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SOLUTIONS OF A CLASS OF NONLINEAR WAVE EQUATIONS

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The Burgers equation, KdV equation and Burgers-KdV equation are real physical mode equations concerning many branches in physics. In this paper, by introducing a new transformation and applying the trial-function method to these three equations, a series of more general explicit and exact travelling wave solutions to them, which include the solitary wave solutions, the singular travelling-wave solutions, and the triangle-function periodic wave solutions, are presented. Among them, some are new travelling-wave solutions.

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1. *Introduction*

At present, more and more problems in branches of modern mathematical physics and other interdisciplinary science are described in terms of suitable nonlinear models, such as the nonlinear Schrödinger equation in plasma physics [1], the KdV equation in shallow water model [2], and so on. Therefore, it is an important and essential topic to seek the explicit and exact solutions of nonlinear evolution equations in nonlinear science, especially in nonlinear-physics science. In recent years, a vast variety of simple and direct approaches have been developed by mathematicians and physicists to find the exact and analytical solutions to nonlinear evolution equations. Some of the most important new approaches are the function-transformation method [3, 4], the homogeneous balance method [5, 6], the hyperbolic-tangent-function expansion method [7, 8], the trial-function method [9 − 12], the nonlinear-transformation method [13, 14], the sine-cosine method [15], the Jacobi-elliptic-function expansion method [16, 17], the superposition method

[18], the auxiliary-equation method [19, 20], and the like. However, not all these techniques are universally applicable for solving all kinds of nonlinear evolution equations directly. As a result, it is still a very significant task to look for more powerful and efficient approaches to solve nonlinear evolution equations.

The primary aim of the present paper is to study the travelling-wave solutions for a class of nonlinear wave equations (NWEs for short) such as the Burgers equation and the KdV equation as well as the Burgers-KdV equation by introducing a new transformation and making use of the trial-function method. As a consequence, rich explicit and exact solutions including the solitary-wave solutions, the singular-travelling-wave solutions and the triangle-function periodic-wave solutions are obtained, some of which are new travelling-wave solutions.

The rest of this paper is organized as follows. In Sect. 2, we introduce a method to construct the explicit and exact solutions of a class of NEWs. In Sect. 3, as illustrative examples, we apply it to solve three well-known NWEs. Conclusions are given in the last section.

2. *Essential approach*

Throughout this paper, we restrict our attention to the investigation of the following class of NWEs

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + \alpha\frac{\partial^2 u}{\partial x^2} + \beta\frac{\partial^3 u}{\partial x^3} + \gamma\frac{\partial^4 u}{\partial x^4} + \cdots = 0\,, \tag{1}$$

where α, β and γ are arbitrary constants.

In order to solve Eq. (1) easily, we introduce a new transformation of the following form

$$u = \frac{\partial v}{\partial x}\,, \qquad v = v(y)\,, \qquad y = y(x,t)\,, \tag{2}$$

where $v(y)$ and $y(x,t)$ are two trial functions.

To begin with, let us determine the trial function $y(x,t)$. As is well known, the solutions of NWEs should contain the phase factor $(kx - \omega t)$. Consequently, we directly choose the trial function $y(x,t)$ in the following form

$$y = \mathrm{e}^{(kx-\omega t)}\,, \tag{3}$$

in which k and ω are the wave number and angular frequency, respectively.

As for the other trial function, $v(y)$, which is not of a given form but varies from equation to equation, it must be flexibly selected according to the specific NWE. After determining the trial functions $v(y)$ and $y(x,t)$, Eq. (1) can be easily solved. In what follows, we shall make use of the technique stated above to solve three physically important NWEs, namely the Burgers equation, the KdV equation as well as the Burgers-KdV equation, and acquire their explicit and exact travelling solutions.

3. *Applications*

3.1. *Burgers equation*

It is commonly known that the Burgers equation is of the following general form

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} - \alpha\frac{\partial^2 u}{\partial x^2} = 0\,, \tag{4}$$

which was first proposed by Burgers [21] to study issues of turbulence. For Eq. (4), we select the trial function in the following form

$$v(y) = a\ln(b + y^n)\,, \tag{5}$$

where a, b and n are constants to be determined later.

Based on Eqs. (2) and (3), together with Eq. (5), it is an easy exercise to show that

$$u = \frac{\partial v}{\partial x} = \frac{akny^n}{b+y^n}\,, \tag{6}$$

$$\frac{\partial u}{\partial t} = \frac{abkn^2\omega y^n}{(b+y^n)^2}\,, \tag{7}$$

$$\frac{\partial u}{\partial x} = \frac{abk^2n^2y^n}{(b+y^n)^2}\,, \tag{8}$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{abk^3n^3y^n(b-y^n)}{(b+y^n)^3}\,. \tag{9}$$

Substituting Eqs. (6) – (9) into Eq. (4), and collecting the coefficients of powers of y with the aid of powerful Mathematica, then setting each of the obtained coefficients to zero, gives rise to a set of algebraic equations with respect to the unknown constants a, b and n as follows

$$-ab^2k^3n^3\alpha - ab^2kn^2\omega = 0\,, \tag{10}$$

$$a^2bk^3n^3 + abk^3n^3\alpha - abkn^2\omega = 0\,. \tag{11}$$

From Eqs. (10) and (11), we find

$$a = -2\alpha, \qquad \omega = -\alpha k^2 n, \qquad b = \textit{arbitrary constant}\,, \tag{12}$$

and we can also find the wave velocity as follows

$$c = \frac{\omega}{k} = -\alpha k n\,. \tag{13}$$

Putting Eq. (12) into Eq. (6) and considering Eq. (3) as well as Eq. (13), we obtain the general travelling-wave solution of the Burgers equation (4) in the following form

$$u = -\frac{2\alpha kn e^{nk(x-ct)}}{b + e^{nk(x-ct)}} + c + \alpha kn \,, \tag{14}$$

with four arbitrary constants b, c, k and n.

Making use of the following identity

$$\frac{e^x}{e^x + 1} = \frac{1}{2}\left(1 + \tanh\frac{x}{2}\right) \,, \tag{15}$$

and setting $b = 1$ in Eq. (14), we get the so-called kink-type solitary-wave solution to the Burgers equation (4) as follows

$$u = -\alpha kn \tanh\frac{nk}{2}(x - ct) + c \,. \tag{16}$$

If choosing $b = -1$ in Eq. (14) and making use of the following identity

$$\frac{e^x}{e^x - 1} = \frac{1}{2}\left(1 + \coth\frac{x}{2}\right), \tag{17}$$

then we acquire the singular travelling-wave solution to the Burgers equation (4) as follows

$$u = -\alpha kn \coth\frac{nk}{2}(x - ct) + c \,, \tag{18}$$

Making use of the following identity

$$\tanh\frac{x}{2} = \frac{\sinh x}{\cosh x + 1} \,, \tag{19}$$

then Eq. (16) can be changed to

$$u = -\alpha kn \frac{\sinh nk(x - ct)}{\cosh nk(x - ct) + 1} + c \,. \tag{20}$$

Making use of the following identity

$$\tanh\frac{x}{2} = \frac{\cosh x - 1}{sinhx} \,, \tag{21}$$

then Eq. (16) can be transformed to

$$u = -\alpha kn \frac{\cosh nk(x - ct) - 1}{\sinh nk(x - ct)} + c \,. \tag{22}$$

Making use of the following identity

$$\coth\frac{x}{2} = \frac{\sinh x}{\cosh x - 1}\,, \tag{23}$$

then Eq. (18) can be rewritten as

$$u = -\alpha kn\frac{\sinh nk(x-ct)}{\cosh nk(x-ct) - 1} + c\,. \tag{24}$$

Making use of the following identity

$$\coth\frac{x}{2} = \frac{\cosh x + 1}{\sinh x}\,, \tag{25}$$

then Eq. (18) can be reduced to

$$u = -\alpha kn\frac{\cosh nk(x-ct) + 1}{\sinh nk(x-ct)} + c\,. \tag{26}$$

Let

$$k = \mathrm{i}k'\,, \tag{27}$$

where i is the imaginary unit and k' a constant.

Making use of the following two identities

$$\tanh(\mathrm{i}x) = \mathrm{i}\tan x, \qquad \coth(\mathrm{i}x) = -\mathrm{i}\cot x\,, \tag{28}$$

then Eq. (16) and Eq. (18) can be simplified as

$$u = \alpha k' n \tan\frac{nk'}{2}(x-ct) + c\,, \tag{29}$$

$$u = -\alpha k' n \cot\frac{nk'}{2}(x-ct) + c\,, \tag{30}$$

which are two triangle-function periodic wave solutions to the Burgers equation (4).

The above solutions can be regarded as the general form solutions to the Burgers equation (4). In the following, let us take into account a special case, and for brevity, only some new solutions are listed as follows

$$u = -\alpha k\frac{\sinh k(x-ct) - 1}{\cosh k(x-ct)} + c\,, \tag{31}$$

$$u = -\alpha k\frac{\cosh k(x-ct) + 1}{\sinh k(x-ct)} + c\,. \tag{32}$$

3.2. KdV equation

The celebrated KdV equation under consideration reads

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + \beta\frac{\partial^3 u}{\partial x^3} = 0\,, \tag{33}$$

which can be used to describe the behaviour of long waves in shallow water [2].

For Eq. (33), we select the trial function $v(y)$ in the following form

$$v(y) = \frac{ay^n}{b + y^n}\,, \tag{34}$$

where a, b and n are constants to be determined later.

From Eq. (2) together with Eq. (3) as well as Eq. (34), it is not difficult to deduce that

$$u = \frac{abkny^n}{(b + y^n)^2}\,, \tag{35}$$

$$\frac{\partial u}{\partial t} = \frac{abkn^2\omega y^n(-b + y^n)}{(b + y^n)^3}\,, \tag{36}$$

$$\frac{\partial u}{\partial x} = \frac{abk^2n^2y^n(b - y^n)}{(b + y^n)^3}\,, \tag{37}$$

$$\frac{\partial^3 u}{\partial x^3} = \frac{abk^4n^4y^n(b^3 - 11b^2y^n + 11by^{2n} - y^{3n})}{(b + y^n)^5}\,. \tag{38}$$

Inserting Eqs. (35) – (38) into Eq. (33), and collecting the coefficients of powers of y with the aid of Mathematica, then letting each of the obtained coefficients to zero, leads to a system of algebraic equations with regard to the unknown constants a, b and n as follows

$$ab^4k^4n^4\beta - ab^4kn^2\omega = 0\,, \tag{39}$$

$$a^2b^3k^3n^3 - 11ab^3k^4n^4\beta - ab^3kn^2\omega = 0\,, \tag{40}$$

Solving the above system of equations, we obtain

$$a = 12k\beta n, \qquad \omega = k^3\beta n^2, \qquad b = \mathit{arbitrary\ constant}\,, \tag{41}$$

from which it follows immediately that

$$c = k^2\beta n^2\,, \tag{42}$$

Putting Eq. (41) into Eq. (35) and utilizing Eq. (3) as well as Eq. (42), we have the general travelling-wave solution for the KdV equation (33) as follows

$$u = \frac{12b\beta k^2 n^2 \mathrm{e}^{nk(x-ct)}}{(b+\mathrm{e}^{nk(x-ct)})^2} + c - \beta k^2 n^2 \,, \tag{43}$$

with four arbitrary constants b, c, k and n.

Making use of the following identity

$$\frac{\mathrm{e}^x}{\mathrm{e}^{2x}+1} = \frac{1}{2}\mathrm{sech} x \,, \tag{44}$$

and setting $b = 1$ in Eq. (43), we obtain the famous bell-type solitary-wave solution to the KdV equation (33) as follows

$$u = 3\beta k^2 n^2 \mathrm{sech}^2 \left[\frac{nk}{2}(x-ct)\right] + c - \beta k^2 n^2 \,. \tag{45}$$

Similarly, making use of the following identity

$$\frac{\mathrm{e}^x}{\mathrm{e}^{2x}-1} = \frac{1}{2}\mathrm{csch}\, x \,, \tag{46}$$

and setting $b = -1$ in Eq. (43), we have the singular travelling-wave solution to the KdV equation (33) as follows

$$u = -3\beta k^2 n^2 \mathrm{csch}^2 \left[\frac{nk}{2}(x-ct)\right] + c - \beta k^2 n^2 \,. \tag{47}$$

Making use of the following identity

$$\mathrm{sech}^2 \frac{x}{2} = \frac{1}{\cosh x + 1} \,, \tag{48}$$

Eq. (45) can be converted to

$$u = 6\beta k^2 n^2 \, \frac{1}{\cosh nk(x-ct) + 1} + c - \beta k^2 n^2 \,. \tag{49}$$

Making use of the following identity

$$\mathrm{csch}^2 \frac{x}{2} = \frac{2}{\cosh x - 1} \,, \tag{50}$$

Eq. (47) can be rewritten as

$$u = -6\beta k^2 n^2 \, \frac{1}{\cosh nk(x-ct) - 1} + c - \beta k^2 n^2 \,. \tag{51}$$

In view of Eq. (27) and making use of the following identities

$$\operatorname{sech}(\mathrm{i}x) = \sec x, \qquad \operatorname{csch}(\mathrm{i}x) = -\mathrm{i}\csc x\,, \tag{52}$$

Eq. (45) and Eq. (47) can be simplified as

$$u = -3\beta k'^2 n^2 \sec^2\left[\frac{nk'}{2}(x-ct)\right] + c + \beta k'^2 n^2\,, \tag{53}$$

$$u = -3\beta k'^2 n^2 \csc^2\left[\frac{nk'}{2}(x-ct)\right] + c + \beta k'^2 n^2\,, \tag{54}$$

which are two triangle function periodic wave solutions to the KdV equation (33).

The above solutions can be thought of as the general-form solutions to the KdV equation (33). In what follows, let us take into consideration the special case $n = 2$, and for simplicity, only some new solutions are given in the following

$$u = 24\beta k^2 \frac{1}{\cosh 2k(x-ct) + 1} + c - 4\beta k^2\,, \tag{55}$$

$$u = -24\beta k^2 \frac{1}{\cosh 2k(x-ct) - 1} + c - 4\beta k^2\,. \tag{56}$$

3.3. *Burgers-KdV equation*

The famous Burgers-KdV equation reads

$$\frac{\partial u}{\partial t} + \frac{\partial u}{\partial x} - \alpha\frac{\partial^2 u}{\partial x^2} + \beta\frac{\partial^3 u}{\partial x^3} = 0\,, \tag{57}$$

which stems from many different physical contexts as a nonlinear model equation incorporating the effects of dispersion, dissipation and nonlinearity. Liu used it to model the inverse energy cascade and intermittent turbulence in which a dispersion effect is taken into consideration [22, 23].

For Eq. (57), we select the trial function $v(y)$ in the following form

$$v(y) = \frac{ay^n}{b + y^n} + d\ln(B + y^n)\,, \tag{58}$$

where a, b, d and n are constants to be determined later.

In view of Eqs. (2) and Eq. (3) as well as Eq. (58), and with the aid of Mathematica, it is not hard to derive that

$$u = \frac{kny^n(ab + bd + dy^n)}{(b + y^n)^2}\,, \tag{59}$$

$$\frac{\partial u}{\partial t} = \frac{bkn^2\omega y^n(-ab-bd+ay^n-dy^n)}{(b+y^n)^3}\,, \tag{60}$$

$$\frac{\partial u}{\partial x} = \frac{bk^2n^2y^n(ab+bd-ay^n+dy^n)}{(b+y^n)^3}\,, \tag{61}$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{bk^3n^3y^n(ab^2+b^2d-4aby^n+ay^{2n}-dy^{2n})}{(b+y^n)^4}\,, \tag{62}$$

$$\frac{\partial^3 u}{\partial x^3} = \frac{bk^4n^4y^n(ab^3+b^3d-11ab^2dy^n-3b^2dy^n+11aby^{2n}-3bdy^{2n}-ay^{3n}+dy^{3n})}{(b+y^n)^5}\,. \tag{63}$$

Putting Eqs. (59) − (63) into Eq. (57), and collecting the coefficients of the same powers of y with the help of Mathematica, then setting each of the obtained coefficients to zero, brings about a set of over-determined algebraic equations with respect to the unknown constants a, b, d and n as follows

$$-ab^4k^3n^3\alpha - b^4dk^3n^\alpha + ab^4k^4n^4\beta - ab^4kn^2\omega - b^4dkn^2\omega = 0\,, \tag{64}$$

$$a^2b^3k^3n^3 + 2ab^2dk^3n^3 + b^3d^2k^3n^3 + 3ab^3k^3n^3\alpha - b^3dk^3n^3\alpha$$
$$-11ab^3k^4n^4\beta - 3b^3dk^4n^4\beta - ab^3kn^2\omega - 3b^3dkn^2 = 0\,, \tag{65}$$

$$-a^2b^2k^3n^3 + ab^2dk^3n^3 + 2b^2d^2k^3n^3 + 3ab^2k^3n^3\alpha + b^2dk^3n^3\alpha$$
$$+11ab^2k^4n^4\beta - 3b^2dk^4n^4\beta + ab^2kn^2\omega - 3b^2dkn^2\omega = 0\,, \tag{66}$$

$$-abdk^3n^3+bd^2k^3n^3-abk^3n^3\alpha+bdk^3n^3\alpha-abk^4n^4\beta+bdk^4n^4\beta-abkn^2\omega-bdkn^2\omega = 0\,. \tag{67}$$

Solving the above system of equations with the help of Mathematica, we get

$$a = -\frac{12}{5}\alpha,\;\; d = -\frac{12}{5}\alpha,\;\; k = -\frac{\alpha}{5n\beta},\;\; \omega = \frac{6\alpha^3}{125n\beta^2},\;\; b = \mathit{arbitrary\ constant}\,, \tag{68}$$

from which it follows immediately that

$$c = \frac{6\alpha^2}{25\beta}\,. \tag{69}$$

Substituting Eq. (68) into Eq. (59) and taking account of Eq. (3) as well as Eq. (69), we obtain the general travelling-wave solution to the Burgers-KdV equation (57) as follows

$$u = \frac{12\alpha^2\left[1+2b\mathrm{e}^{(\alpha/5\beta)(x-ct)}\right]}{25\beta^2\left[1+b\mathrm{e}^{(\alpha/5\beta)(x-ct)}\right]^2} + c - \frac{6\alpha^2}{25\beta}\,, \tag{70}$$

with two arbitrary constants b and c.

Setting $b = 1$ in Eq. (70), and making use of the identity (15), we obtain the solitary-wave solution to the Burgers-KdV equation (57) as follows

$$u = -\frac{3\alpha^2}{25\beta}\left[1 + \tanh\frac{\alpha}{10\beta}(x - ct)\right]^2 + c + \frac{6\alpha^2}{25\beta}\,. \tag{71}$$

Similarly, taking $b = -1$ in Eq. (70) and making use of the identity (17), we get the singular travelling-wave solution to the Burgers-KdV equation (57) as follows

$$u = -\frac{3\alpha^2}{25\beta}\left[1 + \coth\frac{\alpha}{10\beta}(x - ct)\right]^2 + c + \frac{6\alpha^2}{25\beta}\,. \tag{72}$$

Making use of the identity Eq. (19) and the following identity

$$\sinh^2 x = \cosh^2 x - 1\,, \tag{73}$$

then Eq. (71) can be rewritten as

$$u = \frac{6\alpha^2}{25\beta\left[\cosh\frac{\alpha}{5\beta}(x - ct) + 1\right]} - \frac{6\alpha^2\sinh\frac{\alpha}{5\beta}(x - ct)}{25\beta\left[\cosh\frac{\alpha}{5\beta}(x - ct) + 1\right]} + c\,. \tag{74}$$

Likewise, making use of the identity Eq. (21) and Eq. (73), then Eq. (71) can be transformed to

$$u = \frac{6\alpha^2}{25\beta\left[\cosh\frac{\alpha}{5\beta}(x - ct) + 1\right]} - \frac{6\alpha^2\left[\cosh\frac{\alpha}{5\beta}(x - ct) - 1\right]}{25\beta\sinh\frac{\alpha}{5\beta}(x - ct)} + c\,. \tag{75}$$

Similarly, making use of the identities Eq. (23) and Eq. (73), then Eq. (72) can be converted to

$$u = -\frac{6\alpha^2}{25\beta\left[\cosh\frac{\alpha}{5\beta}(x - ct) - 1\right]} - \frac{6\alpha^2\sinh\frac{\alpha}{5\beta}(x - ct)}{25\beta\left[\cosh\frac{\alpha}{5\beta}(x - ct) - 1\right]} + c\,. \tag{76}$$

Likewise, making use of the identities Eq. (25) and Eq. (73), then Eq. (72) can be changed to

$$u = -\frac{6\alpha^2}{25\beta\left[\cosh\frac{\alpha}{5\beta}(x - ct) - 1\right]} - \frac{6\alpha^2\left[\cosh\frac{\alpha}{5\beta}(x - ct) + 1\right]}{25\beta\sinh\frac{\alpha}{5\beta}(x - ct)} + c\,. \tag{77}$$

Let

$$\frac{\alpha}{10\beta} = \mathrm{i}k'\,, \tag{78}$$

and making use of the identity (28), then Eq. (71) and Eq. (72) can be simplified as

$$u = -\frac{6\alpha k'}{5}[1 + \mathrm{i}\tan k'(x - ct)]^2\mathrm{i} + c + \frac{12\alpha k'}{5}\mathrm{i}\,, \tag{79}$$

$$u = -\frac{6\alpha k'}{5}[1 - \mathrm{i}\cot k'(x - ct)]^2\mathrm{i} + c + \frac{12\alpha k'}{5}\mathrm{i}\,, \tag{80}$$

which are two complex-line solutions for the Burgers-KdV equation (57).

Evidently, the solutions (71) and (72) are in complete agreement with those obtained in Ref. [24]. The solutions (74) and (76) can be seen in Ref. [25]. But the rest of the solutions, to our knowledge, are not found in the literature. Finally, it should be remarked that if "c" in the each of the above solutions is replaced by "$-c$", then we can obtain many other explicit and exact travelling wave solutions to the Burgers equation, KdV equation and Burgers-KdV equation. Here we omit them for the reason of brevity.

4. *Conclusions*

To summarize, by introducing a new transformation and using the trial-function method, abundant explicit and exact travelling-wave solutions to the Burgers equation, KdV equation and Burgers-KdV equation, such as the solitary-wave solutions, the singular travelling-wave solutions, and the triangle-function periodic wave solutions are obtained. Among them, some are new travelling-wave solutions. Because these three equations are real physical models, much attention has been paid to solve them by many authors, and quite a few papers have studied their travelling-wave solutions, but the results obtained in this paper are more abundant than others. In addition, compared with the proposed approaches in the literature, the above described technique appears to be advantageous.

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RJEŠENJA KLASE NELINEARNIH VALNIH JEDNADŽBI

Burgersova, KdV-ova i Burgers-KdV-ova jednadžba opisuju stvarne fizičke sustave u mnogim granama fizike. Uvođenjem nove transformacije i primjenom metode pokusnih funkcija za rješavanje tih triju jednadžbi, u ovom se radu izvodi niz općenitih, eksplicitnih i egzaktnih rješenja, koja uključuju rješenja za solitarne valove, singularne putujuće valove i trokutne periodičke valove. Među njima neka su rješenja nova.