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PARAMETRIZATION OF ELECTRON ENERGY DISTRIBUTION FUNCTION IN GLOW DISCHARGE PLASMA UNDER RF ELECTROMAGNETIC FIELDS

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A set of twenty four plasma glow discharge experiments using radio-frequency electromagnetic fields were carried out. The discharge pressure was 27 Pa. Dried air was used as the discharge gas. Axial radio-frequency fields between 0.6–1.2 MHz were applied. Four different values of the RF field intensity were used. For each case, the plasma Langmuir probe characteristics were measured. A computational method was used to extract the electron energy distribution function in each case. Values of the plasma density and plasma temperature were obtained directly from the fits. They compare well with values obtained using the conventional logarithmic method. The Maxwellian behaviour of the distribution function was established to hold in many but not in all cases.

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1. *Introduction*

The study of glow discharge plasmas under the actions of radio-frequency electromagnetic fields has always been of great interest. Such studies are concerned both with theoretical and experimental sides of the subject. Few of the many articles published during the last thirty years may include the study of Gagene and Cantine [1] who used double probes to study radio-frequency discharges in argon gas at a pressure of 93 Pa (0.7 Torr; 1 Torr = 133.3 Pa). Double probes were also used by Godyak and Popov [2] to study plasma parameters under radio-frequency electromagnetic fields up to 15 kHz and in the pressure range of the order of 130 Pa. Difficulties concerning measurements of these parameters are reported due to the distortions in the probe characteristics caused by the RF fields. Chakravarti and Gupta [3] measured the electron energy distribution function using analog electronic differentiation circuits. Langmuir probes were used to study plasma kinetics in plasma etching with radio-frequencies at high E/P values. Pointu [4] used an unsymmetrical probe model to describe glow discharge. Plasma temperatures were measured by Phadke et al. [5] by projecting the probe I–V characteristics on the oscilloscope screen. They used air plasma at pressures ranging between 40 – 120 Pa. Good agreement between temperature values were obtained using this method and those using conventional hand-drawing method. Ohe and Kimura [6] suggested a new method for studying electron velocity distribution functions in a helium positive column. It has been indicated that the distribution function can have two peaks in some cases, however, the possibility that such effect may be due to the electronic circuitry can not be ruled out. In a theoretical and experimental study by Ichikawa et al. [7] of the positive column glow discharge plasma, the Langmuir probe was used to measure the electron temperature. Good agreement between theory and experiment was reported. Kaneda et al. [8] studied the neon plasma positive column in a rectangular tube. A special camera was used to measure optical emissions which are related to the plasma density. Results were compared to those of cylindrical tubes. An energy analyzer was described by Pointu et al. [9] which is capable of measuring plasma charge-particle distribution functions. Langmuir probe analysis was used for comparison purposes in DC- and microwave-induced helium plasmas. Behaviour of the distribution function in air plasma under axial magnetic fields using electronic differentiating circuits was reported by Azooz and Hussien [10]. Recently, the conventional method used to determine the plasma parameters from saturation probe currents has come under criticism [11]. The argument is based on the fact that this method assumes that the plasma energy distribution function is Maxwellian. This is indeed not necessarily true. Instead, it has been suggested to use the Druyvesteyn equation [12] to extract the energy distribution function. That way, the plasma parameters can be derived. The traditional method of evaluating the plasma temperature from plots of $\ln(I_e)$ against the probe voltage can run into errors due to two main reasons, the uncertainty in choosing the value of the ion saturation current and such plots are not linear in many cases. Maresca Me [13] and Heil [14] described a numerical method of analysis which involves smoothing and then further fitting to experimental I–V probe data. A reservation about this method is the large number of free parameters involved both in smoothing the data with a large number of polynomial sectors and carrying out the final fitting. A more detailed description of such smoothing method is given by Andrei et al. [15]. In an attempt to apply the latter procedure to extract the electron energy distribution function, we found that that the results are highly critical on the number of data points per sector used in the smoothing program. Changing this number between 3 and 10 produced distribution functions of variety of shapes. Some of these shapes can be mistaken as to represent actual energy distribution functions.

In this paper, we describe an alternative and easier fitting method based on fitting the experimental data by a tangent hyperbolic function with only four free parameters. The fitting function is then analytically differentiated twice to get the energy distribution function. Distribution functions for twenty four cases of glow discharge air plasma at 27 Pa under varying conditions of axial RF field frequency and intensity have been obtained.

2. Experimental setup and measurements

A schematic diagram of the experimental setup is shown in Fig. 1. It consists of a 28 cm long, 1.4 cm diameter Pyrex glass tube. The two ends are fitted with two discharge DC electrodes. A 0.32 mm diameter insulated tungsten electrode is connected through the glass tubing at the side of the discharge tube forming the Langmuir probe system. The part of the probe exposed to the plasma is 1 mm. The depth to which the probe can be inserted inside the plasma can be controlled by a magnetic mechanism from the outside. The discharge tube is surrounded by a 200 turn insulated copper coil. The coil is usually fed from the output of a vacuum tube radio-frequency amplifier specially built by the author for this purpose. The amplifier is driven from an RC oscillator. The oscillator frequency can be varied between 0.5 and 1.2 MHz. The load voltage of the amplifier can be changed between 0 and 1000 V peak to peak. However, for frequencies below 600 kHz or for output voltages below 150 V, the amplifier operation was rather unstable. Thus measurements in the vicinity of these values were not taken into account. The probe circuit is driven from a dual polarity 300 V DC supply. I–V characteristics were hand measured using an ammeter and a vacuum tube voltmeter.

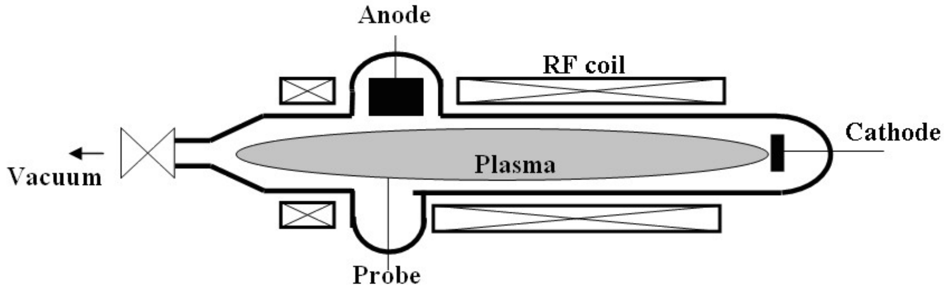

Fig. 1. Schematic diagram of the discharge tube.

The discharge tube is connected at one end to a vacuum gauge and a vacuum rotary pump together with an air release valve. This system allowed a good pressure control above 13 Pa pressure range. Before each experimental run, the system was checked for vacuum leaks for a period of about one hour during which time the pressure should change less than 20 %. It is worth mentioning that a set of data for one measurement of I–V probe characteristics takes about 15 minutes on the average.

In all, twenty four sets of measurements were carried out. These involve four settings of the RF coil voltage. They were 200, 400, 600 and 800 V. For each one of the voltages, which corresponds to a particular relative field strength, six values of the radio-frequencies were used. These were 600, 700, 800, 900, 1000 and 1200 kHz. For each value of the RF field strength at a particular frequency value, two I–V probe characteristics were measured. Results of one such typical measurement are shown in Fig. 2. Other results follow more or less the same pattern. For each curve in these figures, the value of the RF field voltage and frequency are shown. Points in these plots represent experimental data. Solid lines represent fitted functions.

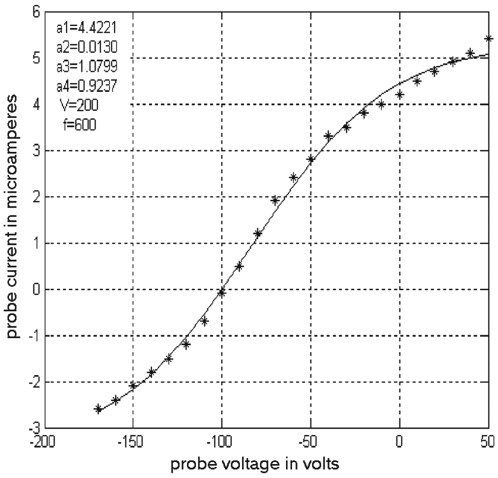

Fig. 2. Langmuir probe I–V characteristics at different frequencies at RF voltage of 200 V *and 600* kHz.

3. Data handling

A close inspection of the shape a typical I–V Langmuir probe characteristics reveals that it is highly similar to the hyperbolic tangent function. Such a variation is known to describe double-asymmetric probe and we decided to use it here. On such grounds, one may choose the following parameterization of the data

$$i = a_1 \tanh(a_2 V_\mathrm{p} + a_3) + a_4, \tag{1}$$

where i represents the probe current and V_p is the probe voltage. The first parameter, a_1, takes care of the maximum span of the current values. It is related to the ion saturation currents. The second parameter, a_2, is important in the sense that it controls the rise and saturation rate of the current values against changing voltage. It has a major physical importance in that, for the special case of the Maxwellian distribution, it is equal to $1/(k_\mathrm{B}T)$. The parameters a_3 and a_4 are related to the position of the curve in the I–V plane. If one assumes that there is some other large probe (counter probe) action taking place due to any reason, then the parameters a_1 and a_4 are related to the probe and counter probe ion currents $I_{i\mathrm{cp}}$ and $I_{i\mathrm{p}}$ in the following way: $a_1 = (I_{i\mathrm{cp}} + I_{i\mathrm{p}})/2$ and $a_4 = (I_{i\mathrm{cp}} - I_{i\mathrm{p}})/2$.

The non-linear curve-fitting facility of the MATLAB6.5 computer program "nlinfit" was used to carry out each fit. All fits produced good convergence. The fitting parameters were a_1, a_2, a_3 and a_4. The fitting program default confidence level was set at 95 %. Table 1 shows the values of χ^2 for all twenty four fits carried out. Values of the fitting parameters were substituted into Eq. (1) and double differentiation was carried out analytically. The distribution function is related to the second differential of the probe characteristics curve through the Druyvesteyn relation

$$F(E) = \frac{\sqrt{8m}\sqrt{V}}{Ae^{3/2}n} \frac{\mathrm{d}^2 I_{\mathrm{prob}}}{\mathrm{d}V_{\mathrm{prob}}^2} \tag{2}$$

where e is the electronic charge, m is the electron mass, $F(E)$ is the energy distribution function, V_{prob} is the voltage applied to the probes, I_{prob} is the probe current, n is the plasma electron density and A is the probe area.

TABLE 1. Values of χ^2 for all twenty four fits of probe I–V characteristics.

	200 V	400 V	600 V	800 V
600 kHz	0.0027	0.0020	0.0028	0.0036
700 kHz	0.0069	0.0084	0.0042	0.0033
800 kHz	0.0056	0.0065	0.0042	0.0022
900 kHz	0.0072	0.0048	0.0032	0.0028
1000 kHz	0.0039	0.0041	0.0027	0.0021
1200 kHz	0.0042	0.0024	0.0025	0.0019

TABLE 2. Plasma density in units of 10^{13} for all cases.

	200 V	400 V	600 V	800 V
600 kHz	6.4	6.6	10.7	11.8
700 kHz	5.0	5.1	10.0	8.3
800 kHz	5.6	6.1	9.9	10.9
900 kHz	6.0	6.8	8.5	6.8
1000 kHz	11.9	18.6	23.1	21.9
1200 kHz	7.8	11.1	16.5	21.5

Due to the fact that our main concern here was the shape of the distribution function, the second differentials were multiplied by the square root of the corresponding voltage value at which they were evaluated. The results are plotted against V in the range between zero and 200 V. The choice of the latter value is based on the fact that the distribution function values tend to approach zero before this value is reached. All plots were normalized to the value of unity through division by the area under the curve for each case. The result is the electron energy distribution function. The normalization constant multiplied by the numerical constant factor $\sqrt{8m}/(e^{3/2}A)$ gives directly the plasma electron charge density. The values of the charge densities are given in Table 2.

4. *Discussion*

A close look at the plots of Fig. 3 may lead one to the quantitative conclusion that for small field strength and field frequency values, the distribution function seems to be less spread over the whole energy spectrum. For each RF field intensity value, the distribution function tends to shift away toward higher energy values as the field strength is increased. However, calculations have shown that the overall average energy remains almost constant. This is consistent with the fact that electrons can not draw energy from the RF field. What the field seems to be doing is that it is concentrating the energy of electrons nearer to the average value from both sides around that average. This is due to the fact that the field is causing electrons to move along more and more complicated trajectories, what induces a larger number of collisions. For high frequency values, especially at 1000 and 1200 kHz, all distribution functions tend to scale to a similar shape. One can thus argue that some type of scaling behaviour of energy distribution functions is taking place when high amplitude or high frequency are applied to the glow discharge plasmas.

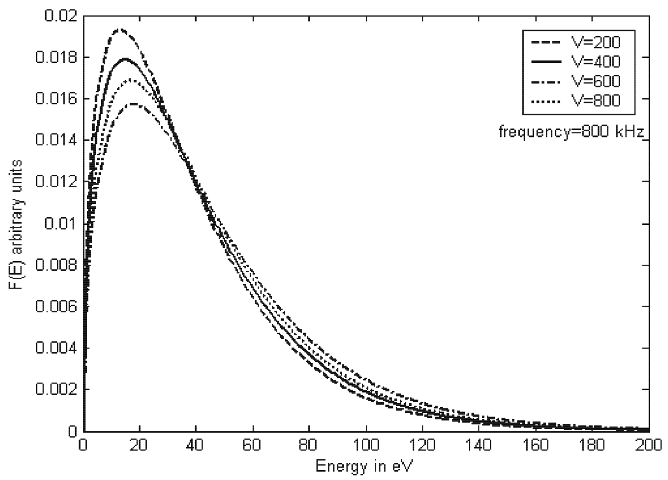

Fig. 3. Calculated plasma energy distribution functions.

A more quantitative approach to this matter may be discussed if one tries to look at the real mathematical shapes of the obtained distribution functions. To the first approximation, one may assume that the distribution functions of glow discharge plasmas at low pressure are of the Maxwellian type. To test that hypothesis, an attempt was made to fit the derived distribution functions by both Maxwellian and non-Maxwellian functions. The Maxwellian function is of the form

$$F(E) = K_1 \sqrt{E}\, \mathrm{e}^{E/K_2}, \tag{3}$$

where K_1 and K_2 are fitting constants. Their values are related to the plasma density and plasma temperature in the usual way.

The non-Maxwellian equation chosen has the form

$$F(E) = K_1 E^{0.5+x}\, \mathrm{e}^{E/K_2}, \tag{4}$$

where x here is another fitting constant that represents some deviation from the Maxwellian form. All distribution functions obtained are fitted with Eqs. (3) and (4).

TABLE 3. Plasma temperature values (in eV*) for all cases using the Maxwellian and the logarithmic fitting method (numbers in bold).*

	200 V		400 V		600 V		800 V	
600 kHz	12.1	**14.0**	13.7	**13.5**	15.7	**15.0**	14.7	**13.5**
700 kHz	21.5	**18.0**	18.3	**17.0**	21.2	**16.7**	22.2	**19.5**
800 kHz	12.6	**13.5**	13.7	**14.0**	15.7	**17.0**	14.6	**15.5**
900 kHz	15.2	**12.4**	14.0	**11.5**	13.3	**11.2**	12.4	**10.5**
1000 kHz	13.2	**12.5**	16.5	**18.0**	17.5	**16.0**	16.5	**17.0**
1200 kHz	13.0	**15.0**	14.5	**17.3**	15.5	**16.5**	16.3	**18.0**

As far as fits with the Maxwellian Eq. (3) are concerned, most fits reproduced the distribution function perfectly well. However, in some cases the residuals were slightly high. This is especially the case at field frequencies of 600 and 1000 kHz. For most other cases, one can not see any distinction between the calculated energy distribution function and the fitted Maxwellian distribution of Eq. (3). The free parameter K_2 in this case is, of course, just half the temperature in eV. Temperature values obtained from the fits, after integration the energy distribution function over the full energy range, are shown in Table 3. For the purpose of comparison, the temperature values obtained using the conventional method of $\ln(I_e)$ versus V plots are given on the same table. I_e values are obtained after subtracting the ion saturation current from the values of the probe current. The slope of the first linear part of these plots represents the temperature. Both sets of temperature values seem to be compatible within the errors that are associated with the conventional method.

This is a good indication that our fitting method gives values of the temperature that are compatible with those obtained using the logarithmic method. It must be pointed out, however, that the temperature values obtained using the logarithmic method can be highly sensitive to the value of the subtracted ion saturation current. Discrepancies between repeated calculations for a particular temperature value of up to 50 % have been noted depending upon the value of the ion saturation current subtracted. However, every effort has been paid to be even-handed in the selection of this value for the ion saturation current. The temperature values obtained using our fitting program do not suffer from any such discrepancies as the fits are unique representations of the energy distribution function. Values shown in bold letters are those obtained using the logarithmic method.

TABLE 4. Values of the deviation parameter x in Eq. (4) for all cases (RF field voltage in V*).*

	200 V	400 V	600 V	800 V
600 kHz	0.12	0.12	0.22	0.24
700 kHz	0.05	0.06	0.10	0.07
800 kHz	0.04	0.06	0.10	0.10
900 kHz	0.06	0.07	0.08	0.06
1000 kHz	0.12	0.23	0.31	0.27
1200 kHz	0.08	0.08	0.19	0.23

In order to get a better insight on the assumption widely used in plasma glow discharge calculation that the energy distribution function is Maxwellian, similar fits were carried out using the modified distribution function of Eq. (4). The deviation parameter, x, from the Maxwellian distribution was evaluated for each case. The values are shown in Table 4. It is clear that, although for most cases the values of this parameter are confined to the second decimal digit, there are cases where this parameter takes a value of 0.3, which indicates a substantial deviation from the Maxwellian distribution.

This result indicates clearly that although the assumption of Maxwellian distribution is justified in most cases, particular care must be taken in applying that assumption in other cases. This is indeed necessary when the plasma is under the action of strong radio-frequency fields. Fig. 4 shows a typical case of deviation from the Maxwellian shape. It is clear that the modified function of Eq. (4) is far better than the Maxwellian function of Eq. (3) in describing the data. From the figure, the full widths at half maximum for both functions are 55 and 45 eV for the Maxwellian and the modified fits, respectively.

It may be interesting to give the counter probe action some further consideration at this stage. In particular, and in order to identify the counter probe through its area, we can compare the two quantities $a_2(V_\mathrm{p} - V_\mathrm{f}) + \log(A_\mathrm{p}/A_\mathrm{cp})$ (where

A_p and A_{cp} are the probe and counter probe areas respectively) and $2V_p + a_3$. Such comparison was carried out for all twenty four cases. Values of the quantity $\log(A_p/A_{cp})$ always ranged between 3 and 3.5. Such result indicates that A_p is at least three orders of magnitudes higher that A_{cp}.

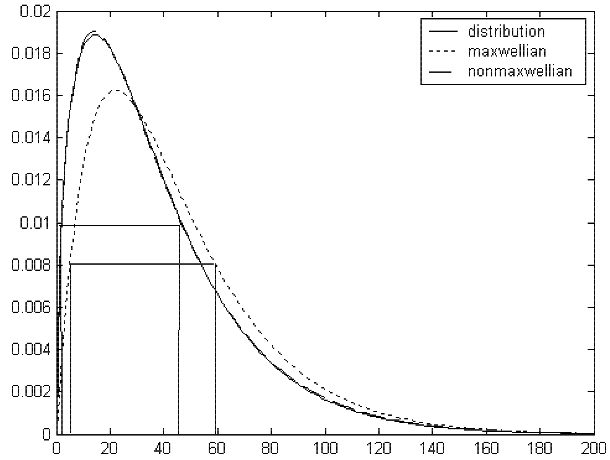

Fig. 4. Comparison between the Maxwellian and modified fits of the energy distribution function.

5. Conclusions

From all above results and discussion we may come to the following conclusions:

1) The tangent hyperbolic function provides a reasonable description of the Langmuir probe data. This in turn overcomes difficulties in data analysis.

2) Any further analysis can be carried out without making the assumption that the energy distribution function is Maxwellian.

3) Plasma electron density and plasma temperature can be easily obtained from the analytically derived distribution function.

4) Our data support the argument that low pressure DC glow discharge energy distribution function under RF fields may be assumed to be Maxwellian in most but not in all cases. Such an assumption may accepted as a first approximation only and for fields of low frequency.

5) The modified distribution function of Eq. (4) provides excellent fits of the data. These fits are at least one order of magnitude superior in terms of χ^2 to the Maxwellian shape of Eq. (3).

References

[1] R. R. J. Gagene and A. Cantine, J. Appl. Phys. **43** (1972) 2639.
[2] V. A. Godyak and O. A. Popov, Tech. Phys. **22** (1977) 461.
[3] P. K. Chakravarti and S. N. Gupta, Rev. Sci. Instrum. **48** (1977) 1344.
[4] A. M. Pointu, Appl. Phys. Lett. **48** (1986) 762.
[5] K. W. Phadke, R. S. Kot and S. V. Deshmukh, Indian J. Pure Appl. Phys. **26** (1988) 678.
[6] K. Ohe and T. Kimura, Japan J. Appl. Phys. **28** (1989) 1997.
[7] Y. Ichikawa, R. L. C. Wu and T. Kaneda, J. Appl. Phys. **67** (1990) 108.
[8] T. Kaneda, Y. Ohsone and J. Chang, Japan J. Appl. Phys. **29** (1990) 1324.
[9] A. M. Pointu, E. Stamate and K. Wiesemann, Czech. J. Phys. **48** (1998) 1147.
[10] A. A. Azooz and A. K. Hussien, Mu'tah Lil-Buhuth wad-Dirasat **16** (2001) 71.
[11] V. A. Godyak, R. B. Piejak and B. M. Alexandroich, J. Appl. Phys. **73** (1993) 3657.
[12] M. J. Druyvesteyn, Z. Phys. **64** (1930) 781.
[13] Maresca Me, Master thesis, University of Minnesota, 2001.
[14] Me. B. Heil, Master thesis, University of Minnesota, 1999.
[15] H. Andrei, V. Covlea, V. V. Covlea and E. Barna, Romanian Reports in Physics **55** (2003) 51.

PARAMETRIZACIJA FUNKCIJE RASPODJELE ELEKTRONSKE ENERGIJE U TINJAVOM IZBOJU U RF ELEKTROMAGNETSKOM POLJU

Izveli smo 24 mjerenja u plazmi u tinjavom izboju pod djelovanjem radiofrekventnog (RF) elektromagnetskog polja. Tlak u izboju bio je 27 Pa. Izbojni plin bio je suhi zrak. Rabili smo osna RF polja u području 0.6 do 1.2 MHz. Primijenili smo četiri jakosti RF polja. U svakom mjerenju odredili smo značajku Langmuirove probe. Računalnom smo metodom izveli funkciju elektronske raspodjele energije za svako mjerenje. Izravno iz prilagodbi izveli smo vrijednosti gustoće i temperature plazme. Te se vrijednosti dobro uspoređuju s vrijednostima dobivenim uobičajenom logaritamskom prilagodbom. Maxwellova funkcija raspodjele se pokazuje dobrom u mnogim, ali ne svim slučajevima.