Printed ISSN 1330–0008
Online ISSN 1333–9125
CD ISSN 1333–8390
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THE PHYSICAL PROBLEM AND THE MODELLING OF THE SHEATH IN COLLISIONAL DUSTY PLASMA

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Received 27 June 2006; Revised manuscript received 7 April 2007
Accepted 4 July 2007 Online 24 September 2007

The aim of this paper is to study the behaviour of sheath structure in plasma with collisions, and simulation of the effects of collisionality on the plasma sheath using the Runge–Kutta routine. We consider the near-wall region of an unmagnetized dusty plasma which consists of electrons, ions, micron-size dust particles and neutral particles. Since the dust particles are much heavier than electrons and ions, the latter are assumed to be out of thermal equilibrium with dust as a cold fluid. The neutrals are taken as immobile. Precise numerical solutions of the model are used to determine the collisional dependence of the sheath width and the impact energy at the wall.

PACS numbers: 52.40.Kh, 52.27.Lw, 52.27.Cm UDC 533.95, 533.93

Keywords: sheath, dusty plasma, sheath width, collisional parameter, impact energy

1. *Introduction*

For many decades, the behaviour of plasma in contact with a negatively biased surface has been qualitatively studied. It is known that the strong localized electric field appears between the plasma and such a surface. The ion-rich boundary layer, called the sheath, confines electrons in and expels ions from the plasma [1].

The sheath is composed of ions, atoms, electrons and dust particles. Sheath formation at the plasma–boundary interface, which is separating the quasi-neutral plasma, is ubiquitous in bounded plasma. Accurate sheath modelling is of considerable interest to the effective design of ionized flow in wide-ranging applications in plasma processing: in the ion cyclotron heating, in electric propulsion devices and in controlled thermonuclear fusion. Recently, the study of dusty plasmas represents one of the most rapidly growing branches of plasma physics research. The relevance of dusty plasma to both the space plasma and industrial plasma communities has led to the rapid growth of a variety of laboratory experiments.

Dust grains in plasma are not neutral, they are charged due to their interaction with electrons, ions and background radiation [2]. The mass of dust grains can have very high value, up to $10^6 - 10^8$ times the proton mass. The presence of the charged dust grains alters many physical properties of the plasma, including its charge distribution and potential distribution. Additionally, dusty plasmas facilitate the investigation of many unique phenomena including the formation of dust crystals and new collective modes. The proprieties of dusty plasma have been the subject of increasing interest [3 − 6]. The basic mechanism by which dust grain interacts with plasma is electrical charge accumulation on the surface of an insulating particle. In the space environment, this charge accumulates via photoionization, secondary electron emission due to impacts with energetic particles and collisions with the background thermal plasma. For the laboratory experiments on dusty plasmas, the principle charging mechanism is the flux of charged particles (electrons, ions) from the plasma to dust particles residing on a plasma exposed surface. Due to the high mobility of electrons, the grains become negatively charged.

Once the surface charge of the dust grain is large enough, the electric force, due to the sheath electric field, can exceed the combination of gravitational and adhesive forces that bind the grain to the surface. Under these conditions, the dust particle is accelerated though the sheath and passes into the main plasma. In laboratory conditions, it is the balance between the gravitational and electrical forces, and various neutral-particle and ion-drag forces that control the transport of individual dust particles in the plasma.

When suspended in the plasma, these charged dust particles interact with ion and electrons in the plasma. In this way, the dust particles modify many of the physical properties of the plasma, including its charge distribution and potential distribution.

The presence of dust grains in a radio-frequency plasma reactor for engraving in micro-electronic devices or for the deposition of a thin layer may be critical. It is thus of practical interest to investigate the interaction between a dusty plasma and solid boundary.

Recently, several authors have considered the effects of collisionality on the sheath [7 − 8]. A. Samarian and S. V. Vladimirov report on the experiments dedicated to clarify the dependence of the dust charge as a function of its size in a rf-discharge plasma for the ion-neutral friction with the ion drift velocity dependence fitting the existing experimental data on the ion mobility in a low temperature plasma [9]. Tsytovich, Vladimirov, Morfil and Goree developed the theory of dust voids in collisional dominated plasma for the one-dimensional case [10]. However, we develop the model for the ion-neutral and dust-neutral collisions in order to determine the sheath thickness and the grain impact energy at the wall.

In this paper, we consider an unmagnetized, dusty plasma which is in contact with a physical boundary like a wall, probe or substract. It is well known that a sheath is created to separate the plasma from the surface.

The second section provides the mathematical formulation, the basic equations that are used to describe the sheath in dusty plasma when both ions and dust grains are out thermal equilibrium. Precise numerical solutions for the sheath thickness, the average dusts and ion energy at wall impact as function of the collisional parameter are presented.

In the last section we comment the results obtained from the couplings equations.

2. Sheath model of a collisional dusty plasma

2.1. Governing equations

The aim of this section is to study the effect of collisionality in the dusty plasma. We consider an unmagnetized, charge–neutral plasma in contact with a planar wall. The potential in the sheath is ϕ and the wall is held at a negative potential ϕ_ω. Consequently, a sheath is formed to separate the plasma from the wall. Ions enter the sheath as a cold beam with a velocity v_0 and strike the wall with a velocity v_ω. In order to estimate the sheath characteristics, we adopt a one-dimensional model of the sheath. The sheath surface is represented by the Oyz plane, and the Ox axis is perpendicular to the surface. We assume that all physical variables (density, velocity, potential, . . .) depend only on the x coordinate and we suppose a steady process (i.e., the variables do not depend on time). The boundary between the plasma and the sheath is at $x = 0$. The sheath thickness is D. That means the wall is at $x = D$. The sheath is assumed to be source-free.

It is well known that dust grains are common both in space and laboratory plasma [3 – 5]. A dusty plasma consists of neutral gas, ions, electrons and micron-size particles that have a negative charge [6]. Since the dust particles are much heavier than electrons and ions, we suppose them as a cold fluid. The neutrals are taken as immobile. Since we assumed the fluid theory for dusts, we can ignore the variations in shape, size and charge separations among the individual dust particles [7]. This is so because these variations are sufficiently small and they can not be distinguished in a fluid element.

The collisions of charged particles in plasma are of two types: collisions with other charged particles and collisions with neutral atoms and molecules. For plasma physicists, the collisions with neutral particles are of practical interest, because they are dominant in low-temperature plasmas where the degree of ionization is only a few per cent [16]. The opposite case, when the degree of ionization is high, is called fully ionized plasma. In that case, the collisions with other charged particles tend to dominate over collisions with neutral particles. We shall assume a partially ionized gas. The plasma-neutral collision usually determines the kinetics of motion [17].

The electric and ion drag forces scale differently with particle size. The electric force will dominate for small particles, while ion drag will dominate for larger particles [18]. In this work, we shall assume that the dust diameter less than 120 nm, so the electric force exceeds the ion drag force.

The dynamics of electrons is treated as a neutral background in plasma and they are assumed to follow the Boltzmann relation [19, 20]

$$n_{\mathrm{e}} = n_0 \exp\left\{ \frac{e\phi}{k_{\mathrm{B}} T_{\mathrm{e}}} \right\}, \tag{1}$$

where e is the elementary charge, k_{B} the Boltzmann constant and T_{e} the electron temperature. The cold dust fluid obeys the source-free, steady-state continuity equation

$$\nabla \cdot \big(n_{\mathrm{d}} v_{\mathrm{d}}\big) = 0\,, \tag{2}$$

and the momentum transfer equation

$$m_{\mathrm{d}} \big(v_{\mathrm{d}} \nabla\big) v_{\mathrm{d}} = Z_{\mathrm{d}} e \nabla\phi - F_{\mathrm{cd}}\,, \tag{3}$$

where m_{d}, n_{d}, v_{d} and Z_{d} are, respectively, the dust particle mass, density, velocity and charge number and n_{d0}, v_{d0} are dust density and velocity at the sheath edge, respectively. The temperature ratio is $\tau \equiv T_{\mathrm{i}}/T_{\mathrm{e}}$ and the negative dust charge is $-eZ_{\mathrm{d}}$, where $e > 0$ and $Z_{\mathrm{d}} > 0$. The dimensionless dust charge [12] is

$$z \equiv \frac{Z_{\mathrm{d}} e^2}{a T_{\mathrm{e}}}.$$

The dimensionless charge z is equivalent to the grain's floating potential $-Z_{\mathrm{d}} e/a$ normalized by T_{e}. Its value must be computed from a charging equation [12].

The dust particle's charge z or "floating potential" is determined in the steady state by a balance of the electron and ion currents collected by the particle. The charging equation for a dust particle is (see Ref. [12])

$$\exp\{-z\} = \frac{n}{n_{\mathrm{e}}} \sqrt{\frac{\pi m_{\mathrm{e}}}{m_{\mathrm{i}}}} \frac{2z}{\tau} \alpha_{\mathrm{ch}},$$

where the left-hand side comes from the electron current (which is suppressed exponentially by electrostatic repulsion), and the right-hand side corresponds to the ion current. The ion charging coefficient α_{ch} depends generally on the ion temperature and drift velocity. In the limit $u_i \gg 1$ that we used for the collection force (see Ref. [12])

$$\alpha_{\mathrm{ch}} = \frac{1}{\sqrt{\tau} u_i} \left(1 + \frac{\tau u_i^2}{z} \right),$$

so that the ion velocity u_i will be taken into account in the charging equation.

The collisional effects between the dust and the neutrals are introduced. We use the collisional force term F_{cd}, which is given by [13]

$$F_{\mathrm{cd}} = m_{\mathrm{d}} n_{\mathrm{n}} v_{\mathrm{d}}^2 \sigma = m_{\mathrm{d}} n_{\mathrm{n}} v_{\mathrm{d}}^2 \sigma_s \left(\frac{v_{\mathrm{d}}}{C_{\mathrm{d}}}\right)^{\gamma}. \tag{4}$$

Here, n_{n} is the neutral gas density and σ is the momentum transfer cross section for collisions between the dust's charged grains and neutrals. $C_{\mathrm{d}} = \sqrt{(k_{\mathrm{B}} T_{\mathrm{e}}/m_{\mathrm{d}})}$ is the dust acoustic speed and γ is dimensionless parameter ranging from 0 to -1. The rate of ion production in dusty plasma is determined by the ionization frequency.

The rate of ionization [9] is given as $S_{\mathrm{i}} = k_{\mathrm{i}} n_{\mathrm{n}} n_{\mathrm{e}}$. Here, k_{i} is the ionization coefficient. We shall use the following generalized process rate that is a sum of all three ionization rates [9]

$$k_{\mathrm{i}} = (-3.2087 \times 10^{-5}\, T_{\mathrm{e}}^3 - 0.0022\, T_{\mathrm{e}}^2 + 0.7101\, T_{\mathrm{e}} - 1.76) \times 10^{-14}.$$

In the high-density, low-pressure plasma, the concentration of molecular ions is low for dissociative recombination to be important. Furthermore, since such a plasma is optically thick and the radiative escape factor is zero. Then the only important recombination mechanism is the three-body recombination. Thus, the probability of recombination [9] is $S_{\mathrm{r}} = k_{\mathrm{r}} n_{\mathrm{i}} n_{\mathrm{e}}$. The recombination coefficient k_{r} can be approximated [9] by

$$k_{\mathrm{r}} = 1.09 \times 10^{-20}\, n_{\mathrm{e}} T_{\mathrm{e}}^{-9/2}\ \mathrm{m^3 s^{-1}}.$$

The ions can be described by as a cold fluid responding to the plasma potential according to the continuity equation

$$\nabla \cdot (n_{\mathrm{i}} v_{\mathrm{i}}) = S_{\mathrm{i}} - S_{\mathrm{r}}, \tag{5}$$

and momentum transfer equation is

$$m_{\mathrm{i}}\,(v_{\mathrm{i}} \nabla)\, v_{\mathrm{i}} = -e \nabla \phi - F_{\mathrm{ci}}. \tag{6}$$

Here, m_{i}, n_{i}, v_{i} are, respectively, the ion particle mass, density and velocity, and n_{i0}, v_{i0} are the ion density and velocity at the sheath edge, respectively. As the ion fluid travels through the sheath, the drag force F_{ci} is given by

$$F_{\mathrm{ci}} = m_{\mathrm{i}} n_{\mathrm{n}} v_{\mathrm{i}}^2 \sigma = m_{\mathrm{i}} n_{\mathrm{n}} v_{\mathrm{i}}^2 \sigma_s \left(\frac{v_{\mathrm{i}}}{C_{\mathrm{s}}}\right)^{\gamma}, \tag{7}$$

where $C_{\mathrm{s}} = \sqrt{(k_{\mathrm{B}} T_{\mathrm{e}}/m_{\mathrm{i}})}$ is the acoustic speed. The speed of ions is assumed to be great enough to satisfy Bohm's criteria ($v_{\mathrm{i}} \geq C_{\mathrm{s}}$).

The electron and ion densities are then included in the Poisson equation [14]

$$\nabla^2 \phi = -4\pi e \big(n_{\mathrm{i}} - n_{\mathrm{e}} - Z_{\mathrm{d}} n_{\mathrm{d}}\big)\,. \tag{8}$$

For strong dust-neutral and ion-neutral collisions, the movements of dust grains are mobility-limited. Therefore, we are interested only about the constant dust mobility and ion mobility case (i.e., $\gamma = -1$). Hence, we do not consider the case of the constant dust mean free path (i.e., $\gamma = 0$).

Combining Eqs. (1) to (8), we find four coupled differential equations describing the sheath structure as

$$v_{\rm i}\frac{\rm d}{\rm dx}v_{\rm i} = -\frac{e}{m_{\rm i}}\frac{\rm d}{\rm dx}\phi - n_{\rm n}\sigma_{\rm s}\frac{v_{\rm i}^{2+\gamma}}{C_{\rm s}^{\gamma}}, \tag{9}$$

$$\nabla\cdot\big(n_{\rm i}v_{\rm i}\big) = k_{\rm i}n_{\rm n}n_{\rm e} - k_{\rm r}n_{\rm i}n_{\rm e}, \tag{10}$$

$$v_{\rm d}\frac{\rm d}{\rm dx}v_{\rm d} = \frac{Z_{\rm d}e}{m_{\rm d}}\frac{\rm d}{\rm dx}\phi - n_{\rm n}\sigma_{\rm s}\frac{v_{\rm d}^{2+\gamma}}{C_{\rm d}^{\gamma}}, \tag{11}$$

$$\nabla^2\phi = -4\pi e\big(n_{\rm i} - n_{\rm e} - Z_{\rm d}n_{\rm d}\big). \tag{12}$$

The governing equations can be transformed to the dimensionless form by an appropriate choice of variables [15]. The electric potential ϕ is scaled by the electron temperature, $\eta = -e\phi/(k_{\rm B}T_{\rm e})$ and the distance x is scaled by the Debye length, $\xi \equiv x/\lambda_{\rm D}$, where $\lambda_{\rm D} = \sqrt{\varepsilon_0 k_{\rm B}T_{\rm e}/(n_0e^2)}$. The ion velocity $v_{\rm i}$ is scaled by the ion acoustic speed $u_{\rm i} \equiv v_{\rm i}/C_{\rm s}$. Additionally, the dust velocity $v_{\rm d}$ is transformed to the dimensionless parameter by the dust acoustic speed, $u_{\rm d} \equiv v_{\rm d}/C_{\rm d}$. Also the parameters $Z_{\rm d}(n_{\rm d0}/n_{\rm e0}) = \delta - 1$ and $d = D/\lambda_{\rm D}$, where d is the dimensionless sheath thickness. The degree of collisionality in the sheath is parameterized by $\alpha = \lambda_{\rm D}/\lambda_{\rm mfp} = \lambda_{\rm D}n_{\rm n}\sigma_{\rm s}$, where $\lambda_{\rm mfp} = 1/(n_{\rm n}\sigma_{\rm S})$ is the mean free path of dust, and α is proportional to the neutral gas density $n_{\rm n}$. The collisionless case ($\alpha = 0$) is the limit of zero gas density. If the gas density is high enough, or the Debye length short enough, the ion mean free path is one Debye length, and then $\alpha = 1$. The average number of collisions in the sheath, which will prove to be a useful quantity, is given by $D/\lambda_{\rm mfp} = \alpha d$.

Based on these non-dimensional parameters, the basic equations reduce to

$$u_{\rm i}u_{\rm i}' = \eta' - \alpha u_{\rm i}^{2+\gamma}, \tag{13}$$

$$\frac{{\rm d}n_{\rm i}}{{\rm d}\varepsilon} = -\frac{\eta' n_{\rm i}}{u_{\rm i}^2} + \alpha n_{\rm i}u_{\rm i}^{\gamma} + \frac{\lambda_{\rm D}}{c_{\rm S}u_{\rm i}}\,k_{\rm i}n_{\rm n}n_0\exp\{-\eta\} - \frac{\lambda_{\rm D}}{u_{\rm i}c_{\rm S}}\,k_{\rm r}n_{\rm i}n_0^2\exp\{-2\eta\}, \tag{14}$$

$$u_{\rm d}u_{\rm d}' = -Z_{\rm d}\eta' - \alpha u_{\rm d}^{2+\gamma}, \tag{15}$$

$$\exp\{-z\} = \frac{n}{n_{\rm e}}\sqrt{\frac{\pi m_{\rm e}}{m_{\rm i}}}\frac{2z}{\tau}\alpha_{\rm ch}, \tag{16}$$

$$\eta'' = \frac{n_{\rm i}}{n_{\rm e0}} - \exp\{-\eta\} + (1-\delta)\frac{u_{\rm d0}}{u_{\rm d}}. \tag{17}$$

The prime represents the derivative with respect to the spatial coordinate ξ.

2.2. Numerical solutions

The governing equations were solved precisely for the electric potential η_ω, the ion velocity $u_{\rm i}(\xi)$ and the dust velocity $u_{\rm d}(\xi)$ by integrating them numerically with a Runge–Kutta routine [16]. The plasma parameters used in the numerical computation are given in Table 1.

In Figs. 1 − 3 we plot the sheath thickness d, the ion impact energy and the dust impact energy as functions of the collision parameter α and wall potential η_ω. These plots show three regimes of sheath collisionality. For small α, collisions are negligible, and d, $\varepsilon_{\omega\rm d}$ and $\varepsilon_{\omega\rm i}$ are nearly independent of α. For large α the ion and the dust motion is collisionally dominated, d, $\varepsilon_{\omega\rm d}$ and $\varepsilon_{\omega\rm i}$ decrease and approach to a limiting asymptote. Between the collisionless and collisional regimes, there is a transition regime. Approximate analytic expressions for d and $\varepsilon_{\omega\rm i}$ can be derived.

TABLE 1. Main plasma parameters in the numerical computation [17].

Definition	Notation	Value
Electron temperature	$T_{\rm e}$	2.0 eV
Electron density	$n_{\rm e}$	10^9 cm^{-3}
Neutral density	$n_{\rm n}$	3×10^{15} cm^{-3}
Ion temperature	$T_{\rm i}$	0.025 eV
Electron Debye length	$\lambda_{\rm D}$	400 μm
Ion mass	$m_{\rm i}$	$40.1836\, m_{\rm e}$

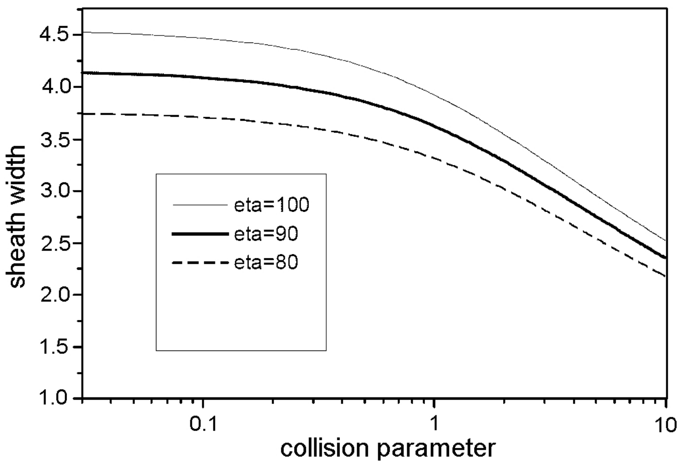

Fig. 1. Precise numerical solutions of the governing equations for the dimensionless sheath thickness as function of the collision parameter α for various wall potentials. Three regimes are evident. We have assumed $\gamma = -0.5$.

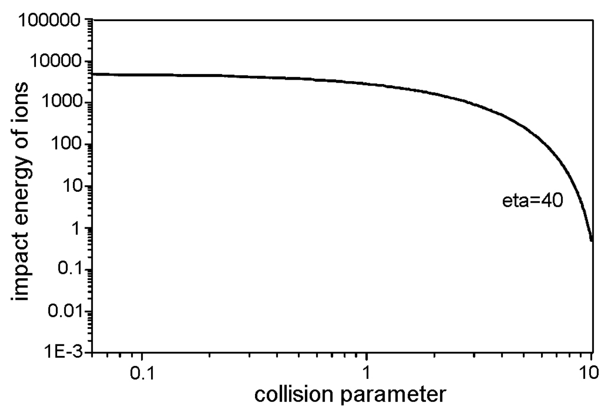

Fig. 2. Precise numerical solution for the average ion impact energy at the wall as function of the collisional parameter. We show results for $\gamma = -0.5$; three regimes are evident: a collisionless regime where $\varepsilon_{\omega i}$ is nearly independent of collisional parameter, a collisionally dominated regime where $\varepsilon_{\omega i}$ approaches to a limiting asymptote, and a transition regime. We have assumed that $u_{0i} = 100$ and $\gamma = -0.5$.

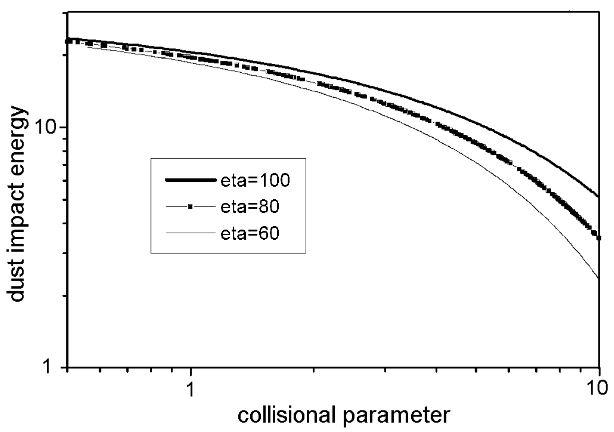

Fig. 3. Precise numerical solution for the average dust impact energy at the wall as function of the collisional parameter for various wall potentials. In this figure, three regimes are evident: a collisionless regime where $\varepsilon_{\omega i}$ is nearly independent of collisional parameter, a collisionally dominated regime where $\varepsilon_{\omega i}$ approaches to a limiting asymptote, and a transition regime. We also show that the dust impact energy increases when the electric potential increases.

The transition regime is much more difficult to treat analytically. Consequently, the numerical results in Figs. 1, 2 and 3 are most valuable for their accuracy in the transition regime.

In the special case of thermal equilibrium without both ionization and recombination, the ions are assumed to follow the Boltzmann relations [10, 11]

$$n_\mathrm{i} = n_\mathrm{i0} \exp\left\{ -\frac{e\phi}{k_\mathrm{B} T_\mathrm{i}} \right\}. \tag{18}$$

We find two coupled differential equations describing the sheath structure,

$$v_\mathrm{d} \frac{\mathrm{d}}{\mathrm{dx}} v_\mathrm{d} = \frac{Z_\mathrm{d} e}{m_\mathrm{d}} \frac{\mathrm{d}}{\mathrm{dx}} \phi - n_\mathrm{n} \sigma_\mathrm{s} \frac{v_\mathrm{d}^{2+\gamma}}{C_\mathrm{d}^\gamma} \tag{19}$$

and

$$\nabla^2 \phi = -4\pi e (n_\mathrm{i} - n_\mathrm{e} - Z_\mathrm{d} n_\mathrm{d}) \,. \tag{20}$$

Using the precedent non-dimensional parameters, the basic equations reduce to

$$u_\mathrm{d} u_\mathrm{d}' = -Z_\mathrm{d} \eta' - \alpha u_\mathrm{d}^{2+\gamma}, \tag{21}$$

$$\eta'' = \delta \exp\{\eta\theta\} - \exp\{-\eta\} + (1-\delta) \frac{u_\mathrm{d0}}{u_\mathrm{d}}. \tag{22}$$

The governing equations were solved precisely by integrating them numerically with a Runge–Kutta routine.

In Figs. 4 and 5, we plot the sheath thickness d and the dust impact energy as functions of the collision parameter α for various wall potentials η_ω. These plots show three regimes of sheath collisionality: collisionless, collisional and transition regime. We show also that both d and $\varepsilon_{\omega\mathrm{d}}$ decrease and approach a limiting asymptote with increasing collisionality.

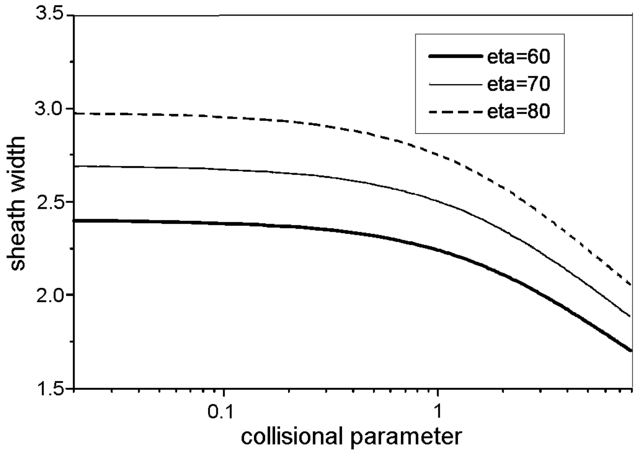

Fig. 4. Precise numerical solutions of the governing equations for the dimensionless sheath thickness as function of the collisional parameter α for various wall potentials. Three regimes are evident. We have assumed that $\gamma = -0.5$.

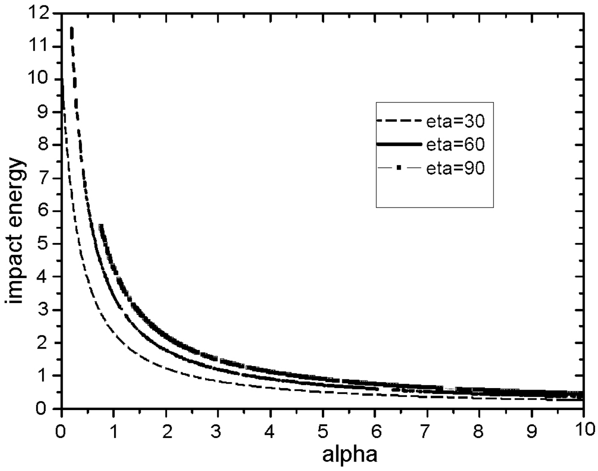

Fig. 5. Precise numerical solution for the average dust impact energy at the wall as function of the collisional parameter for various wall potentials. In this figure, three regimes are evident: a collisionless regime, a collisionally dominated regime where $\varepsilon_{\omega \mathrm{d}}$ approaches to a limiting asymptote, and a transition regime. We also show that, dust impact energy increases when the electric potential increases.

2.3. Comparison

The dust impact energy profile for the two cases of ions is shown in Fig. 6. We note that the evolution of dust impact energy is correlated with the evolution of the collision parameter. This plot shows that a presence of dust charged grains in

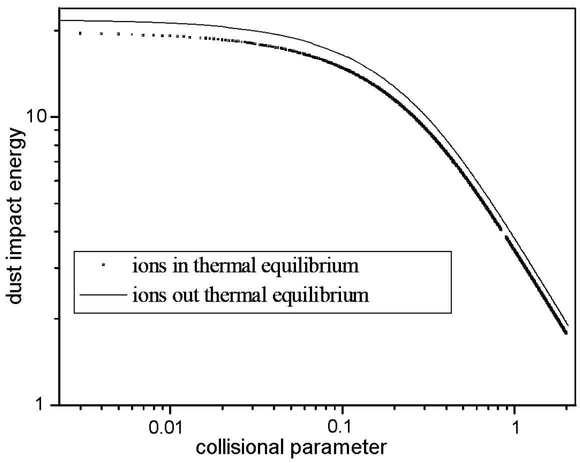

Fig. 6. Comparison between the precise numerical solutions for the dust impact energy at the wall in thermal equilibrium and out of thermal equilibrium. This plot shows that the dust impact energy at the wall is higher in the case of ions out of thermal equilibrium.

plasma influenced the characteristic behaviour of the plasma sheath. The presence of three regimes of sheath collisionality is also shown in this figure. In the limit of strong collisions, the decreases in the dust impact energy approach a limiting asymptote. We also find that the dust impact energy at the wall is greater in the case of ions out thermal equilibrium than in the case of ions in thermal equilibrium.

3. *Summary*

The precise numerical solution of our model provides information about the behaviour of dust impact energy at the wall. We have demonstrated that the sheath thickness d, the dust impact energy and the ion impact energy at the wall decrease with increasing collisionality. Our plots show three different regimes: a collisionless regime, a collisionally dominated regime where $\varepsilon_{\omega \mathrm{d}}$ approaches to a limiting asymptote and a transition regime. We show that the dust impact energy increases when the electric potential increases. It is observed also that the dust impact energy at the wall is larger in the case of ions out thermal equilibrium than in the case of ions in thermal equilibrium.

We note that the evolution of the sheath width is correlated with the evolution of the collisional parameter. In the limit of strong collisions, the decreases in the sheath thickness approach a limiting asymptote.

Our model can be applied to study the sheath in various material plasma processing techniques where negatively charged dust particles are usually found to be present.

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FIZIČKI PROBLEM I TEORIJSKI OPIS SLOJA U SUDARNOJ PRAŠNJAVOJ PLAZMI

Cilj ovog rada je proučavanje svojstava graničnog sloja u sudarnoj plazmi i sročenje učinaka sudara u sloju primjenom Runge–Kutta-ovog postupka. Razmatrali smo područje blizu zida u nemagnetiziranoj prašnjavoj plazmi koja se sastoji od elektrona, iona, čestica prašine mikroskopske veličine i neutralnih čestica. Čestice prašine su mnogo masivnije nego elektroni i ioni, pa se ovi mogu pretpostaviti izvan termičke ravnoteže, dok je prašina hladna tekućina. Uzeli smo da su neutralne čestice nepokretne. Primijenili smo točna numerička rješenja za određivanje sudarne ovisnosti debljine sloja i udarne energije pri zidu.