omega = 10^6,\ \omega_{\mathrm{il}} = 10^7,\ \omega_{\mathrm{ih}} = 10^8,\ v_p = 10^9,\ \omega_{\mathrm{eh}} = 10^8,\ w_{\mathrm{el}} = 10^7,\ k = 10^{-3},\ \omega_{0h} = 10^3,\ \omega_{0l} = 10^3$ we calculate the damping rate (γ/ω) for different models plasma. Fig. 1 shows the variation of $\log(\gamma/\omega)$ with C_{il}/v_p for different values of collision. Fig. 1 shows the

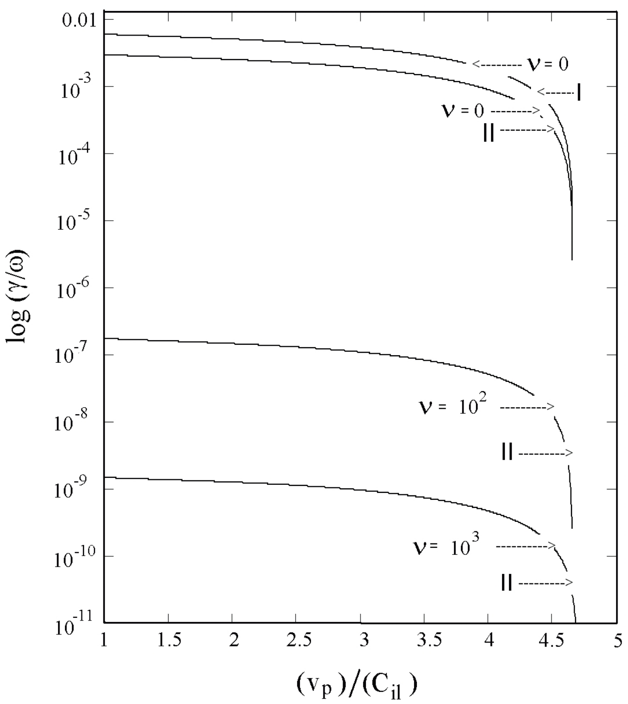

Fig. 1. Plot of γ/ω (I) for the kinetic model and (II) for the fluid model.

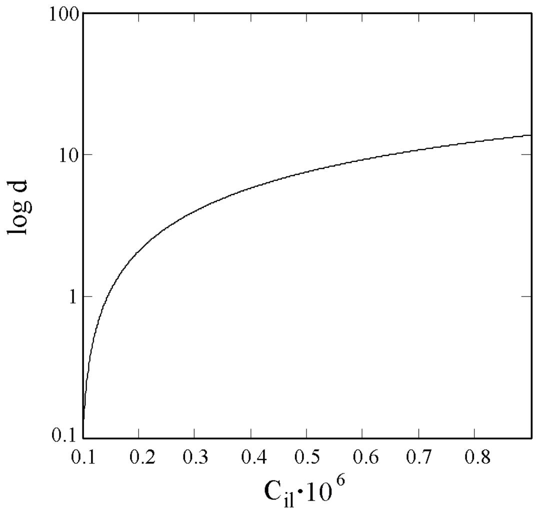

Fig. 2. Variation of damping length with thermal velocity $\langle C_{\rm il} \rangle$ *of helium ions.*

damping rate for kinetic theory models is greater than the fluid models. This is because kinetic theory provides more information than the fluid models. Fig. 1 also shows that damping rate is very low for fluid models plasma if we consider the collisional effects in the plasma. In Fig. 2, we observed that the damping length increases with $C_{\rm il}$, i.e. the ion-thermal velocity of light nuclei when fluid model is considered for the motion of electrons.

4. *Concluding remarks*

A theory has been developed for the study of Landau damping of the ion-acoustic waves in a dope plasma in a light inert gas (helium) in the presence of plasma of a heavy inert gas (argon) considering (i) the motion of electrons and ions are represented by fluid motion and distribution function respectively, (ii) both the motion of the electrons and ions are represented by distribution function. Damping rates of the ion-acoustic waves in dope plasma are different for the two models which may be experimentally investigated by imposing the basic conditions. This theory can be further generalized for the study of the Landau damping of various types of longitudinal waves (e.g. electro-acoustic wave) in multicomponent Vlasov plasma considering the collisional effects.

References

[1] V. N. Tsytovich, *Nonlinear Effects in Plasmas*, Plenum, New York (1970).

[2] P. K. Kaw and J. M. Dawson, Phys. Fluids, **13** (1970) 472.

[3] J. A. Stamper, Laser and Particle Beams **9** (1991) 841.

[4] B. Chakraborty, S. N. Paul, M. Khan and B. Bhattacharya, Phys. Reports **141** (1984) 181.

[5] K. E. Lonngren, Plasma Phys. **25** (1983) 943.

[6] F. Veerhest, Space Sci. Rev. **77** (1996) 267.

[7] S. K. Bhattacharya, S. N. Paul and K. K. Mondal, Phys. Plasmas **9** (11) (2002) 4439.

[8] K. K. Mondal, S. N. Paul and S. K. Bhattachaya, Fizika A (Zagreb) **13** (2004) 77.

[9] N. Bloembergen, Proc. IEEE **51** (1963) 124.

[10] B. Chakraborty, M. Khan and B. Bhattacharya, Phys.Rev. A **23** (1981) 345.

[11] C. Das, B. Bera, B. Chakraborty and M. Khan, J. Plasma Phys. **51** (1993) 191.

[12] S. Mondal, S. Sarkar, A. M. Basu, M. Khan, R. Bhattacharyya and B. Chakraborty, Phys. Plasmas **8** (3) (2001) 713.

[13] B. Chakraborty, S. K. Bhattacharya and S. N. Paul, Fizika A (Zagreb) **11** (2002) 121.

[14] B. Chakraborty, S. K. Bhattacharya and S. N. Paul, Ind. J. Phys. B **77** (3) (2003) 327.

[15] S. N. Paul, S. K. Bhattacharya, S. Mondal and B. Chakraborty, Fizika A (Zagreb) **13** (2004) 161.

[16] L. D. Landau, J. Phys. (USSR) **10** (1946) 25.

[17] J. D. Jackson, J. Nucl. Energy, Part-C: Plasma Phys. **1** (1960) 171.

[18] J. Dawson, Phys. Fluids **4** (1961) 809.

[19] A. Y. Wong, R. W. Motley and N. D'Anjelo, Phys. Rev. A **133** (1964) 436.

[20] F. Ott and R. N. Sudan, Phys. Fluids **12** (1967) 2388.

[21] F. Ott and R. N. Sudan, Phys. Fluids **13** (1970) 1432.

[22] A. Roychowdhury, G. Pakira and S. N. Paul, Physica C (Netherlands) **151** (1988) 518.

[23] J. Mukherjee and A. Roychowdhury, J. Plasma Phys. **52** (1994) 55.

[24] I. Alexeff, W. D. Jones and D. Montgomery, Phys. Rev. Lett. **19** (1967) 422.

PROUČAVANJE LANDAUOVOG PRIGUŠENJA IONSKO-ZVUČNIH VALOVA U PUNJENOJ PLAZMI

Proučavamo teorijski Landauovo prigušenje ionsko-zvučnih valova u plazmi teških inertnih iona, npr. argonskoj, punjenoj lakim inertnim ionima, npr. helijevim, razmatrajući fluidni i kinetički model. Prigušenje ionsko-zvučnih valova u punjenoj plazmi je različito za ta dva modela, što se u nekim uvjetima može eksperimentalno proučavati. Razmatraju se i drugi slučajevi Landauovog prigušenja u Vlasovoj plazmi. Ovim se radom želi potaknuti eksperimentalno istraživanje Landauovog prigušenja s isključenjem utjecaja drugih sudarnih prigušenja i sudarnih prigušenja.