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MEDIUM-ASSISTED VACUUM FORCE

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Dedicated to the memory of Professor Vladimir Šips

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We discuss some implications of a very recently obtained result for the force on a slab in a planar cavity based on the calculation of the vacuum Lorentz force [C. Raabe and D.-G. Welsch, Phys. Rev. A **71** (2005) 013814]. We demonstrate that, according to this formula, the total force on the slab consists of a medium-screened Casimir force and, in addition to it, a medium-assisted force. The sign of of the medium-assisted force is determined solely by the properties of the cavity mirrors. In the Lifshitz configuration, this force is proportional to $1/d$ at small distances and is very small compared with the corresponding van der Waals force. At large distances, however, it is proportional to $1/d^4$ and comparable with the Casimir force, especially for denser media. The exponents in these power laws decrease by 1 in the case of a thin slab. The formula for the medium-assisted force also describes the force on a layer of the cavity medium, which has similar properties. For dilute media, it implies an atom-mirror interaction of the Coulomb type at small and of the Casimir–Polder type at large atom-mirror distances. For a perfectly reflecting mirror, the latter force is effectively only three-times smaller than the Casimir–Polder force.

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1. *Introduction*

A number of approaches to the Casimir effect [1] in material systems lead to the conclusion that the Casimir force on the medium between two bodies (mirrors) vanishes and that the only existing force is that between the mirrors [2 – 4] (see also text books [5, 6] and references therein). It is well known, however, that an atom (or a molecule) in the vicinity of a mirror experiences the Casimir–Polder force [7] and, at smaller distances, its nonretarded counterpart, the van der Waals force. Consequently, being a collection of atoms, every piece of a medium in front of a mirror should experience the corresponding force. To resolve this puzzling situation and overcome the above "unphysical" result, usually derived by calculating the Minkowski stress tensor [2,4] but also obtained using other methods [2,3,5,6], Raabe and Welsch [8] very recently suggested an approach based on the calculation of the vacuum Lorentz force (see also Ref. [9]). In this approach, the force on a body is simply the sum of the Lorentz forces acting on its constituents. Evidently, this should lead to a nonzero force on the medium between the mirrors.

As an application of their approach, Raabe and Welsch calculated the force on a magnetodielectric slab in a magnetodielectric planar cavity. The aim of this work is to demonstrate several straightforward implications of their formula. The paper is organized as follows. For completeness, in Sec. 2 we (re)derive the Raabe and Welsch formula and demonstrate that, according to it, the force on the slab naturally splits into two rather different components: a medium-screened and a medium-assisted force. The latter force, being genuinely related to the Lorentz-force approach, is discussed in more detail in Sec. 3. Our conclusions are summarized in Sec. 4. The necessary mathematical background is given in the Appendices.

2. *Preliminaries*

Consider a multilayered system described by permittivity $\varepsilon(\boldsymbol{r},\omega) = \varepsilon'(\boldsymbol{r},\omega) + \mathrm{i}\varepsilon''(\boldsymbol{r},\omega)$ and permeability $\mu(\boldsymbol{r},\omega) = \mu'(\boldsymbol{r},\omega) + \mathrm{i}\mu''(\boldsymbol{r},\omega)$ defined in a stepwise fashion, as depicted in Fig. 1. The force per unit area acting on a stack of layers between a plane z in a jth layer and a plane z' in an $l > j$ layer is then given by

$$f_{jl}(z,z') = \widetilde{T}_{l,zz}(z') - \widetilde{T}_{j,zz}(z), \tag{1}$$

where $\widetilde{\overleftrightarrow{\boldsymbol{T}}}_j \equiv \overleftrightarrow{\boldsymbol{T}}_j - \overleftrightarrow{\boldsymbol{T}}^0_j$, with $\overleftrightarrow{\boldsymbol{T}}_j$ being the corresponding stress tensor and $\overleftrightarrow{\boldsymbol{T}}^0_j$ its infinite-medium counterpart.

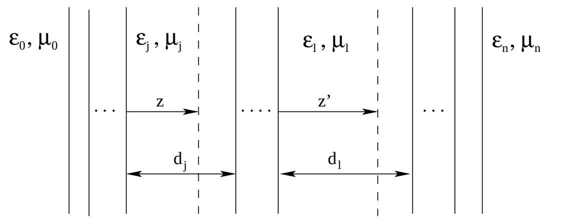

Fig. 1. System considered schematically. The dashed lines represent the planes where the stress tensor is calculated.

2.1. *Stress tensor*

The Lorentz-force approach to the Casimir effect eventually leads to the calculation of the stress tensor (component) [8,9]

$$T_{j,zz}(z) = \frac{1}{8\pi}\left\langle E_z E_z - \boldsymbol{E}_\parallel \cdot \boldsymbol{E}_\parallel + B_z B_z - \boldsymbol{B}_\parallel \cdot \boldsymbol{B}_\parallel \right\rangle_{\boldsymbol{r}\in(j)}, \tag{2}$$

where the brackets denote the average over the vacuum state of the field. The correlation functions that appear here can be straightforwardly calculated using the fluctuation-dissipation theorem [10,11]. Decomposing the field operators into the positive-frequency and negative-frequency parts according to

$$\boldsymbol{E}(\boldsymbol{r},t) = \int_0^\infty \mathrm{d}\omega\, \boldsymbol{E}(\boldsymbol{r},\omega)\mathrm{e}^{-\mathrm{i}\omega t} + \int_0^\infty \mathrm{d}\omega\, \boldsymbol{E}^\dagger(\boldsymbol{r},\omega)\mathrm{e}^{\mathrm{i}\omega t}, \tag{3}$$

we have (in the dyadic form) [10]

$$\left\langle \boldsymbol{E}(\boldsymbol{r},\omega)\boldsymbol{E}^\dagger(\boldsymbol{r}',\omega')\right\rangle = \frac{\hbar}{\pi}\frac{\omega^2}{c^2}\mathrm{Im}\overleftrightarrow{\boldsymbol{G}}(\boldsymbol{r},\boldsymbol{r}';\omega)\delta(\omega-\omega'), \tag{4}$$

and the magnetic-field correlation function is obtained from this expression using $\boldsymbol{B}(\boldsymbol{r},\omega) = (-\mathrm{i}c/\omega)\nabla\times\boldsymbol{E}(\boldsymbol{r},\omega)$. Here $\overleftrightarrow{\boldsymbol{G}}(\boldsymbol{r},\boldsymbol{r}';\omega)$ is the classical Green function satisfying

$$\left[\nabla\times\frac{1}{\mu(\boldsymbol{r},\omega)}\nabla\times -\varepsilon(\boldsymbol{r},\omega)\frac{\omega^2}{c^2}\overleftrightarrow{\boldsymbol{I}}\cdot\right]\overleftrightarrow{\boldsymbol{G}}(\boldsymbol{r},\boldsymbol{r}';\omega) = 4\pi\overleftrightarrow{\boldsymbol{I}}\delta(\boldsymbol{r}-\boldsymbol{r}'), \tag{5}$$

with the outgoing wave condition at the infinity. Applying these results to the jth layer, for the relevant correlation functions we find

$$\langle\boldsymbol{E}(\boldsymbol{r},t)\boldsymbol{E}(\boldsymbol{r},t)\rangle_{\boldsymbol{r}\in(j)} = \frac{\hbar}{\pi}\,\mathrm{Im}\int_0^\infty \mathrm{d}\omega\,\frac{\omega^2}{c^2}\overleftrightarrow{\boldsymbol{G}}_j(\boldsymbol{r},\boldsymbol{r};\omega), \tag{6a}$$

$$\langle\boldsymbol{B}(\boldsymbol{r},t)\boldsymbol{B}(\boldsymbol{r},t)\rangle_{\boldsymbol{r}\in(j)} = \frac{\hbar}{\pi}\,\mathrm{Im}\int_0^\infty \mathrm{d}\omega\,\overleftrightarrow{\boldsymbol{G}}{}_j^B(\boldsymbol{r},\boldsymbol{r};\omega), \tag{6b}$$

where $\overleftrightarrow{\boldsymbol{G}}_j(\boldsymbol{r},\boldsymbol{r}';\omega)$ is the Green function element for $\boldsymbol{r}$ and $\boldsymbol{r}'$ both in the layer j, and

$$\overleftrightarrow{\boldsymbol{G}}{}_j^B(\boldsymbol{r},\boldsymbol{r}';\omega) = \nabla\times\overleftrightarrow{\boldsymbol{G}}_j(\boldsymbol{r},\boldsymbol{r}';\omega)\times\overleftarrow{\nabla}' \tag{7}$$

is the corresponding Green function element for the magnetic field.

With the above equations inserted in Eq. (2), the stress tensor $\widetilde{T}_{j,zz}$ is formally obtained by replacing the Green function with its scattering part

$$\overleftrightarrow{\boldsymbol{G}}{}_j^{\mathrm{sc}}(\boldsymbol{r},\boldsymbol{r}';\omega) = \overleftrightarrow{\boldsymbol{G}}_j(\boldsymbol{r},\boldsymbol{r}';\omega) - \overleftrightarrow{\boldsymbol{G}}{}_j^0(\boldsymbol{r},\mathbf{r}';\omega), \tag{8}$$

where $\overleftrightarrow{\boldsymbol{G}}{}_j^0(\boldsymbol{r},\boldsymbol{r}';\omega)$ is the infinite-medium Green function. In this way, from Eq. (2) we have

$$\begin{aligned}\widetilde{T}_{j,zz}(z) = \frac{\hbar}{4\pi}\,\mathrm{Im}\int_0^\infty \frac{\mathrm{d}\omega}{2\pi}\Big\{&\frac{\omega^2}{c^2}\left[G^{\mathrm{sc}}_{j,zz}(\boldsymbol{r},\boldsymbol{r};\omega) - G^{\mathrm{sc}}_{j,\parallel}(\boldsymbol{r},\boldsymbol{r};\omega)\right]\\ &+G^{B,\mathrm{sc}}_{j,zz}(\boldsymbol{r},\boldsymbol{r};\omega) - G^{B,\mathrm{sc}}_{j,\parallel}(\boldsymbol{r},\boldsymbol{r};\omega)\Big\},\end{aligned} \tag{9}$$

where $G^{\rm sc}_{j,\|}(\boldsymbol{r},\boldsymbol{r}';\omega) = G^{\rm sc}_{j,xx}(\boldsymbol{r},\boldsymbol{r}';\omega) + G^{\rm sc}_{j,yy}(\boldsymbol{r},\boldsymbol{r}';\omega)$. In Appendix A, we derive the Green function $\overleftrightarrow{\boldsymbol{G}}^{\rm sc}_j(\boldsymbol{r},\boldsymbol{r}';\omega)$ for a magnetodielectric multilayer and, in Appendix B, we calculate the expression in the curly brackets of the above equation. We find that

$$\{\ldots\} = -2\pi{\rm i}\mu_j \int \frac{{\rm d}^2\boldsymbol{k}}{(2\pi)^2}\frac{1}{\beta_j}\sum_{q=p,s} g_{qj}(\omega,k;z)\,, \tag{10}$$

where $\boldsymbol{k}$ and $\beta_j(\omega,k) = \sqrt{n_j^2(\omega)\omega^2/c^2 - k^2}$, with $n_j(\omega) = \sqrt{\varepsilon_j(\omega)\mu_j(\omega)}$, are, respectively, the parallel and the perpendicular component of the wave vector in the layer, and the functions $g_{qj}(\omega,k;z)$ are in the shifted-z representation (see Appendix A) given by

$$g_{qj}(\omega,k;z) = \frac{2r^q_{j-}r^q_{j+}{\rm e}^{2{\rm i}\beta_j d_j}}{D_{qj}}\left[\beta_j^2(1+n_j^{-2}) + \Delta_q k^2(1-n_j^{-2})\right] \tag{11}$$

$$+\Delta_q\frac{r^q_{j-}{\rm e}^{2{\rm i}\beta_j z} + r^q_{j+}{\rm e}^{2{\rm i}\beta_j(d_j-z)}}{D_{qj}}(\beta_j^2+k^2)(1-n_j^{-2}), \quad 0\le z\le d_j.$$

Here $\Delta_q = \delta_{qp} - \delta_{qs}$,

$$D_{qj}(\omega,k) = 1 - r^q_{j-}r^q_{j+}{\rm e}^{2{\rm i}\beta_j d_j}, \tag{12}$$

and $r^q_{j\pm}(\omega,k)$ are the reflection coefficients of the right and left stack bounding the layer, respectively. Specially, noting that $r^q_{0-} = r^q_{n+} = 0$ and recalling that $d_0 = 0$ (see Appendix A), for the outmost (semi-infinite) layers we have

$$g_{q0}(\omega,k;z) = \Delta_q r^q_{0+}{\rm e}^{-2{\rm i}\beta_0 z}(\beta_0^2+k^2)(1-n_0^{-2}), \quad -\infty < z \le 0, \tag{13a}$$

$$g_{qn}(\omega,k;z) = \Delta_q r^q_{n-}{\rm e}^{2{\rm i}\beta_n z}(\beta_n^2+k^2)(1-n_n^{-2}), \quad 0 \le z < \infty. \tag{13b}$$

Converting the integral over the real ω-axis in Eq. (9) to that along the imaginary ω-axis in the usual way, letting $\omega = {\rm i}\xi$,

$$\beta_j({\rm i}\xi,k) \equiv {\rm i}\kappa_j(\xi,k) = {\rm i}\sqrt{n_j^2({\rm i}\xi)\frac{\xi^2}{c^2} + k^2}, \tag{14}$$

and noticing the reality of the integrand, we finally obtain for the stress tensor in the layer [8]

$$\widetilde{T}_{j,zz}(z) = -\frac{\hbar}{8\pi^2}\int_0^\infty {\rm d}\xi\,\mu_j\int_0^\infty \frac{k\,{\rm d}k}{\kappa_j}\sum_{q=p,s} g_{qj}({\rm i}\xi,k;z)\,. \tag{15}$$

As seen, the standard expression for the (Minkowski) stress tensor obtained with [12]

$$g^{\rm M}_{qj}({\rm i}\xi,k;z) = -4\kappa_j^2\frac{r^q_{j-}r^q_{j+}{\rm e}^{-2\kappa_j d_j}}{\mu_j D_{qj}} \tag{16}$$

is recovered from the above result only in the case of the empty space between the stacks, i.e., only if $\varepsilon_j(\omega) = \mu_j(\omega) = 1$. We also note that, according to Eq. (13), the stress tensor is discontinuous across the boundary between two semi-infinite media (in this case, 0 and n). This implies the existence of a force acting on a layer around the interface between the media ($f_{\rm int} \equiv f_{0n}(-a_0, a_n)$)

$$\begin{aligned} f_{\rm int} &= -\frac{\hbar}{8\pi^2 c^2}\int\limits_0^\infty {\rm d}\xi\,\xi^2 \int\limits_0^\infty {\rm d}k\,k\left[\frac{\mu_0}{\kappa_0}(n_0^2-1){\rm e}^{-2\kappa_0 a_0}\right. \\ &\qquad\qquad \left. +\frac{\mu_n}{\kappa_n}(n_n^2-1){\rm e}^{-2\kappa_n a_n}\right]\sum_{q=p,s}\Delta_q r_{0n}^q({\rm i}\xi,k;z), \end{aligned} \tag{17}$$

where $a_0 + a_n$ is the layer thickness and where we have used $r_{0+}^q = -r_{n-}^q = r_{0n}^q$ (Eq. (6a)). Since $\widetilde{T}_{zz}^{\rm M} = 0$ in semi-infinite layers, as follows from Eq. (16), such a force does not appear in the approach based on the calculation of the Minkowski stress tensor [13] and in other equivalent approaches leading to the Lifshitz-like expression [14] for the force.

2.2. *Force in a planar cavity*

Owing to the z-dependence of $\widetilde{T}_{j,zz}(z)$, Eqs. (11) and (15) imply the nonzero force on a slice of the medium between the stacks, contrary to the Lifshitz-like result (Eqs. (15) and (16)) obtained previously by many authors [2−6]. In order to calculate this force, we consider a slightly more general configuration consisting of a slab with refraction index $n_{\rm s}$ and thickness $d_{\rm s}$ embedded in a material cavity with refraction index n and length L, as depicted in Fig. 2. The cavity walls are conveniently described by the reflection coefficients r_1^q and r_2^q.

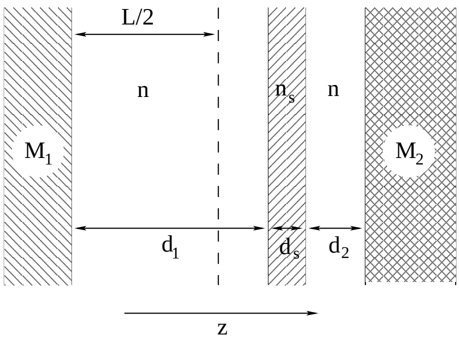

Fig. 2. A slab in a planar cavity shown schematically. The refraction index of the slab is $n_{\rm s}(\omega) = \sqrt{\varepsilon_{\rm s}(\omega)\mu_{\rm s}(\omega)}$ and that of the cavity $n(\omega) = \sqrt{\varepsilon(\omega)\mu(\omega)}$. The cavity walls are described by their reflection coefficients $r_1^q(\omega, k)$ and $r_2^q(\omega, k)$, with k being the in-plane wave vector of a wave. The arrow indicates the direction of the force on the slab.

According to Eqs. (1) and (15), the force on the slab $f_{\rm s} = \widetilde{T}_{2,zz}(0) - \widetilde{T}_{1,zz}(d_1)$ in this configuration is given by

$$f_{\rm s}(d_1, d_2) = -\frac{\hbar}{8\pi^2}\int\limits_0^\infty {\rm d}\xi\,\mu\int\limits_0^\infty \frac{{\rm d}k\,k}{\kappa}\sum_{q=p,s}\left[g_{q2}({\rm i}\xi, k; 0) - g_{q1}({\rm i}\xi, k; d_1)\right]. \tag{18}$$

The functions D_{q1} and D_{q2} (Eq. (12)) are straightforwardly obtained using Eq. (65) to determine the reflection coefficients at the right boundary of region 1 (r^q_{1+}) and the left boundary of region 2 (r^q_{2-}). With $r^q_{1-} = r^q_1$ and $r^q_{2+} = r^q_2$, we find

$$\begin{aligned} D_{q1} &= 1 - r_1^q\left(r^q + \frac{t^{q2} r_2^q {\rm e}^{2{\rm i}\beta d_2}}{1 - r^q r_2^q {\rm e}^{2{\rm i}\beta d_2}}\right){\rm e}^{2{\rm i}\beta d_1}, \\ D_{q2} &= 1 - \left(r^q + \frac{t^{q2} r_1^q {\rm e}^{2{\rm i}\beta d_1}}{1 - r^q r_1^q {\rm e}^{2{\rm i}\beta d_1}}\right) r_2^q {\rm e}^{2{\rm i}\beta d_2}. \end{aligned} \tag{19}$$

Here $r^q = r^q_{1/2} = r^q_{2/1}$ and $t^q = t^q_{1/2} = t^q_{2/1}$ are the Fresnel coefficients for the (whole) slab which are related through (Eq. (65))

$$r^q = \rho^q \frac{1 - {\rm e}^{2{\rm i}\beta_{\rm s} d_{\rm s}}}{1 - \rho^{q2}{\rm e}^{2{\rm i}\beta_{\rm s} d_{\rm s}}}, \qquad t^q = \frac{(1 - \rho^{q2}){\rm e}^{{\rm i}\beta_{\rm s} d_{\rm s}}}{1 - \rho^{q2}{\rm e}^{2{\rm i}\beta_{\rm s} d_{\rm s}}} \tag{20}$$

to the *single-interface* medium-slab Fresnel reflection coefficient $\rho^q = r^q_{1s} = r^q_{2s}$, given by (see Eq. (6a))

$$\rho^q = \frac{\beta - \gamma^q \beta_{\rm s}}{\beta + \gamma^q \beta_{\rm s}}, \qquad \gamma^p = \frac{\varepsilon}{\varepsilon_{\rm s}}, \qquad \gamma^s = \frac{\mu}{\mu_{\rm s}}. \tag{21}$$

This gives

$$\begin{aligned} g_{q2}(\omega, k; 0) - g_{q1}(\omega, k; d_1) = \Bigg\{ & 4\beta^2\left(\delta_{qs} + \frac{1}{n^2}\delta_{qp}\right) r^q \\ & + \frac{\omega^2}{c^2}(n^2 - 1)[(1 + r^q)^2 - t^{q2}]\Delta_q \Bigg\} \frac{r_2^q {\rm e}^{2{\rm i}\beta d_2} - r_1^q {\rm e}^{2{\rm i}\beta d_1}}{N^q}, \end{aligned} \tag{22}$$

where

$$N^q = 1 - r^q(r_1^q {\rm e}^{2{\rm i}\beta d_1} + r_2^q {\rm e}^{2{\rm i}\beta d_2}) + (r^{q2} - t^{q2}) r_1^q r_2^q {\rm e}^{2{\rm i}\beta(d_1 + d_2)}. \tag{23}$$

Combining Eqs. (18) and (22), we see that f_s naturally splits into two rather different components

$$f_{\rm s}(d_1, d_2) = f^{(1)}(d_1, d_2) + f^{(2)}(d_1, d_2), \tag{24}$$

where

$$f^{(1)}(d_1,d_2) = \frac{\hbar}{2\pi^2}\int_0^\infty \mathrm{d}\xi \int_0^\infty \mathrm{d}k\, k\kappa \sum_{q=p,s}\left(\mu\delta_{qs}+\frac{1}{\varepsilon}\delta_{qp}\right) r^q \frac{r_2^q \mathrm{e}^{-2\kappa d_2} - r_1^q \mathrm{e}^{-2\kappa d_1}}{N^q}, \tag{25}$$

and

$$f^{(2)}(d_1,d_2) = \frac{\hbar}{8\pi^2 c^2}\int_0^\infty \mathrm{d}\xi\, \xi^2 \mu(n^2-1)\int_0^\infty \frac{\mathrm{d}k\, k}{\kappa}\sum_{q=p,s}[(1+r^q)^2 - t^{q2}] \times \Delta_q \frac{r_2^q \mathrm{e}^{-2\kappa d_2} - r_1^q \mathrm{e}^{-2\kappa d_1}}{N^q}. \tag{26}$$

Equation (25) differs in two respects from the formula for the Casimir force in a dielectric cavity obtained through the Minkowski tensor calculation [4]. First, the Fresnel coefficients refer to a magnetodielectric system [12]. Another new feature in Eq. (25) is the (effective) screening of the force through the multiplication of the contributions coming from TE- and TM-polarized waves by μ and $1/\varepsilon$, respectively. This gives a simple recipe how to adapt the traditionally obtained formulas for the Casimir force to the Lorentz-force approach, as we illustrate below.

Clearly, $f^{(2)}$ owes its appearance to the cavity medium (note that it vanishes when $n = 1$) and is therefore a genuine consequence of the Lorentz force approach, so that below we consider this force in more detail.

3. *Medium-assisted force*

3.1. *Force on a slab*

Assuming, for simplicity, a large (semi-infinite) cavity obtained formally by letting $d_1 \to \infty$ (or $r_1^q = 0$), from Eq. (26), we have

$$f^{(2)}(d) = \frac{\hbar}{8\pi^2 c^2}\int_0^\infty \mathrm{d}\xi\, \xi^2 \mu(n^2-1)\int_0^\infty \frac{\mathrm{d}k\, k}{\kappa}\sum_{q=p,s}\Delta_q \frac{[(1+r^q)^2 - t^{q2}]R^q e^{-2\kappa d}}{1 - r^q R^q e^{-2\kappa d}}, \tag{27}$$

where we have changed the notation so that $d_2 \equiv d$ and $r_2^q \equiv R^q$. Another remarkable feature of the medium-assisted force is that its sign depends only on the properties of the mirror. Indeed, assuming an ideally reflecting mirror and letting $R^q = \pm\Delta_q$ (the minus sign is for an infinitely permeable mirror, see Eq. (38) below), we clearly see that $f^{(2)}$ is attractive or repulsive, depending on whether the mirror is (dominantly) conducting (dielectric) or permeable, irrespective of the properties of the slab.

3.1.1. **Small distances**

The integral over ξ in Eq. (27) effectively extends up to a frequency Ω beyond which the mirror becomes transparent. Accordingly, at small mirror-slab distances

$d \ll \Lambda = 2\pi c/\Omega$, the main contribution to $f^{(2)}$ comes from large k's ($k \sim 1/d$). In this region, the nonretarded (quasistatic) approximation applies to the integrand obtained formally by letting $\kappa = \kappa_l = k$ everywhere. Thus, for example, for a structureless mirror consisting of a semi-infinite medium with refraction index n_m, we have (from Eq. (6a))

$$R^p_{\mathrm{nr}\infty}(\mathrm{i}\xi, k) = \frac{\varepsilon_\mathrm{m} - \varepsilon}{\varepsilon_m + \varepsilon} \equiv \rho(\varepsilon_\mathrm{m}, \varepsilon), \qquad R^s_{\mathrm{nr}\infty}(\mathrm{i}\xi, k) = \frac{\mu_\mathrm{m} - \mu}{\mu_\mathrm{m} + \mu}, \tag{28}$$

and the nonretarded Fresnel coefficients of the slab are from Eq. (20) given by

$$r^q_\mathrm{nr}(\mathrm{i}\xi, k) = \rho^q_\mathrm{nr} \frac{1 - \mathrm{e}^{-2kd_\mathrm{s}}}{1 - [\rho^q_\mathrm{nr}]^2 \mathrm{e}^{-2kd_\mathrm{s}}}, \qquad t^q_\mathrm{nr}(\mathrm{i}\xi, k) = \frac{(1 - \rho^q_\mathrm{nr})^2 \mathrm{e}^{-kd_\mathrm{s}}}{1 - [\rho^q_\mathrm{nr}]^2 \mathrm{e}^{-2kd_\mathrm{s}}}, \tag{29}$$

with $\rho^p_\mathrm{nr} = \rho(\varepsilon_\mathrm{s}, \varepsilon)$ and $\rho^\mathrm{s}_\mathrm{nr} = \rho(\mu_\mathrm{s}, \mu)$ [see Eq. (21)]. With the substitution $u = 2kd$, this gives

$$f^{(2)}(d \ll \Lambda) = \frac{\hbar}{16\pi^2 c^2 d} \int_0^\infty \mathrm{d}\xi\, \xi^2 \mu(n^2 - 1) \int_0^\infty \mathrm{d}u \sum_{q=p,s} \Delta_q \frac{[(1 + r^q)^2 - t^{q2}]_\mathrm{nr} R^q_\mathrm{nr} \mathrm{e}^{-u}}{1 - r^q_\mathrm{nr} R^q_\mathrm{nr} \mathrm{e}^{-u}}, \tag{30}$$

where the (nonretarded) reflection coefficients are now functions of $(\mathrm{i}\xi, u/2d)$.

The medium-assisted force on a thick, $d_\mathrm{s} \to \infty$, slab at small distances is obtained from the above equation when letting $t^q_\mathrm{nr} = 0$ and $r^q_\mathrm{nr} = \rho^q_\mathrm{nr}$ (see Eq. (29)). Specially, in the case of a single-medium mirror, corresponding to the classical Lifshitz (L) configuration [14], all reflection coefficients in Eq. (30) are independent of u so that the entire dependence of $f^{(2)}$ on d is given by the factor in front of the integral. Using Eq. (28), in this case we find

$$f^{(2)}_\mathrm{L}(d \ll \Lambda; d_\mathrm{s} \gg d) = \frac{\hbar}{16\pi^2 c^2 d} \int_0^\infty \mathrm{d}\xi\, \xi^2 \mu(n^2 - 1) \int_0^\infty \mathrm{d}u \Bigg\{ \left(\frac{2\varepsilon_\mathrm{s}}{\varepsilon_\mathrm{s} + \varepsilon} \right)^2 \times \left[\frac{\varepsilon_\mathrm{m} + \varepsilon}{\varepsilon_\mathrm{m} - \varepsilon} \mathrm{e}^u - \frac{\varepsilon_\mathrm{s} - \varepsilon}{\varepsilon_\mathrm{s} + \varepsilon} \right]^{-1} - \left(\frac{2\mu_\mathrm{s}}{\mu_\mathrm{s} + \mu} \right)^2 \left[\frac{\mu_\mathrm{m} + \mu}{\mu_\mathrm{m} - \mu} e^u - \frac{\mu_\mathrm{s} - \mu}{\mu_\mathrm{s} + \mu} \right]^{-1} \Bigg\}. \tag{31}$$

We compare this with the screened Casimir force in the Lifshitz configuration which, by applying the recipe embodied in Eq. (25) directly to the Lifshitz formula [14], reads

$$\begin{aligned} f^{(1)}_\mathrm{L}(d \ll \Lambda; d_\mathrm{s} \gg d) \quad &= \quad \frac{\hbar}{16\pi^2 d^3} \int_0^\infty \mathrm{d}\xi \int_0^\infty \mathrm{d}u\, u^2 \Bigg\{ \frac{1}{\varepsilon} \left[\frac{\varepsilon_\mathrm{s} + \varepsilon}{\varepsilon_\mathrm{s} - \varepsilon} \frac{\varepsilon_\mathrm{m} + \varepsilon}{\varepsilon_\mathrm{m} - \varepsilon} \mathrm{e}^u - 1 \right]^{-1} \\ &\quad + \mu \left[\frac{\mu_\mathrm{s} + \mu}{\mu_\mathrm{s} - \mu} \frac{\mu_\mathrm{m} + \mu}{\mu_\mathrm{m} - \mu} \mathrm{e}^u - 1 \right]^{-1} \Bigg\}. \end{aligned} \tag{32}$$

If we scale the frequency in the above integrals with Ω, we see that $f_{\rm L}^{(2)}/f_{\rm L}^{(1)} \sim (\Omega d/c)^2 \ll 1$. Accordingly, the medium-assisted force at small distances is very small when compared with the screened Casimir force.

Of interest is also the medium-assisted force on a thin, $d_{\rm s} \ll d$, slab. From Eqs. (20) and (21), we find that to the first order in $\kappa_{\rm s} d_{\rm s}$

$$r^q(\mathrm{i}\xi,k) \simeq 2\rho^q \kappa_{\rm s} d_{\rm s}, \qquad [(1+r^q)^2 - t^{q2}](\mathrm{i}\xi,k) \simeq 2\frac{\kappa d_{\rm s}}{\gamma^q}. \tag{33}$$

Making here the nonretarded approximation ($\kappa_{\rm s} = \kappa = k$) and letting $k \to u/2d$, from Eq. (30) we find that to the first order in $d_{\rm s}/d$

$$\begin{aligned} f^{(2)}(d \ll \Lambda; d_{\rm s} \ll d) \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad (34)\\ = \frac{\hbar d_{\rm s}}{16\pi^2 c^2 d^2} \int_0^\infty \mathrm{d}\xi\, \xi^2 \mu (n^2-1) \int_0^\infty \mathrm{d}u\, u \mathrm{e}^{-u} \left[\frac{\varepsilon_{\rm s}}{\varepsilon} R^p_{\rm nr}(\mathrm{i}\xi, \frac{u}{2d}) - \frac{\mu_{\rm s}}{\mu} R^s_{\rm nr}(\mathrm{i}\xi, \frac{u}{2d}) \right], \end{aligned}$$

which, for a single-medium (s-m) mirror, reduces to

$$f^{(2)}_{\text{s-m}}(d \ll \Lambda; d_{\rm s} \ll d) = \frac{\hbar d_{\rm s}}{16\pi^2 c^2 d^2} \int_0^\infty \mathrm{d}\xi\, \xi^2 \mu (n^2-1) \left(\frac{\varepsilon_{\rm s}}{\varepsilon} \frac{\varepsilon_{\rm m} - \varepsilon}{\varepsilon_{\rm m} + \varepsilon} - \frac{\mu_{\rm s}}{\mu} \frac{\mu_{\rm m} - \mu}{\mu_{\rm m} + \mu} \right). \tag{35}$$

3.1.2. Large distances

To find $f^{(2)}$ for large d, we use the standard substitution $\kappa = n\xi p/c$ in Eq. (27). This gives

$$f^{(2)}(d) = \frac{\hbar}{8\pi^2 c^3} \int_0^\infty \mathrm{d}\xi\, \xi^3 \mu n (n^2-1) \int_1^\infty \mathrm{d}p \sum_{q=p,s} \Delta_q \frac{[(1+r^q)^2 - t^{q2}] R^q \mathrm{e}^{-2n\xi p d/c}}{1 - r^q R^q \mathrm{e}^{-2n\xi p d/c}}, \tag{36}$$

where the reflection coefficients as functions of $(\mathrm{i}\xi, p)$ are obtained from their $(\mathrm{i}\xi, k)$-counterparts by letting

$$\kappa_l \to n\frac{\xi}{c} s_l, \qquad s_l = \sqrt{p^2 - 1 + n_l^2/n^2} \tag{37}$$

for all relevant layers. Thus, for example, for a single-medium mirror we have (from (Eq. (6a))

$$R^p_\infty(\mathrm{i}\xi, p) = \frac{\varepsilon_{\rm m} p - \varepsilon s_{\rm m}}{\varepsilon_{\rm m} p + \varepsilon s_{\rm m}} \equiv \rho(\varepsilon_{\rm m}, \varepsilon; p), \qquad R^s_\infty(\mathrm{i}\xi, p) = \frac{\mu_{\rm m} p - \mu s_{\rm m}}{\mu_{\rm m} p + \mu s_{\rm m}}. \tag{38}$$

Now, since $p \geq 1$, for large d the contributions from the $\xi \simeq 0$ region dominate the integral in Eq. (36). Consequently, we may approximate the frequency-dependent quantities with their static values (which we denote by the subscript 0). With the substitution $v = 2n_0\xi pd/c$, this leads to

$$f^{(2)}(d \gg \Lambda) = \frac{\hbar c\mu_0(n_0^2-1)}{2^7\pi^2 n_0^3 d^4}\int_0^\infty \mathrm{d}v\, v^3 \int_1^\infty \frac{\mathrm{d}p}{p^4}\sum_{q=p,s}\Delta_q \frac{[(1+r^q)^2 - t^{q2}]_0 R_0^q \mathrm{e}^{-v}}{1-r_0^q R_0^q \mathrm{e}^{-v}}. \quad (39)$$

For the Lifshitz configuration ($t^q = 0$, $r^p = \rho(\varepsilon_\mathrm{s},\varepsilon;p)$, $r^\mathrm{s} = \rho(\mu_\mathrm{s},\mu;p)$ and $R^q = R^q_\infty$, see Eq. (38)), we now obtain

$$\begin{aligned} &f_\mathrm{L}^{(2)}(d \gg \Lambda; d_\mathrm{s} \gg d) \qquad (40)\\ &= \frac{\hbar c\mu_0(n_0^2-1)}{2^7\pi^2 n_0^3 d^4}\int_0^\infty \mathrm{d}v\, v^3 \int_1^\infty \frac{\mathrm{d}p}{p^4}\Bigg\{ \left(\frac{2\varepsilon_s p}{\varepsilon_\mathrm{s} p + \varepsilon s_\mathrm{s}}\right)_0^2 \left[\frac{\varepsilon_\mathrm{m} p + \varepsilon s_\mathrm{m}}{\varepsilon_\mathrm{m} p - \varepsilon s_\mathrm{m}}\mathrm{e}^v - \frac{\varepsilon_\mathrm{s} p - \varepsilon s_\mathrm{s}}{\varepsilon_\mathrm{s} p + \varepsilon s_\mathrm{s}}\right]_0^{-1} \\ &\qquad - \left(\frac{2\mu_\mathrm{s} p}{\mu_\mathrm{s} p + \mu s_\mathrm{s}}\right)_0^2 \left[\frac{\mu_\mathrm{m} p + \mu s_\mathrm{m}}{\mu_\mathrm{m} p - \mu s_\mathrm{m}}\mathrm{e}^v - \frac{\mu_\mathrm{s} p - \mu s_\mathrm{s}}{\mu_\mathrm{s} p + \mu s_\mathrm{s}}\right]_0^{-1}\Bigg\}, \end{aligned}$$

which is to be compared with the screened Casimir force at large distances [14]

$$\begin{aligned} &f_\mathrm{L}^{(1)}(d \gg \Lambda; d_\mathrm{s} \gg d) \qquad (41)\\ &= \frac{\hbar c}{2^5\pi^2 n_0 d^4}\int_0^\infty \mathrm{d}v\, v^3 \int_1^\infty \frac{\mathrm{d}p}{p^2}\Bigg\{ \frac{1}{\varepsilon_0}\left[\frac{\varepsilon_\mathrm{s} p + \varepsilon s_\mathrm{s}}{\varepsilon_\mathrm{s} p - \varepsilon s_\mathrm{s}}\frac{\varepsilon_\mathrm{m} p + \varepsilon s_\mathrm{m}}{\varepsilon_\mathrm{m} p - \varepsilon s_\mathrm{m}}\mathrm{e}^v - 1\right]_0^{-1} \\ &\qquad + \mu_0 \left[\frac{\mu_\mathrm{s} p + \mu s_\mathrm{s}}{\mu_\mathrm{s} p - \mu s_\mathrm{s}}\frac{\mu_\mathrm{m} p + \mu s_\mathrm{m}}{\mu_\mathrm{m} p - \mu s_\mathrm{m}}\mathrm{e}^v - 1\right]_0^{-1}\Bigg\}. \end{aligned}$$

The relative magnitude of $f^{(2)}$ and $f^{(1)}$ is best estimated if we consider the force in a cavity with ideally reflecting mirrors, corresponding to the classical Casimir configuration. Letting $\varepsilon_{\mathrm{s}0} \to \infty$ and $\varepsilon_{\mathrm{m}0} \to \infty$, the integrals in Eqs. (40) and (41) become elementary and we find

$$f_\mathrm{id}^{(2)}(d \gg \Lambda) = \frac{\hbar c\pi^2}{45\cdot 2^5 d^4}\sqrt{\frac{\mu_0}{\varepsilon_0}}\left(1 - \frac{1}{n_0^2}\right), \quad (42)$$

$$f_\mathrm{id}^{(1)}(d \gg \Lambda) = \frac{\hbar c\pi^2}{15\cdot 2^5 d^4}\sqrt{\frac{\mu_0}{\varepsilon_0}}\left(1 + \frac{1}{n_0^2}\right), \quad (43)$$

It is seen that at large distances $f^{(2)}$ is comparable in magnitude with $f^{(1)}$, especially for optically denser media where, ideally, $f^{(2)}$ is only three times smaller than $f^{(1)}$.

To find the force on a thin slab at large distances, we note that according to Eq. (33)

$$r^q(\mathrm{i}\xi,p) = 2\rho^q \frac{n\xi s_\mathrm{s} d_\mathrm{s}}{c}, \qquad [(1+r^q)^2 - t^{q2}](\mathrm{i}\xi,p) \simeq 2\frac{n\xi p d_\mathrm{s}}{c\gamma^q}. \tag{44}$$

Inserting this into Eq. (36) and proceeding in the same way as above, we find to the first order in d_s/d

$$f^{(2)}(d \gg \Lambda; d_\mathrm{s} \ll d) = \frac{3\hbar c\mu_0(n_0^2-1)d_\mathrm{s}}{16\pi^2 n_0^3 d^5} \int_1^\infty \frac{\mathrm{d}p}{p^4}\left[\frac{\varepsilon_{\mathrm{s}0}}{\varepsilon_0}R^p(0,p) - \frac{\mu_{\mathrm{s}0}}{\mu_0}R^s(0,p)\right]. \tag{45}$$

3.2. Force on the cavity medium

Clearly, when $n_\mathrm{s} = n$, $f_\mathrm{s}^{(2)}$ describes the force on a layer of the medium in the cavity f_m. Since in this case $\rho^q = 0$ in Eq. (20), the corresponding results for f_m are straightforwardly obtained from the above formulas when letting $r^q(\mathrm{i}\xi,k) = 0$ and $t^q(\mathrm{i}\xi,k) = \mathrm{e}^{-\kappa d_\mathrm{s}}$. Thus, from Eq. (27) we find that f_m is generally given by

$$f_\mathrm{m}(d) = \frac{\hbar}{8\pi^2c^2}\int_0^\infty \mathrm{d}\xi\, \xi^2\mu(n^2-1)\int_0^\infty \frac{\mathrm{d}k\, k}{\kappa}(1-\mathrm{e}^{-2\kappa d_\mathrm{s}})\mathrm{e}^{-2\kappa d}\sum_{q=p,s}\Delta_q R^q(\mathrm{i}\xi,k). \tag{46}$$

The small-distance behavior of f_m from Eq. (30) is described by

$$f_\mathrm{m}(d \ll \Lambda) = \frac{\hbar}{16\pi^2c^2 d}\int_0^\infty \mathrm{d}\xi\, \xi^2\mu(n^2-1)\int_0^\infty \mathrm{d}u\,(1-\mathrm{e}^{-ud_\mathrm{s}/d})\mathrm{e}^{-u}\sum_{q=p,s}\Delta_q R^q_\mathrm{nr}(\mathrm{i}\xi,\frac{u}{2d}), \tag{47}$$

and, as follows from Eq. (39) (upon performing the integration over v), at large distances f_m behaves as

$$f_\mathrm{m}(d \gg \Lambda) = \frac{3\hbar c\mu_0(n_0^2-1)}{64\pi^2 n_0^3}\left[\frac{1}{d^4} - \frac{1}{(d+d_\mathrm{s})^4}\right]\int_1^\infty \frac{\mathrm{d}p}{p^4}\sum_{q=p,s}\Delta_q R^q(0,p). \tag{48}$$

Note that for an ideally reflecting mirror, the value of the above integral is $\pm 2/3$. Accordingly, the force on the medium is attractive or repulsive depending on whether the mirror is (dominantly) dielectric or permeable, resembling, in this respect, the force on an (electrically polarizable) atom [15 – 17] near a mirror.

The thick-layer results are easily recognized from the above formulas when letting $d_\mathrm{s} \gg d$. Similarly, the force on a thin layer is given by these equations in the limit $d_\mathrm{s} \ll d$. At small distances, from Eq. (47) we find

$f_\mathrm{m}(d \ll \Lambda; d_s \ll d)$

$$= \frac{\hbar d_{\rm s}}{16\pi^2 c^2 d^2} \int\limits_0^\infty {\rm d}\xi\, \xi^2 \mu(n^2-1) \int\limits_0^\infty {\rm d}u\, u{\rm e}^{-u} \sum_{q=p,s} \Delta_q R^q_{\rm nr}({\rm i}\xi, \frac{u}{2d}), \quad (49a)$$

$$= \frac{\hbar d_{\rm s}}{16\pi^2 c^2 d^2} \int\limits_0^\infty {\rm d}\xi\, \xi^2 \mu(n^2-1) \left(\frac{\varepsilon_{\rm m}-\varepsilon}{\varepsilon_{\rm m}+\varepsilon} - \frac{\mu_{\rm m}-\mu}{\mu_{\rm m}+\mu} \right) \quad (49b)$$

in agreement with Eq. (34). Here the second line corresponds to the system with a structureless mirror. Finally, the force on a thin layer at large distances is from Eq. (48) found to be

$$f_{\rm m}(d \gg \Lambda; d_{\rm s} \ll d) = \frac{3\hbar c(n_0^2-1)d_{\rm s}}{16\pi^2 n_0 \varepsilon_0 d^5} \int\limits_1^\infty \frac{{\rm d}p}{p^4} \left[R^p(0,p) - R^s(0,p) \right], \qquad (50)$$

in agreement with Eq. (45).

We end this short discussion by noting that for a dilute medium, $f_{\rm m}$ is the sum of the forces $f_{{\rm a}i}$ acting on each atom i in the layer. Accordingly, the force on an atom $f_{\rm a}$ at distance d from a mirror is obtained from $f_{\rm m}$ for a thin layer as $f_{\rm a} = f_{\rm m}/Nd_{\rm s}$, where N is the atomic number density. Since for dilute media $n^2 - 1 = 4\pi N(\alpha_e + \alpha_m)$, it follows that f_a is given by the above thin-layer results upon making the formal replacement

$$\frac{n^2({\rm i}\xi)-1}{4\pi} d_{\rm s} \to \alpha_{\rm e}({\rm i}\xi) + \alpha_{\rm m}({\rm i}\xi), \qquad (51)$$

where $\alpha_{\rm e(m)}$ is the electric (magnetic) polarizability of the atom. Thus, expanding the integrand in Eq. (46) for small $2\kappa d_s \sim d_s/d$ and using the above recipe, we find that generally

$$f_{\rm a}(d) = \frac{\hbar}{\pi c^2} \int\limits_0^\infty {\rm d}\xi\, \xi^2 \mu(\alpha_{\rm e}+\alpha_{\rm m}) \int\limits_0^\infty {\rm d}k\, k\, {\rm e}^{-2\kappa d} \left[R^p({\rm i}\xi,k) - R^s({\rm i}\xi,k) \right]. \qquad (52)$$

We also observe that Eq. (49) then implies a Coulomb-like force on an atom at small distances from a mirror rather than the common van der Waals force [3]. At large atom-mirror distances, however, Eq. (50) implies a screened Casimir–Polder force on the atom. Of course, in accordance with the above mentioned unique property of the medium-assisted force, the sign of $f_{\rm a}$ is insensitive to the polarizability type (electric or magnetic) of the atom, contrary to the standard Casimir–Polder force [18]. Note also that, since $n_0\varepsilon_0 \simeq 1$ for dilute media, $f_{\rm a}$ at large distances from an ideally reflecting dielectric mirror is effectively three times smaller than the Casimir–Polder force. We stress, however, that, as a medium-assisted force, $f_{\rm a}$ is a collective property of the atomic system and this (perhaps) explains its unusual properties.

It is natural to compare the above medium-assisted atomic force with the familiar force $\tilde{f}_{\mathrm{a}}$ acting on an atom in vacuum near a mirror. This *single-atom* force can be obtained in the same way as above by considering the force on a thin dilute slab in an empty semi-infinite cavity. We find

$$\tilde{f}_{\mathrm{a}}(d) = \frac{\hbar}{\pi c^2}\int\limits_0^\infty \mathrm{d}\xi\,\xi^2 \int\limits_0^\infty \mathrm{d}k\,k\,\mathrm{e}^{-2\kappa d}\Bigg\{\left[\alpha_{\mathrm{e}}\left(2\frac{\kappa^2 c^2}{\xi^2}-1\right)-\alpha_{\mathrm{m}}\right]R^p(\mathrm{i}\xi,k) + \left[\alpha_{\mathrm{m}}\left(2\frac{\kappa^2 c^2}{\xi^2}-1\right)-\alpha_{\mathrm{e}}\right]R^s(\mathrm{i}\xi,k)\Bigg\}, \qquad (53)$$

which generalizes (in different directions) earlier results obtained for $\tilde{f}_{\mathrm{a}}$ in various systems [2,3,7,15 – 18]. This expression correctly reproduces the dependence of the Casimir–Polder force on the polarizability type of the atom [18] and the dielectric/magnetic properties of the mirror [15 – 17]. Also, for structureless mirrors, $\tilde{f}_{\mathrm{a}} \sim 1/d^4$ at small and $\tilde{f}_{\mathrm{a}} \sim 1/d^5$ at large distances. Apparently, this asymptotic behaviour of the atom-mirror force is well supported experimentally [19 – 26]. However, we note that the results presented in these works do not definitely disqualify the medium-assisted force. Indeed, being a collective property, f_{a} is expected to show up at higher atomic densities, whereas most experiments were usually performed with low-density atomic beams [20 – 24,26], i.e. under the conditions in favour of the single-atom force. Besides, a number of these experiments probed the d^{-5} tail of the force [20,21,23,25,26], which is common to both f_{a} and $\tilde{f}_{\mathrm{a}}$. Actually, there were also spectroscopic evidences showing that the characteristic features due to the d^{-4} tail of $\tilde{f}_{\mathrm{a}}$ disappear from the spectra at higher atomic densities [19]. Accordingly, to test the existence of f_{a}, one must design an experiment involving a higher-density homogeneous atomic system close to a mirror and probing the nonretarded atom-mirror interaction, where f_{a} substantially differs from $\tilde{f}_{\mathrm{a}}$. On the theoretical side, to understand the properties of the medium-assisted force, a microscopic consideration of the atom-mirror interaction is needed, for an atom of the medium in the vicinity of a mirror.

4. *Summary*

In summary, in this work we have discussed a formula for the force on a slab in a planar cavity, as derived very recently by Raabe and Welsch using the Lorentz-force approach [8]. We have shown that this result naturally splits into a formula for a medium-screened Casimir force and a formula for a medium-assisted force. A remarkable feature of the latter force is that its sign depends only on the properties of the cavity mirrors. In the classical Lifshitz configuration, at small distances the medium-assisted force is proportional to d^{-1} and is generally very small compared with the screened Casimir ($\sim d^{-3}$). At large distances, however, the medium-assisted force is proportional to d^{-4} and is comparable with the screened Casimir force, especially for denser media (actually, for a dense medium in a cavity with ideally reflecting mirrors, it is only three times smaller). As usual, the exponents in these power laws decrease by 1 in the case of a thin slab. The formula for the medium-assisted force also describes the force on the cavity medium. For dilute media, it predicts the atom-mirror interaction of the Coulomb type at small and of the Casimir–Polder type at large atom-mirror distances. In a semi-infinite cavity with an ideally reflecting mirror, the predicted medium-assisted force on an atom is effectively only three times smaller at large distances than the Casimir–Polder force.

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Appendix A. Green function

Following the derivation presented in Ref. [27] for a purely dielectric multilayer, for clarity, we consider the field

$$\boldsymbol{E}(\boldsymbol{r},\boldsymbol{r}';\omega)=\frac{\omega^2}{c^2}\overleftrightarrow{\boldsymbol{G}}(\boldsymbol{r},\boldsymbol{r}';\omega)\cdot\boldsymbol{p} \tag{54}$$

of an oscillating point dipole $\boldsymbol{p}\exp(-\mathrm{i}\omega t)$ at a position $\boldsymbol{r}'$, rather than the Green function itself. Assuming the dipole in the jth layer, its field $\mathbf{E}_l^{(j)}(\boldsymbol{r},\boldsymbol{r}';\omega)$ in an lth layer is given by

$$\boldsymbol{E}_l^{(j)}(\boldsymbol{r},\boldsymbol{r}';\omega)=\boldsymbol{E}_j^0(\boldsymbol{r},\boldsymbol{r}';\omega)\delta_{lj}+\boldsymbol{E}_l^h(\boldsymbol{r},\boldsymbol{r}';\omega), \tag{55}$$

where $\boldsymbol{E}_j^0(\boldsymbol{r},\boldsymbol{r}';\omega)$ is the field of the dipole as it would be in the infinite medium j, and $\boldsymbol{E}_l^h(\boldsymbol{r},\boldsymbol{r}';\omega)$ describe the propagation of this source field through the system. Specially, $\boldsymbol{E}_j^h(\boldsymbol{r},\boldsymbol{r}';\omega)\equiv\boldsymbol{E}_j^{\mathrm{sc}}(\boldsymbol{r},\boldsymbol{r}';\omega)$ represents the scattered (reflected) field in the jth layer.

According to Eq. (5), $\boldsymbol{E}_j^0(\boldsymbol{r},\boldsymbol{r}';\omega)$ is of the same form as the dipole field in a purely dielectric medium multiplied by μ_j, except that this time the wave vector is given by $k_j=n_j\omega/c=\sqrt{\varepsilon_j\mu_j}\omega/c$. In the plane-wave representation

$$\boldsymbol{E}(\boldsymbol{r},\boldsymbol{r}';\omega)=\int\frac{\mathrm{d}^2\boldsymbol{k}}{(2\pi)^2}\,\boldsymbol{E}(\boldsymbol{k},\omega;z,z')\mathrm{e}^{\mathrm{i}\boldsymbol{k}\cdot(\boldsymbol{r}_\parallel-\boldsymbol{r}'_\parallel)}\,. \tag{56}$$

We therefore have [27]

$$\begin{aligned}\boldsymbol{E}_j^0(\boldsymbol{k},\omega;z,z') \;=\; & -4\pi\frac{\mu_j}{\varepsilon_j}\hat{\boldsymbol{z}}\hat{\boldsymbol{z}}\cdot\boldsymbol{p}\delta(z-z')+\sum_{q=p,s}\big[\hat{\boldsymbol{e}}_{qj}^{+}(\boldsymbol{k})\mathrm{e}^{\mathrm{i}\beta_j z}E_{qj}^{0+}\theta(z-z') \\ & +\hat{\boldsymbol{e}}_{qj}^{-}(\boldsymbol{k})\mathrm{e}^{-\mathrm{i}\beta_j z}E_{qj}^{0-}\theta(z'-z)\big],\end{aligned} \tag{57}$$

where $\beta_j = \sqrt{k_j^2 - k^2}$,

$$E_{qj}^{0\pm} = \mu_j \frac{2\pi i}{\beta_j} \frac{\omega^2}{c^2} \xi_q \hat{\boldsymbol{e}}_{qj}^{\mp}(-\boldsymbol{k}) \cdot \boldsymbol{p}\, \mathrm{e}^{\mp \mathrm{i}\beta_j z'}, \tag{58}$$

with $\xi_q = \delta_{qp} - \delta_{qs}$, and

$$\hat{\boldsymbol{e}}_{pj}^{\pm}(\boldsymbol{k}) = \frac{1}{k_j}(k\hat{\boldsymbol{z}} \mp \beta_j \hat{\boldsymbol{k}}), \qquad \hat{\boldsymbol{e}}_{sj}^{\pm}(\boldsymbol{k}) = \hat{\boldsymbol{k}} \times \hat{\boldsymbol{z}} \equiv \hat{\boldsymbol{n}}, \tag{59}$$

are unit polarization vectors for $q = p$ (TM) and $q = s$ (TE) polarized waves, respectively.

The fields $\boldsymbol{E}_l^h(\boldsymbol{r}, \boldsymbol{r}'; \omega)$ obey homogeneous Maxwell equations. In analogy to Eq. (57), $\mathbf{E}_l^h(\boldsymbol{k}, \omega; z, z')$ can therefore be written as

$$\boldsymbol{E}_l^h(\boldsymbol{k}, \omega; z, z') = \sum_{q=p,s} \left[\hat{\boldsymbol{e}}_{ql}^{+}(\boldsymbol{k}) \mathrm{e}^{\mathrm{i}\beta_l z} E_{ql}^{+} + \hat{\boldsymbol{e}}_{ql}^{-}(\boldsymbol{k}) \mathrm{e}^{-\mathrm{i}\beta_l z} E_{ql}^{-} \right]. \tag{60}$$

Since only the outgoing waves should exist in the external layers, $E_{q0}^{+} = E_{qn}^{-} = 0$ and the remaining coefficients $E_{ql}^{\pm}$ can be expressed in terms of the generalized reflection and transmission coefficients of the corresponding stacks of layers. A reflection coefficient r^q of a stack is defined as the ratio of the reflected to incoming wave (electric-field) amplitude (factors multiplying $\hat{\boldsymbol{e}}$'s) at the corresponding stack's boundary. Similarly, a transmission coefficient t^q of a stack is defined as the ratio of the transmitted to incident wave amplitude calculated at the corresponding stack's boundary. In calculating these coefficients, it is convenient to adopt a (shifted-z) representation for the field [27] in which $0 \le z \le d_l$ in any finite layer, whereas $-\infty < z \le 0$ ($l = 0$) and $0 \le z < \infty$ ($l = n$), respectively, in the external layers.

According to the above definitions, the coefficients $E_{qj}^{\pm}$ of the field in the jth layer are given by

$$E_{qj}^{+} = r_{j-}^{q}(E_{qj}^{0-} + E_{qj}^{-}), \qquad \mathrm{e}^{-\mathrm{i}\beta_j d_j} E_{qj}^{-} = r_{j+}^{q} \mathrm{e}^{\mathrm{i}\beta_j d_j}(E_{qj}^{0+} + E_{qj}^{+}), \tag{61}$$

where we have introduced the notation $r_{j-}^{q} \equiv r_{j/0}^{q}$ and $r_{j+}^{q} \equiv r_{j/n}^{q}$ for the reflection coefficients of the bounding stacks. With Eq. (58), we find

$$\begin{aligned} E_{qj}^{+} &= \mu_j \frac{2\pi \mathrm{i}}{\beta_j} \frac{\omega^2}{c^2} \xi_q \frac{r_{j-}^{q} \mathrm{e}^{\mathrm{i}\beta_j d_j}}{D_{qj}} [\hat{\mathbf{e}}_{qj}^{+}(-\boldsymbol{k}) \mathrm{e}^{-\mathrm{i}\beta_j z'_+} + r_{j+}^{q} \hat{\mathbf{e}}_{qj}^{-}(-\boldsymbol{k}) \mathrm{e}^{\mathrm{i}\beta_j z'_+}] \cdot \boldsymbol{p}, \quad (62a) \\ E_{qj}^{-} &= \mu_j \frac{2\pi \mathrm{i}}{\beta_j} \frac{\omega^2}{c^2} \xi_q \frac{r_{j+}^{q} \mathrm{e}^{2\mathrm{i}\beta_j d_j}}{D_{qj}} [\hat{\mathbf{e}}_{qj}^{-}(-\boldsymbol{k}) \mathrm{e}^{-\mathrm{i}\beta_j z'_-} + r_{j-}^{q} \hat{\mathbf{e}}_{qj}^{+}(-\boldsymbol{k}) \mathrm{e}^{\mathrm{i}\beta_j z'_-}] \cdot \boldsymbol{p}, \quad (62b) \end{aligned}$$

where $z'_+ \equiv d_j - z'$ and $z'_- \equiv z'$ are the distances of the dipole from the layer's boundaries and

$$D_{qj} = 1 - r_{j-}^{q} r_{j+}^{q} \mathrm{e}^{2\mathrm{i}\beta_j d_j}. \tag{63}$$

Repeating the same considerations for the dipole embedded in the layer 0 (n), we find that its field $\boldsymbol{E}_0^{\rm sc}(\boldsymbol{r},\boldsymbol{r}';\omega)$ ($\boldsymbol{E}_n^{\rm sc}(\boldsymbol{r},\boldsymbol{r}';\omega)$) is also given by the above equations, with $j = 0$ (n), provided that we let $r_{0-}^q = 0$ ($r_{n+}^q = 0$) and put d_0 (d_n), which appears formally in Eq. (62), equal to zero.

Collecting the equations and using Eq. (54), we obtain the Green function for the scattered field in the jth layer in the form

$$\overleftrightarrow{\boldsymbol{G}}_j^{\rm sc}(\boldsymbol{r},\boldsymbol{r}';\omega) = \mu_j \frac{\rm i}{2\pi}\int \frac{{\rm d}^2\boldsymbol{k}}{\beta_j} {\rm e}^{{\rm i}\boldsymbol{k}\cdot(\boldsymbol{r}_\parallel - \boldsymbol{r}'_\parallel)} \sum_{q=p,s} \xi_q \frac{{\rm e}^{{\rm i}\beta_j d_j}}{D_{qj}} \tag{64}$$

$$\times\Big\{ r_{j-}^q \hat{\boldsymbol{e}}_{qj}^+(\boldsymbol{k}){\rm e}^{{\rm i}\beta_j z_-}\left[\hat{\boldsymbol{e}}_{qj}^+(-\boldsymbol{k}){\rm e}^{-{\rm i}\beta_j z'_+} + r_{j+}^q \hat{\boldsymbol{e}}_{qj}^-(-\boldsymbol{k}){\rm e}^{{\rm i}\beta_j z'_+}\right]$$

$$+ r_{j+}^q \hat{\boldsymbol{e}}_{qj}^-(\boldsymbol{k}){\rm e}^{{\rm i}\beta_j z_+}\left[\hat{\boldsymbol{e}}_{qj}^-(-\boldsymbol{k}){\rm e}^{-{\rm i}\beta_j z'_-} + r_{j-}^q \hat{\boldsymbol{e}}_{qj}^+(-\mathbf{k}){\rm e}^{{\rm i}\beta_j z'_-}\right]\Big\}, \quad 0 \le z, z' \le d_j.$$

Apparently, except for the multiplication by μ_j, $\overleftrightarrow{\boldsymbol{G}}_j^{\rm sc}(\boldsymbol{r},\boldsymbol{r}';\omega)$ is formally the same as for a purely dielectric system. This time, however, the wave vectors in the layers are given by $k_l = \sqrt{\varepsilon_l \mu_l}\,\omega/c$. As follows from their definition, for local stratified media the Fresnel coefficients satisfy recurrence and symmetry relations

$$r_{i/j/k}^q = r_{i/j}^q + \frac{t_{i/j}^q t_{j/i}^q r_{j/k}^q {\rm e}^{2{\rm i}\beta_j d_j}}{1 - r_{j/i}^q r_{j/k}^q {\rm e}^{2{\rm i}\beta_j d_j}}, \tag{65a}$$

$$t_{i/j/k}^q = \frac{t_{i/j}^q t_{j/k}^q {\rm e}^{{\rm i}\beta_j d_j}}{1 - r_{j/i}^q r_{j/k}^q {\rm e}^{2{\rm i}\beta_j d_j}} = \frac{\mu_k \beta_i}{\mu_i \beta_k} t_{k/j/i}^q, \tag{65b}$$

and, for a single $i - j$ interface, reduce to

$$r_{ij}^q = \frac{\beta_i - \gamma_{ij}^q \beta_j}{\beta_i + \gamma_{ij}^q \beta_j} = -r_{ji}^q, \tag{66a}$$

$$t_{ij}^q = \sqrt{\frac{\gamma_{ij}^q}{\gamma_{ij}^s}}(1 + r_{ij}^q) = \frac{\mu_j \beta_i}{\mu_i \beta_j} t_{ji}^q, \tag{66b}$$

where $\gamma_{ij}^p = \varepsilon_i/\varepsilon_j$ and $\gamma_{ij}^s = \mu_i/\mu_j$.

Appendix B. Calculation of Eq. (10)

Performing the derivations indicated in Eq. (7) and using

$$\boldsymbol{K}_j^\pm(\boldsymbol{k}) \times \hat{\boldsymbol{e}}_{qj}^\pm(\boldsymbol{k}) = k_j \xi_q \hat{\boldsymbol{e}}_{q'j}^\pm(\boldsymbol{k}), \qquad p' = s, \qquad s' = p, \tag{67}$$

we find that $\overleftrightarrow{\boldsymbol{G}}_j^{B,\mathrm{sc}}(\boldsymbol{r},\boldsymbol{r}';\omega)$ is given by Eq. (64) multiplied by $-k_j^2$ and with $\hat{\boldsymbol{e}}_{qj}^{\pm} \to \hat{\mathbf{e}}_{q'j}^{\pm}$. Noting that the equal-point Green function dyadics consist only of diagonal elements, we easily find

$$
\begin{aligned}
\overleftrightarrow{\boldsymbol{G}}_j^{\mathrm{sc}}(\boldsymbol{r},\boldsymbol{r};\omega) \quad = \quad & \frac{\mathrm{i}\mu_j}{2\pi k_j^2}\int\frac{\mathrm{d}^2\boldsymbol{k}}{\beta_j}\Bigg\{\hat{\boldsymbol{k}}\hat{\boldsymbol{k}}\frac{\beta_j^2}{D_{pj}}\left[2r_{j-}^p r_{j+}^p \mathrm{e}^{2\mathrm{i}\beta_j d_j} - r_{j-}^p\mathrm{e}^{2\mathrm{i}\beta z_-} - r_{j+}^p\mathrm{e}^{2\mathrm{i}\beta z_+}\right] \\
& + \hat{\boldsymbol{n}}\hat{\boldsymbol{n}}\frac{k_j^2}{D_{sj}}\left[2r_{j-}^s r_{j+}^s \mathrm{e}^{2\mathrm{i}\beta_j d_j} + r_{j-}^s\mathrm{e}^{2\mathrm{i}\beta_j z_-} + r_{j+}^s\mathrm{e}^{2\mathrm{i}\beta_j z_+}\right] \\
& + \hat{\boldsymbol{z}}\hat{\mathbf{z}}\frac{k^2}{D_{pj}}\left[2r_{j-}^p r_{j+}^p \mathrm{e}^{2\mathrm{i}\beta_j d_j} + r_{j-}^p\mathrm{e}^{2\mathrm{i}\beta_j z_-} + r_{j+}^p\mathrm{e}^{2\mathrm{i}\beta_j z_+}\right]\Bigg\}, \qquad (68)
\end{aligned}
$$

and $\overleftrightarrow{\boldsymbol{G}}_j^{B,\mathrm{sc}}(\boldsymbol{r},\boldsymbol{r};\omega)$ is given by this equation multiplied by k_j^2 and with $p \leftrightarrow s$. The traces $G_{j,\parallel}^{\mathrm{sc}}(\mathbf{r},\boldsymbol{r};\omega)$ and $G_{j,\parallel}^{B,\mathrm{sc}}(\boldsymbol{r},\mathbf{r};\omega)$ can be easily recognized from these equations and one has, for example,

$$
\begin{aligned}
& \frac{\omega^2}{c^2}\left[G_{j,zz}^{\mathrm{sc}}(\boldsymbol{r},\boldsymbol{r};\omega) - G_{j,\parallel}^{\mathrm{sc}}(\boldsymbol{r},\boldsymbol{r};\omega)\right] \\
& \quad = \frac{\mathrm{i}\mu_j}{2\pi n_j^2}\int\frac{\mathrm{d}^2\boldsymbol{k}}{\beta_j}\Bigg\{\frac{k^2}{D_{pj}}\left[2r_{j-}^p r_{j+}^p \mathrm{e}^{2\mathrm{i}\beta_j d_j} + r_{j-}^p\mathrm{e}^{2\mathrm{i}\beta_j z_-} + r_{j+}^p\mathrm{e}^{2\mathrm{i}\beta_j z_+}\right] \\
& \quad - \frac{\beta_j^2}{D_{pj}}\left[2r_{j-}^p r_{j+}^p \mathrm{e}^{2\mathrm{i}\beta_j d_j} - r_{j-}^p\mathrm{e}^{2\mathrm{i}\beta z_-} - r_{j+}^p\mathrm{e}^{2\mathrm{i}\beta z_+}\right] \\
& \quad - \frac{k_j^2}{D_{sj}}\left[2r_{j-}^s r_{j+}^s \mathrm{e}^{2\mathrm{i}\beta_j d_j} + r_{j-}^s\mathrm{e}^{2\mathrm{i}\beta_j z_-} + r_{j+}^s\mathrm{e}^{2\mathrm{i}\beta_j z_+}\right]\Bigg\}, \qquad (69)
\end{aligned}
$$

while $G_{j,zz}^{B,\mathrm{sc}}(\boldsymbol{r},\boldsymbol{r};\omega) - G_{j,\parallel}^{B,\mathrm{sc}}(\boldsymbol{r},\boldsymbol{r};\omega)$ is given by this equation multiplied by n_j^2 and with $p \leftrightarrow s$. Adding these two quantities, one obtains Eq. (10) for the expression in the curly bracket of Eq. (9).

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VAKUUMSKA SILA POTPOMOGNUTA SREDSTVOM

Raspavljamo implikacije nedavno dobivenog rezultata za silu na neku ploču u planarnom rezonatoru baziranom na proračunu vakuumske Lorentzove sile [C. Raabe and D.-G. Welsch, Phys. Rev. A 71 (2005) 013814]. Prema toj formuli, ukupna se sila na ploču sastoji od zasjenjene Casimirove sile i sredstvom potpomognute sile čiji je predznak određen isključivo svojstvima zidova rezonatora. U Lifshitzovoj konfiguraciji, ova sila je proporcionalna $1/d$ na malim udaljenostima i jako mala u usporedbi s odgovarajućom van der Waalsovom silom. Međutim, na velikim udaljenostima ona je proporcionalna $1/d^4$ i usporediva s Casimirovom silom, posebno za gušća sredstva u rezonatoru. U slučaju tankog sloja, ova asimptotska ponašanja prelaze u $1/d^2$ odnosno $1/d^5$. Formula za sredstvom potpomognutu silu opisuje također silu na neki sloj sredstva u rezonatoru, koja ima slična svojstva. Za razrijeđena sredstva, ona implicira interakciju atom-zid Coulombovog tipa na malim i Casimir-Polder tipa na velikim udaljenostima. U slučaju idealno reflektirajućeg zida, ova sila je efektivno samo tri puta slabija od Casimir-Polderove sile.