

EFFECTS OF IONIC TEMPERATURES ON PHASE VELOCITIES OF  
ION-ACOUSTIC SOLITARY WAVES IN A DRIFT NEGATIVE-ION PLASMA  
WITH SINGLE TEMPERATURE ELECTRON

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Propagation of ion-acoustic solitary waves in a collisionless, unmagnetised single-electron-temperature plasma containing warm positive and negative ion drifts of equal temperatures is studied. It is found that, if the ions have finite temperatures, then there exist two modes, namely the slow and the fast ion-acoustic mode. The effect of ionic temperatures and negative ions on phase velocities (the fast and slow ion-acoustic modes) of the solitary waves are discussed for the plasmas having ( $\text{He}^+$ ,  $\text{O}^-$ ), ( $\text{He}^+$ ,  $\text{Cl}^-$ ) and ( $\text{H}^+$ ,  $\text{O}^-$ ) ions.

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## 1. Introduction

In the last few decades or so, ion-acoustic solitons of various types of plasmas have been extensively studied by many authors [1–7]. Two-types of soliton modes were found. Tagare and Verheest [8,9] studied ion-acoustic solitons in an unmagnetized two-ion plasma consisting of isothermal electrons and adiabatic positive and negative ions. Both their studies were restricted to some particular parameters. In fact, they have investigated mainly the slow ion-acoustic solitons. On the other hand, Tran, Tagare, Verheest, Das and Verheest [10,11] studied the fast ion-acoustic solitons in an unmagnetized and in a magnetized multi-ion-species plasma, including any number of adiabatic positive and negative-ion species and electrons. In a two-component unmagnetized collisionless plasma, only the compressive ion-acoustic solitons exist. This was found theoretically and afterwards confirmed experimentally. But in this system, rarefactive ion-acoustic solitons will not arise, although compressive and rarefactive solitons co-exist in the presence of

negative ions at the critical concentration of negative ions. Also, it is possible to obtain both compressive and rarefactive solitons in a plasma with more than two components. The presence of negative ions in the plasma has been found to give more interesting results than that of two-electron-temperature plasma. In a two-component plasma, if the temperature of the positive ion species ( $\sigma_i$ ) is smaller than that of the negative-ion species ( $\sigma_j$ ), compressive solitons generally exist for all values of the mass ratio  $Q$  other than  $Q \ll 1$ . Again, it is found that there is a critical value of the negative-ion concentration ( $n_{jc}$ ) below which compressive solitons ( $\phi > 0$ ) exist and above which rarefactive solitons ( $\phi < 0$ ) will occur. In this paper, we investigate the effects of drift velocities, ionic temperatures and ratio of negative to positive ion masses on the formation of ion-acoustic solitons with warm positive and negative ions, and of the phase velocities ( $V'_1$  and  $V'_2$ ) on the negative-ion concentration ( $n_{j0}$ ), where  $V'_1$  and  $V'_2$  are the fast and slow ion-acoustic modes of the solitary waves.

The paper is organized in the following way. The basic set of equations is given in Sec. 2 from which we get phase velocities after some calculations. Section 3 is devoted to the discussion of the phase velocities (fast and slow ion-acoustic modes) for three different types of plasma with respective variation. Concluding remarks are given in Sec. 4.

## 2. Basic equations and solutions

We consider a plasma consisting of warm adiabatic positive and negative-ion species with drift velocities and warm isothermal electrons. We assume that the temperature of ions is much lower than that of electrons (i.e.,  $T_i \ll T_e$ ). Under these conditions, the nonlinear behaviour of ion-acoustic waves may be described by the following set of equations in dimensionless form [12]

$$\frac{\partial n_\alpha}{\partial t} + \frac{\partial}{\partial x}(n_\alpha u_\alpha) = 0, \quad (1)$$

$$\frac{\partial u_\alpha}{\partial t} + u_\alpha \frac{\partial u_\alpha}{\partial x} + \frac{\sigma_\alpha}{Q_\alpha n_\alpha} \frac{\partial p_\alpha}{\partial x} = -\frac{Z_\alpha}{Q_\alpha} \frac{\partial \phi}{\partial x}, \quad (2)$$

$$\frac{\partial p_\alpha}{\partial t} + u_\alpha \frac{\partial p_\alpha}{\partial x} + 3p_\alpha \frac{\partial u_\alpha}{\partial x} = 0, \quad (3)$$

$$\frac{\partial^2 \phi}{\partial x^2} = n_e - \sum_\alpha Z_\alpha n_\alpha, \quad (4)$$

where  $n_e = \exp\{\phi\}$ ,  $n_\alpha$ ,  $u_\alpha$  and  $p_\alpha$  are the number density, velocity and pressure of the ions, respectively.  $Q_\alpha$ ,  $Z_\alpha$ ,  $\sigma_\alpha$  and  $\phi$  are the mass ratio of negative to positive ions, charge, temperature of ions and electrostatic potential. Here  $Q_\alpha = m_j/m_i$ ,  $\sigma_\alpha = T_\alpha/T_e$ ,  $Z_\alpha = 1$ ,  $Q_\alpha = 1$  for positive ions (i); also  $Z_\alpha = -Z$ ,  $Q_\alpha = Q$  for negative ions (j), and  $\alpha = i$  for positive ions and  $\alpha = j$  for negative ions.

In the above equations, we normalized the velocity by the characteristic velocity  $\sqrt{KT_e/m}$ , all densities by the equilibrium value  $n_0$ , and the length by the Debye

length  $\sqrt{KT_e/(4\pi e^2 n_0)}$ , whereas the potential is normalized to  $KT_e/e$ . Therefore, the equations appear entirely in the dimensionless form.

For the solitary wave solution, we make the transformation to the stationary wave frame so that we make all dependent variables depend on a single independent variable  $\eta$  defined by

$$\eta = x - Vt, \quad (5)$$

where  $V$  is the velocity of the solitary waves.

The boundary conditions are

$$u_\alpha \rightarrow u_{\alpha 0}, \quad n_\alpha \rightarrow n_{\alpha 0}, \quad p_\alpha \rightarrow p_{\alpha 0} \quad \text{and} \quad \phi \rightarrow 0, \quad \text{as} \quad x \rightarrow \pm\infty. \quad (6)$$

The charge neutrality condition of the plasma is

$$\sum_{\alpha} n_{\alpha 0} Z_{\alpha} = 1. \quad (7)$$

From Eqs. (1) to (4), after using (5), (6) and (7) we get finally

$$\frac{d^2\phi}{d\eta^2} = n_e - \sum_{\alpha} Z_{\alpha} \sqrt{\frac{Q_{\alpha} n_{\alpha 0}^3}{6 p_{\alpha 0} \sigma_{\alpha}}} \sqrt{b_{1\alpha} - \frac{2Z_{\alpha}}{Q_{\alpha}}\phi - \sqrt{\left(b_{1\alpha} - \frac{2Z_{\alpha}}{Q_{\alpha}}\phi\right)^2 - b_{2\alpha}^2}}, \quad (8)$$

where

$$\begin{aligned} b_{1\alpha} &= (V - u_{\alpha 0})^2 + \frac{3\sigma_{\alpha} p_{\alpha 0}}{Q_{\alpha} n_{\alpha 0}}, \\ b_{2\alpha}^2 &= \frac{12\sigma_{\alpha} p_{\alpha 0}}{Q_{\alpha} n_{\alpha 0}} (V - u_{\alpha 0})^2. \end{aligned} \quad (9)$$

Equation (8) can be written in the form

$$\frac{d^2\phi}{d\eta^2} = -\frac{\partial\psi}{\partial\phi}, \quad (10)$$

where

$$\begin{aligned} \psi(\phi) &= 1 - e^{\phi} - \frac{1}{6} \sum_{\alpha} \sqrt{\frac{n_{\alpha 0}^3 Q_{\alpha}^3}{3 p_{\alpha 0} \sigma_{\alpha}}} \left[ \left( b_{1\alpha} - \frac{2Z_{\alpha}}{Q_{\alpha}}\phi + b_{2\alpha} \right)^{3/2} \right. \\ &\quad \left. - \left( b_{1\alpha} - \frac{2Z_{\alpha}}{Q_{\alpha}}\phi - b_{2\alpha} \right)^{3/2} + (b_{1\alpha} - b_{2\alpha})^{3/2} - (b_{1\alpha} + b_{2\alpha})^{3/2} \right]. \end{aligned} \quad (11)$$

For the solitary wave solution, the pseudopotential  $\psi(\phi)$  in Eq. (11) must satisfy the following conditions [13]

$$\begin{aligned} \psi(\phi) = \frac{\partial\psi(\phi)}{\partial\phi} = 0, \quad \frac{\partial^2\psi(\phi)}{\partial\phi^2} < 0, \quad \text{for } \phi = 0, \\ \psi(\phi) = 0, \quad \frac{\partial\psi}{\partial\phi} > 0, \quad \text{for } \phi = \phi_m \quad (\text{Compressive soliton}), \\ \psi(\phi) < 0, \quad \text{in } 0 < |\phi| < |\phi_m|, \end{aligned} \quad (12)$$

where  $|\phi_m|$  is the amplitude of the solitary wave.

From (11) and (12) we get

$$\left. \frac{\partial^2\psi(\phi)}{\partial\phi^2} \right|_{\phi=0} < 0,$$

which implies

$$\sum_{\alpha} \frac{Z_{\alpha}^2 n_{\alpha 0}^2}{Q_{\alpha} (V - u_{\alpha 0})^2 n_{\alpha 0} - 3\sigma_{\alpha} p_{\alpha 0}} < 1. \quad (13)$$

From inequality (13), the phase velocity is given by the equation

$$\frac{Z^2}{Q(V - u_{j0})^2 n_{j0} - 3\sigma_j p_{j0}} + \frac{n_{i0}^2}{(V - u_{i0})^2 n_{i0} - 3\sigma_i p_{i0}} = 1. \quad (14)$$

This equation can be written in the form

$$\begin{aligned} QV^4 - 2Q(u_{i0} + u_{j0})V^3 + \left( Qu_{j0}^2 - Z^2 n_{j0} + 4Qu_{i0}u_{j0} + Qu_{i0}^2 \right. \\ \left. - \frac{3\sigma_j p_{j0}}{n_{j0}} - \frac{3Q\sigma_i p_{i0}}{n_{i0}} - Qn_{i0} \right) V^2 + \left( 2Z^2 n_{j0} u_{i0} - 2Qu_{i0}u_{j0}^2 - 2Qu_{i0}^2 u_{j0} \right. \\ \left. + \frac{6\sigma_j p_{j0} u_{i0}}{n_{j0}} + \frac{6Q\sigma_i p_{i0} u_{j0}}{n_{i0}} + 2Qn_{i0} u_{j0} \right) V \\ + \left( \frac{3Z^2 \sigma_i p_{i0} n_{j0}}{n_{i0}} - Z^2 n_{j0} u_{i0}^2 + Qu_{i0}^2 u_{j0}^2 - \frac{3\sigma_j p_{j0} u_{i0}^2}{n_{j0}} \right. \\ \left. - \frac{3Q\sigma_i p_{i0} u_{j0}^2}{n_{i0}} + \frac{9\sigma_i \sigma_j p_{i0} p_{j0}}{n_{i0} n_{j0}} - Qn_{i0} u_{j0}^2 + \frac{3\sigma_j p_{j0} n_{i0}}{n_{j0}} \right) = 0. \end{aligned} \quad (14a)$$

Equation (14a) is a modified form of the phase velocity ( $V$ ) expression [14]. It is a quadratic in  $V^2$  and shows that the inclusion of a finite ion-temperature gives rise to two ion-acoustic modes [15] propagating with different phase velocities. For  $\sigma_i = \sigma_j = \sigma$ ,  $p_{i0} = n_{i0}$ ,  $p_{j0} = n_{j0}$ ,  $u_{i0} = u_{j0} = 0$  and  $Z = 1$ , Eq. (14a) exactly reduces to the result of Ref. [14]. For  $\sigma_i = \sigma_j = \sigma = 0$ , Eq. (14a) reduces to the

result of Ref. [16] and for  $u_{i0} = u_{j0} = 0$ ,  $\sigma_i = \sigma_j = \sigma = 0$ , Eq. (14a) exactly reduces to the result of Refs. [12] and [17]. For  $\sigma_i = \sigma_j = \sigma$ ,  $p_{i0} = n_{i0}$ ,  $p_{j0} = n_{j0}$ ,  $u_{i0} = u_{j0} = 0$ ,  $Z = 1$  and  $Q = 1$ , Eq. (14a) reduces to the result of Ref. [18] from which we get the phase velocity ( $V$ ) of the fast ion-acoustic solitons only, but it is to be noted here that both fast ion-acoustic solitons and slow ion-acoustic solitons may appear for  $Q \neq 1$ , and for slow ion-acoustic solitons negative ions play an important role. It is also found that as negative-ion concentration increases, the amplitude (both the first and the second order) of the solitary waves decreases which occurs only in the case of slow ion-acoustic solitons.

Equation (14) may be changed when  $V - u_{i0} = V - u_{j0} = V'$ ,  $\sigma_i = \sigma_j = \sigma$ ,  $p_{i0} = p_{j0} = 1$  and  $Z = 1$  so that the above equation takes the following form

$$a_0 V'^4 - a_1 V'^2 + a_2 = 0, \quad (15)$$

where

$$\begin{aligned} a_0 &= Qn_{i0}n_{j0}, \\ a_1 &= 3\sigma n_{i0} + n_{i0}n_{j0}^2 + 3Q\sigma n_{j0} + Qn_{i0}^2n_{j0}, \\ a_2 &= 9\sigma^2 + 3\sigma n_{j0}^2 + 3\sigma n_{i0}^2. \end{aligned} \quad (16)$$

From Eq. (15), the roots are as follows

$$\begin{aligned} V_1'^2 &= \frac{1}{2a_0} \left[ a_1 + \sqrt{a_1^2 - 4a_0a_2} \right], \\ V_2'^2 &= \frac{1}{2a_0} \left[ a_1 - \sqrt{a_1^2 - 4a_0a_2} \right], \end{aligned} \quad (17)$$

where  $V_1'$  and  $V_2'$  are assumed to be the phase velocities named the fast and the slow ion-acoustic mode of the solitary waves.

### 3. Discussion

In this investigation of the existence of ion-acoustic solitary waves, we have found two types of phase velocities (the fast and the slow ion-acoustic mode) for the following three types of plasma:

- i) ( $\text{He}^+$ ,  $\text{O}^-$ ) – Plasma corresponding to  $Q = 4$ ,
- ii) ( $\text{He}^+$ ,  $\text{Cl}^-$ ) – Plasma corresponding to  $Q = 8.875$ ,
- iii) ( $\text{H}^+$ ,  $\text{O}^-$ ) – Plasma corresponding to  $Q = 16$ .

The results of our calculations are shown in Figs. 1–8.

In Fig. 1a, the phase velocity  $V_1'$  (fast ion-acoustic mode) is plotted against the negative-ion concentration ( $n_{j0}$ ) for  $\sigma_i = \sigma_j = \sigma = 0$ . As negative-ion concentration ( $n_{j0}$ ) increases, the phase velocity ( $V_1'$ ) also increases for a particular value of  $Q$ .

When  $Q$  increases ( $Q = 4, 8.875, 16$ ), the phase velocity ( $V_1'$ ) gradually decreases for a particular value of  $Z$  ( $Z = 1$ ) which supports our cold-ion case [16].

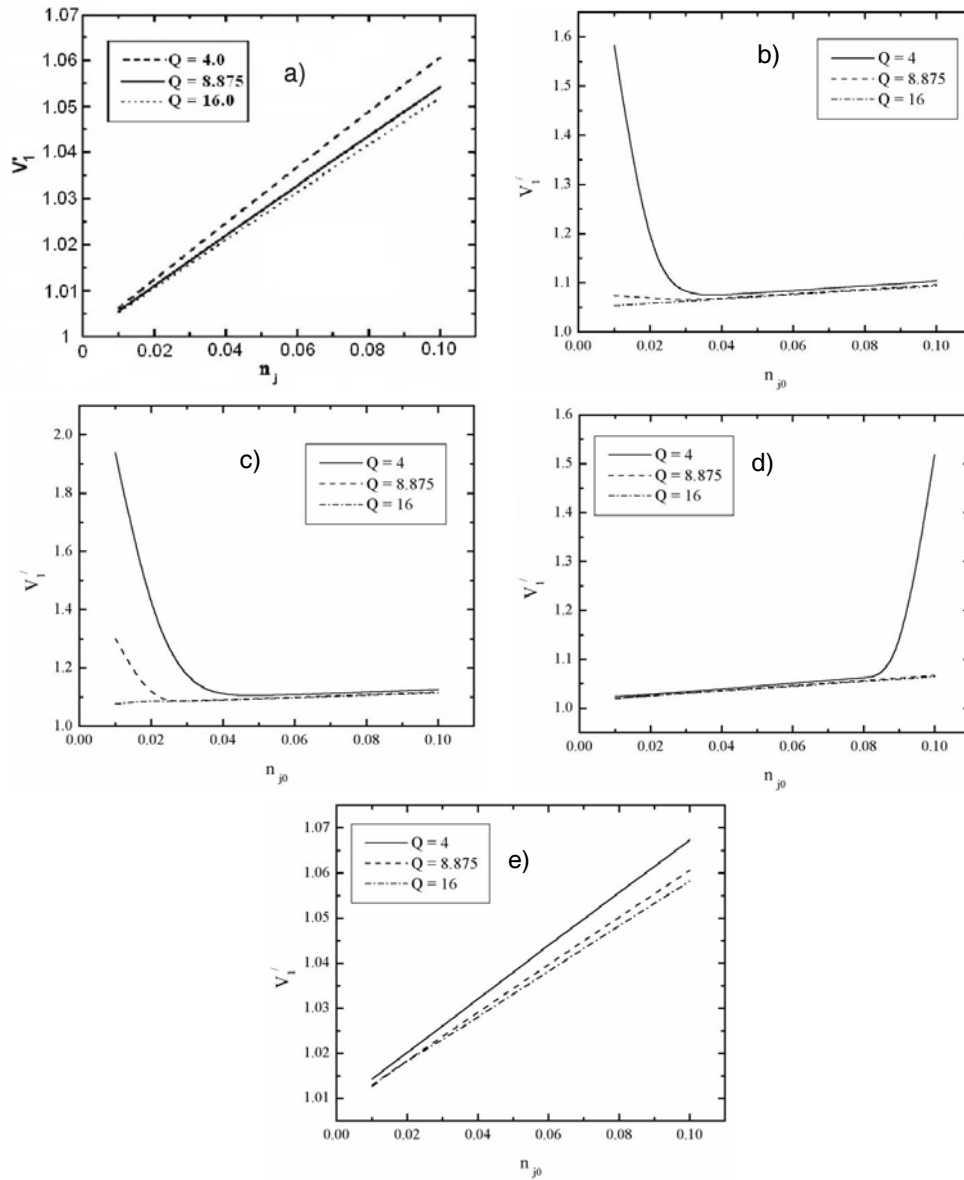


Fig. 1. Variation of phase velocity  $V_1'$  (fast ion-acoustic mode) with the negative-ion concentration ( $n_{j0}$ ) and variation of  $Q$  (ratio of negative to positive ion masses) for ionic temperatures: (a)  $\sigma = 0$ , (b)  $\sigma = 1/30$ , (c)  $\sigma = 1/20$ , (d)  $\sigma = 1/100$ , (e)  $\sigma = 1/200$ .

For  $\sigma_i = \sigma_j = \sigma = 1/30$ , the phase velocity  $V_1'$  (fast acoustic mode) first decreases until a certain negative-ion concentration ( $n_{j0} = 0.03$ ) is reached, and then increases as  $n_{j0}$  increases for the mass ratios  $Q = 4$  and  $Q = 8.875$  with  $Z = 1$ . But for  $Q = 16$  and  $Z = 1$ , the phase velocity  $V_1'$  (fast ion acoustic mode) gradually increases with the increasing value of  $n_{j0}$ . This is shown in Fig. 1b. Fig. 1c shows the variation of the phase velocity  $V_1'$  (fast ion-acoustic mode) with the negative-ion concentration ( $n_{j0}$ ) for  $\sigma_i = \sigma_j = \sigma = 1/20$  and with the variation of mass ratio ( $Q$ ) when  $Z = 1$ . Now for  $Q = 4$ , the phase velocity  $V_1'$  decreases as  $n_{j0}$  increases upto  $n_{j0} = 0.05$  and then increases with the increasing value of  $n_{j0}$ . Also for  $Q = 8.875$ , the fast ion-acoustic mode  $V_1'$  decreases to a certain value of  $n_{j0}$  ( $n_{j0} = 0.02$ ) and then increases with the increasing value of  $n_{j0}$ . But for  $Q = 16$ , the phase velocity  $V_1'$  (fast ion-acoustic mode) decreases with increasing value of  $n_{j0}$  (from  $n_{j0} = 0.01$  to 0.03) and after that  $V_1'$  increases with increasing value of  $n_{j0}$ .

The phase velocity  $V_1'$  (fast acoustic mode) shown in Fig. 1d is a more interesting case than those shown in the previous three figures. In this case, for  $\sigma_i = \sigma_j = \sigma = 1/100$  (i.e. when the temperature of ions is smaller than the previous set) with the variation of mass ratio  $Q$  ( $Q = 4, 8.875, 16$ ), the phase velocity  $V_1'$  (fast acoustic mode) increases with increasing value of  $n_{j0}$  with  $Z = 1$ . Also, it is observed that as  $Q$  increases, the phase velocity  $V_1'$  also increases for any particular value of  $n_{j0}$  which is rather different from the previous case.

If the temperature of ions  $\sigma$  ( $\sigma_i = \sigma_j = \sigma = 1/200$ ) is smaller than in the case shown in Fig. 1d (i.e.  $\sigma_i = \sigma_j = \sigma = 1/100$ ), the phase velocity  $V_1'$  (fast acoustic mode) also increases with the increasing value of negative-ion concentration ( $n_{j0}$ ) for three different values of mass ratio  $Q$  ( $Q = 4, 8.875, 16$ ) when  $Z = 1$ . This is shown in Fig. 1e. The phase velocity  $V_1'$  is larger for small value of mass ratio  $Q$  for a fixed value of  $\sigma$  ( $\sigma_i = \sigma_j = \sigma = 1/200$ ) with  $Z = 1$  which is also shown in Fig. 1e.

In Fig. 2a, the variation of phase velocity  $V_1'$  (fast ion-acoustic mode) is plotted against negative-ion concentration ( $n_{j0}$ ) for different values of ionic temperature  $\sigma$  ( $\sigma_i = \sigma_j = \sigma$ ) with the mass ratio  $Q$  ( $Q = 4$ ) and  $Z = 1$ . As  $\sigma$  increases ( $\sigma = 0, 1/200, 1/100, 1/30, 1/20$ ), the phase velocity  $V_1'$  (fast ion-acoustic mode) also increases with increasing value of  $n_{j0}$  when the mass ratio  $Q$  ( $Q = 4$ ) is fixed and  $Z = 1$ . It is important to note here that temperature of ions ( $\sigma$ ) and negative-ion concentration ( $n_{j0}$ ) control the phase velocity  $V_1'$  for a fixed mass ratio  $Q$ .

Fig. 2b shows the phase velocity  $V_1'$  (fast mode) against negative-ion concentration ( $n_{j0}$ ) with the variation of temperature of ions ( $\sigma_i = \sigma_j = \sigma$ ) for the mass ratio  $Q = 8.875$  and  $Z = 1$ . Similarly, the phase velocity  $V_1'$  also increases as in Fig. 2a with the increasing value of  $\sigma$  ( $\sigma_i = \sigma_j = \sigma$ ) and  $n_{j0}$  when mass ratio  $Q$  is fixed.

The phase velocity  $V_1'$  of the solitary waves for the fast ion-acoustic mode is shown in Fig. 2c against negative-ion concentration ( $n_{j0}$ ) for different increasing values of temperature of ions  $\sigma$  ( $\sigma_i = \sigma_j = \sigma$ ) with mass ratio  $Q = 16$  and  $Z = 1$ . In this case, similarly to the previous cases, the phase velocity  $V_1'$  is directly proportional to the temperature of ions ( $\sigma$ ) for a fixed mass ratio ( $Q$ ) when  $Z = 1$ .

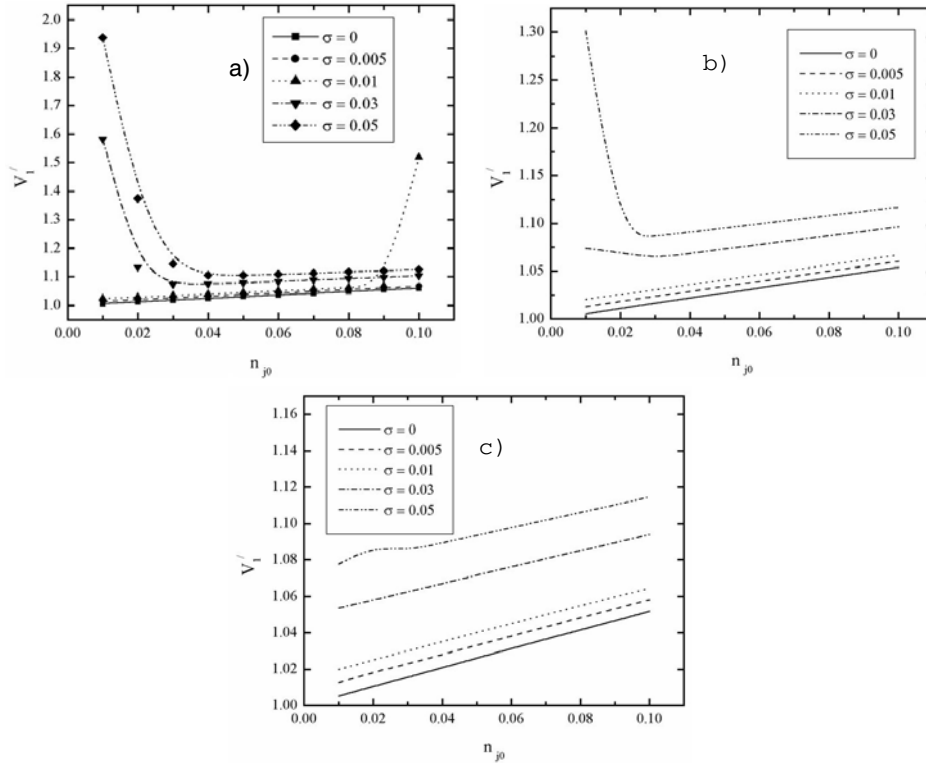


Fig. 2. Variation of phase velocity  $V'_1$  (fast ion-acoustic mode) against negative-ion concentration ( $n_{j0}$ ) with variation of ionic temperature ( $\sigma$ ) for the ratio of negative to positive ion masses: (a)  $Q = 4$ , (b)  $Q = 8.875$ , (c)  $Q = 16$ .

In Fig. 3a, the phase velocity  $V'_2$  (slow ion-acoustic mode) is plotted against the negative-ion concentration  $n_{j0}$  for three different values of mass ratio  $Q$  with ionic temperature  $\sigma = 1/30$  ( $\sigma_i = \sigma_j = \sigma$ ) and  $Z = 1$ . It is seen that as  $Q$  increases (i.e.  $Q = 4, 8.875, 16$ ), the phase velocity  $V'_2$  decreases. In this case, it is also observed from the figure that the phase velocity  $V'_2$  (slow ion-acoustic mode) shows large values for small value of the mass ratio ( $Q$ ).

Figure 3b shows the phase velocity  $V'_2$  (slow ion-acoustic mode) against negative-ion concentration  $n_{j0}$  with the variation of mass ratio ( $Q$ ) for ionic temperature  $\sigma = 1/20$  ( $\sigma_i = \sigma_j = \sigma$ ) and  $Z = 1$ . As  $n_{j0}$  increases,  $V'_2$  decreases and  $V'_2$  is maximum for small values of the mass ratio  $Q$ .

The phase velocity  $V'_2$  (slow mode) is plotted in Fig. 3c against negative-ion concentration  $n_{j0}$  with the variation of mass ratio  $Q$  for ionic temperature  $\sigma_i = \sigma_j = \sigma = 1/100$  and  $Z = 1$ . The phase velocity  $V'_2$  for the slow ion-acoustic mode decreases with the increasing value of the negative ion concentration  $n_{j0}$  for three different values of the mass ratio  $Q$  ( $Q = 4, 8.875, 16$ ). But the value of the phase velocity  $V'_2$  for the slow ion-acoustic mode increases for  $Q = 4$  and  $8.875$  with  $\sigma = 1/100$  for  $n_{j0} = 0.01$ , but not for  $Q = 16$ .



Figure 3d shows the phase velocity  $V_2'$  (slow ion-acoustic mode) against the negative-ion concentration ( $n_{j0}$ ) with different values of the mass ratio  $Q$  for the ionic temperature  $\sigma_i = \sigma_j = \sigma = 1/200$  and  $Z = 1$ . For smaller values of  $Q$ , the value of the phase velocity  $V_2'$  is larger for larger values of  $n_{j0}$ .

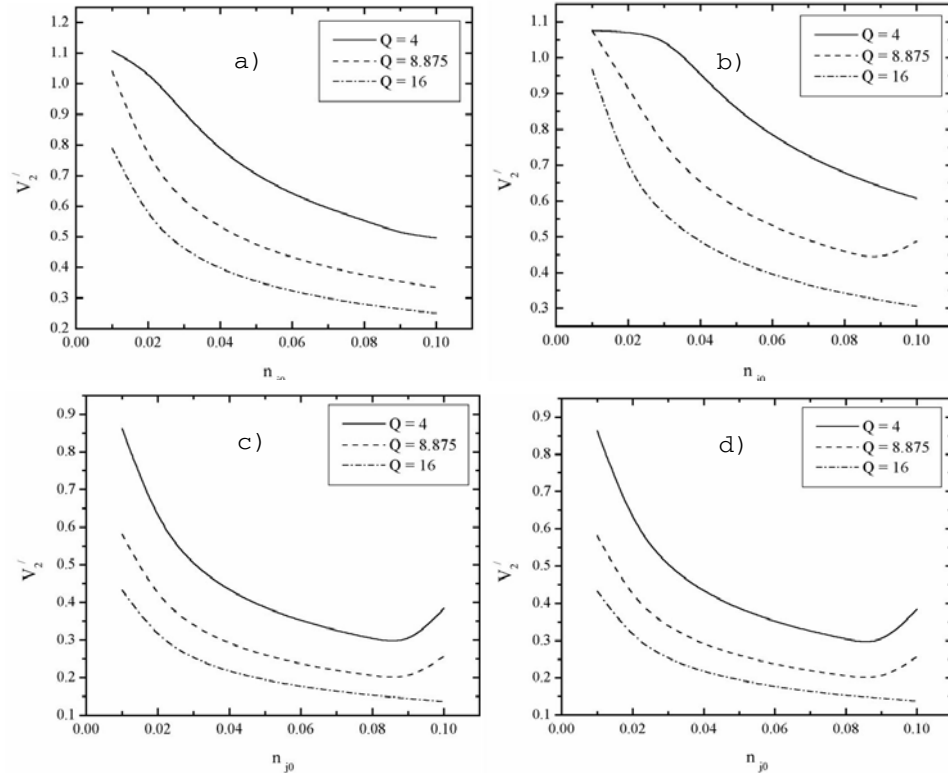


Fig. 3. Variation of phase velocity  $V_2'$  (slow ion-acoustic mode) against negative-ion concentration ( $n_{j0}$ ) with variation of the ratio of negative to positive ion masses ( $Q$ ) for the ionic temperatures: (a)  $\sigma = 1/30$ , (b)  $\sigma = 1/20$ , (c)  $\sigma = 1/100$ , (d)  $\sigma = 1/200$ .

Figure 4a displays the phase velocity  $V_2'$  of the solitary waves for slow-ion acoustic mode against the negative-ion concentration ( $n_{j0}$ ) with the variation of the ionic temperature  $\sigma_i = \sigma_j = \sigma$  for the mass ratio  $Q = 4$  and  $Z = 1$ . The value of the phase velocity  $V_2'$  for the slow ion-acoustic mode increases when the temperature of ions  $\sigma$  increases ( $\sigma_i = \sigma_j = \sigma = 1/200, 1/100, 1/30$ ), except for  $\sigma = 1/20$  at  $n_{j0} < 0.02$ . Moreover,  $V_2'$  decreases for increasing values of  $n_{j0}$ .

Figure 4b presents the phase velocity  $V_2'$  (slow ion-acoustic mode) against the negative-ion concentration ( $n_{j0}$ ) for different values of the temperature of ions ( $\sigma_i = \sigma_j = \sigma = 1/200, 1/100, 1/30, 1/20$ ) for the mass ratio  $Q = 8.875$  and  $Z = 1$ . From this figure it is seen that the phase velocity  $V_2'$  for the slow ion-acoustic mode

decreases except for the case of ion temperatures  $\sigma = 1/100$  and  $1/20$ . Also, this phase velocity varies inversely with the concentration of negative ions ( $n_{j0}$ ) and varies directly with the temperature of ions ( $\sigma$ ).

Figure 4c displays the phase velocity  $V_2'$  of the solitary waves for the slow ion-acoustic mode against the negative-ion concentration  $n_{j0}$  with the variation of the ionic temperature  $\sigma$  ( $\sigma_i = \sigma_j = \sigma = 1/200, 1/100, 1/30, 1/20$ ) for the mass ratio  $Q = 16$  and  $Z = 1$ . The value of the phase velocity  $V_2'$  for the slow ion-acoustic mode increases when the temperature of ions  $\sigma$  ( $= \sigma_i = \sigma_j$ ) increases and decreases when the concentration of negative ions increases.

Figures 5 – 8 show the two-types of phase velocities ( $V_1'$  and  $V_2'$ ) against the ionic temperature  $\sigma$  ( $\sigma_i = \sigma_j = \sigma$ ) for a particular value of the negative-ion concentration  $n_{j0}$  ( $n_{j0} = 0.01$ ) with the variation of mass ratio ( $Q$ ) and for a particular value of the mass ratio  $Q$  ( $Q = 4$ ) with the variation of the negative-ion concentration ( $n_{j0}$ ).

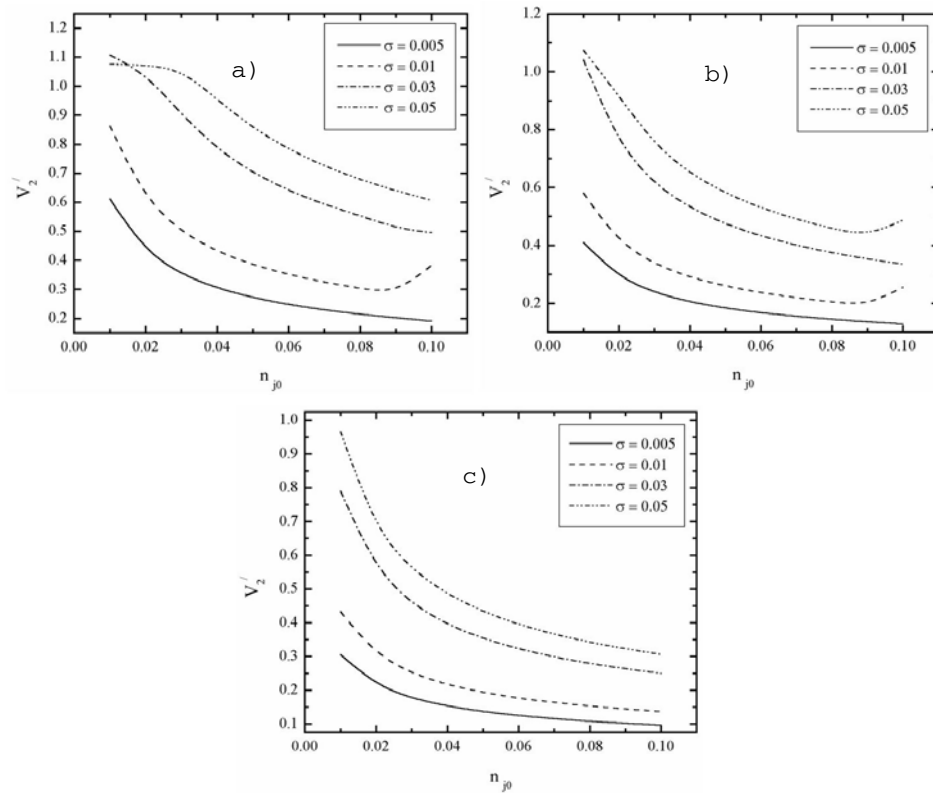


Fig. 4. Variation of phase velocity  $V_2'$  (slow ion-acoustic mode) against negative-ion concentration ( $n_{j0}$ ) with variation of the ionic temperature ( $\sigma$ ) for the ratios of negative to positive ion masses: (a)  $Q = 4$ , (b)  $Q = 8.875$ , (c)  $Q = 16$ , .

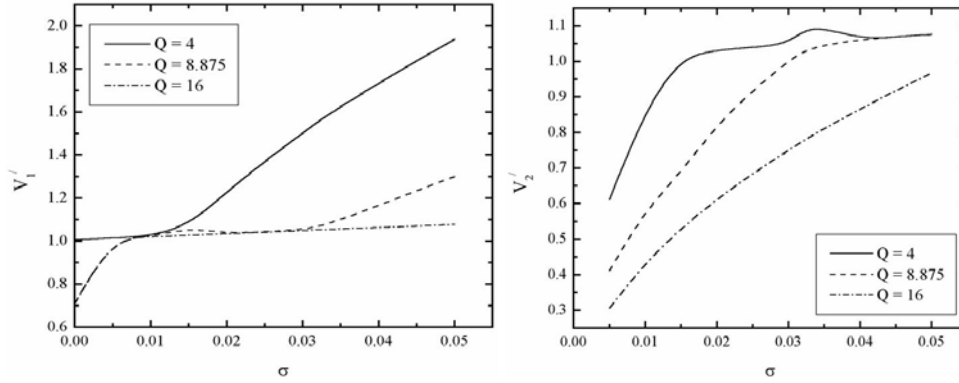


Fig. 5 (left). Variation of phase velocity  $V_1'$  (fast ion-acoustic mode) against ionic temperature ( $\sigma$ ) with the variation of mass ratio  $Q$  for the negative-ion concentration  $n_{j0} = 0.01$ .

Fig. 6. Variation of phase velocity  $V_2'$  (slow ion-acoustic mode) against ionic temperature ( $\sigma$ ) with the variation of mass ratio  $Q$  for the negative-ion concentration  $n_{j0} = 0.01$ .

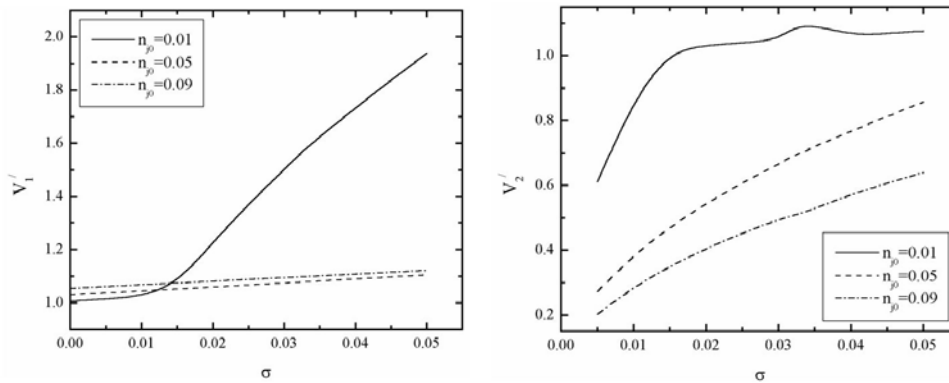


Fig. 7 (left). Variation of phase velocity  $V_1'$  (fast ion-acoustic mode) against ionic temperature ( $\sigma$ ) with the variation of negative-ion concentration ( $n_{j0}$ ) for the ratio of negative to positive ion masses  $Q = 4$ .

Fig. 8. Variation of phase velocity  $V_2'$  (slow ion-acoustic mode) against ionic temperature ( $\sigma$ ) with the variation of negative-ion concentration ( $n_{j0}$ ) for the ratio of negative to positive ion masses  $Q = 4$ .

From Figs. 5–8, it is clear that the phase velocities (fast ion-acoustic mode  $V_1'$  and slow ion-acoustic mode  $V_2'$ ) of the solitary waves increase due to the increase of the temperature of ions [19]  $\sigma_i = \sigma_j = \sigma$  for a particular mass ratio ( $Q$ ) and negative-ion concentration ( $n_{j0}$ ). But as the mass ratio  $Q$  increases, the phase

velocity (both types of modes) of the solitary waves decreases for a particular value of  $n_{j0}$ . That does not support the results of Ref. [19] as shown in Fig. 5 and in Fig. 6. Moreover, the effect of negative ions ( $n_{j0}$ ) (that is, when the concentration ( $n_{j0}$ ) of negative ions increases) increases the phase velocity of the solitary waves which contradicts the results of Ref. [19] as shown in Fig. 7 and in Fig. 8, respectively.

#### 4. Concluding remarks

From our present study based on the application of the pseudopotential technique, we observe the effect of warm positive and negative ions with hot electrons on the existence of solitary waves from which the phase velocities are derived. In this case, two types of modes are found due to the finite temperature of two ion species. One is the fast ion-acoustic mode ( $V_1'$ ) and the other is the slow ion-acoustic mode ( $V_2'$ ). In the case of the fast ion-acoustic mode, for a given set of parameter values, there is a critical concentration of negative ions ( $n_{jc}$ ), below which the compressive solitons exist. The temperature of ions ( $\sigma_i = \sigma_j = \sigma$ ) and the mass ratio ( $Q$ ) are the two important parameters regarding the phase velocities (fast ion-acoustic and slow ion-acoustic modes) of the solitary waves. The phase velocities (fast ion-acoustic mode  $V_1'$  and slow ion-acoustic mode  $V_2'$ ) increase when increasing the value of the negative-ion concentration ( $n_{j0}$ ) for a particular value of the mass ratio ( $Q$ ), when temperature ( $\sigma_i = \sigma_j = \sigma$ ) of ions varies. A similar case will arise for a particular value of the temperature ( $\sigma_i = \sigma_j = \sigma$ ) of ions when the mass ratio ( $Q$ ) varies. Again, the effect of ionic temperatures ( $\sigma = \sigma_i = \sigma_j$ ) on two types of phase velocities (the fast ion-acoustic mode  $V_1'$  and the slow ion-acoustic mode  $V_2'$ ) is observed from Figs. 5–8 with the variation of mass ratio  $Q$  and negative-ion concentration  $n_{j0}$ . In all above cases, we have taken equal temperatures of the two ion species (positive and negative). Our future plan is to take two different temperatures of the two ion species with single temperature electrons and two-temperature electrons, with electron and ion drifts.

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UČINCI IONSKE TEMPERATURE NA FAZNE BRZINE  
IONSKO-AKUSTIČKIH SOLITONSKIH VALOVA U POSMIČNOJ  
NEGATIVNOJ IONSKOJ PLAZMI S ELEKTRONIMA JEDNE  
TEMPERATURE

Proučavamo širenje ionsko-akustičkih solitonskih valova u bez-sudarnoj, nemagnetiziranoj plazmi s elektronima jednake temperature, u kojoj su vrući i posmični pozitivni ioni iste temperature. Nalazimo da ako ioni imaju konačne temperature, tada postoje dva moda, brz i spor ionsko-akustički mod. Proučavamo učinke ionske temperature i negativnih iona na fazne brzine (za brz i spor ionsko-akustički mod) solitarnih valova za plazme koje sadrže  $(\text{He}^+, \text{O}^-)$ ,  $(\text{He}^+, \text{Cl}^-)$  i  $(\text{H}^+, \text{O}^-)$  ione.