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ELECTRON AND HOLE IMPACT IONIZATION COEFFICIENTS AT VERY HIGH ELECTRIC FIELDS IN SEMICONDUCTORS

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We have fitted the soft lucky drift model of impact ionization of Ridley to experimental data for GaAs, InP, Si, Ge and $In_{0.47}Ga_{0.53}As$ semiconductors. Excellent fits of the theory to experimental data were obtained by using least-squares fitting algorithm. A generalized Keldysh formula has been used to introduce a soft threshold factor. Generalized Keldysh formula originates from realistic energy bands in semiconductors at high electric field which reflects the density of states of energy bands. Keldysh factor and a new mean free path are calculated. A comparison with reported values of both Ridley and Marsland showed reasonable agreement for mean free path, but there are still large differences among Keldysh factors.

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1. Introduction

The performance of avalanche photodiodes (APDs) depend critically on the ratio of the electron and hole ionization coefficients. Carrier multiplication through impact ionization is the basic mechanism in their operation. It is essential to identify materials which have electron and hole ionization coefficients greatly different to achieve their optimal performance in their applications in various devices.

The impact ionization in semiconductors offers a mechanism for carrier multiplication, which can be used to amplify small signals. When the energy of an electron in the semiconductor exceeds the impact-ionization threshold energy, the electron can excite another electron from the valence band to the conduction band, creating thereby a hole, and lose most of its energy. The threshold energy for impact ionization is determined by the width of the forbidden energy gap, details of band structure and consideration of momentum and energy conservation. Each excited electron can proceed to gain energy from the field and induce further impact ionization. The consequence is an avalanche, which can damage the sample if uncontrolled.

Experimental measurement of ionization coefficient is difficult. Therefore, a simple theoretical calculation is useful. Wolff [1] applied the gas discharge theory to solve the Boltzmann equation. His results showed that electrons in the high-energy tail of the distribution function cause the impact ionization. On the other hand, Shockley [2] argued that an electrons which gain sufficient energy from the field cause ionization. Few electrons are lucky to avoid several collisions and reach the critical ionizating energy ($E_{\rm I}$). In order to bridge the two theories, Baraff [3] solved the time-independent Boltzmann equation. His numerical results agree with the Shockley model for low field but converge to the Wolff results for high field. Although the above method was successful in describing the impact ionization coefficients in Si and Ge, the physical mechanism by which carriers ionize was unknown. Monte Carlo simulation of Shichijo and Hess [4], carried with the real band structure in GaAs, was consistent with Baraff's theory; in addition, the obtained results offered very clear physical picture of the impact ionization mechanisms. The experimental determination of the impact ionization coefficient was important for the evaluation of the physical parameters used in these theoretical calculations. The experimental determination of the electron and hole impact ionization coefficients is not an easy task as is evident from the wide discrepancies in the literature data [5] and still remains one of the most challenging subjects of investigation of semiconductors devices.

Monte Carlo simulation using the full band structure, calculated using the pseudopotential method [6 – 10] and the realistic analytical band structure, have been developed to investigate carrier transport in silicon [11 – 12].

The Monte Carlo models are categorized into several groups in terms of description of the energy band, the scattering formulae, and the adjustable parameters involved in these formulae, such as deformation potential for electron-phonon scattering rate and prefactors in the Keldysh formula for the rate of impact ionization.

Numerical calculations have been used successfully, especially by Baraff [3], Chwang [13] and in various Monte Carlo calculations [6 – 12]. Due to a large amount of computer power required for Monte Carlo calculations, there is a need for a simple analytical model. This is may be achieved by the lucky-drift model of impact ionization, which was first proposed by Ridley [14] and has been developed by Burt and McKenzie [15 – 16]. The lucky-drift theory gave good agreement with Baraff's numerical solution. In addition, its agreement with Monte Carlo calculations is impressive. There was a growing conviction that impact-ionization threshold was soft. Ridley [17] and Marsland [18] have presented the argument for soft ionization using the lucky drift. The excellent fit to the experimental data confirmed that the threshold was soft. The non-local nature of impact ionization was modeled using the lucky drift [19].

Due to the existence of the soft threshold, Ridley [17] suggested the use of the Keldysh formula, which is valid only near the threshold energy and, therefore, requires a more carefull treatment.

Theoretical prediction suggests that the power exponent of the Keldysh formula [20] is higher than 2. It originates from the realistic energy bands of silicon at high electric fields and reflects the density of states [10 − 11, 21 − 23].

2. *The lucky-drift formula*

Ridley [14] has developed a simple analytic theory of impact ionization, which is known as the lucky drift model. This theory is based on a physical picture similar to that of Shichijo and Hess [4]. It gave results that are in good agreement with the Baraff theory [3]. This approach is based on the distinction between the rates of momentum and energy relaxation $\tau_{\rm m}$ and $\tau_{\rm E}$. If $\tau_{\rm m} \ll \tau_{\rm E}$, which is true for many semiconductors at very high electric field, it is possible for carriers to drift in an electric field with a drift velocity determined by momentum-relaxing collision. Meanwhile, there is no significant change in the energy relaxation. This state is called the lucky drift. Carriers are perceived to suffer collisions, which soon send them from ballistic mode into a drift mode. In this drift mode, carriers still gain energy from the field and some are lucky enough to reach the ionization threshold.

In Ridley's theory, carriers in the lucky-drift mode cause the dominant component of the ionization rate that start from the average energy once a steady state is reached for times greater than $\tau_{\rm E}$ and the energy has also relaxed.

The ionization coefficient for electron (α) depends on five parameters, namely the mean free path at 0 K ($\lambda(0)$), the phonon energy ($\hbar\omega$), the threshold energy ($E_{\rm I}$), the ionization path at threshold and the temperature. A similar set of five parameters determines the ionization for holes (β). Lucky drift model gives the following expression for α [14]

$$\alpha\lambda = \frac{1}{x}\left\{ {\rm e}^{-x} + \frac{{\rm e}^{-2rx^2} - {\rm e}^{-x}}{1-2rx} + P_{\rm T}\left({\rm e}^{-x(1-\xi)} + \frac{{\rm e}^{-2rx^2(1-\xi)} - {\rm e}^{-x(1-\xi)}}{1-2rx} \right) \right\} \tag{1}$$

where

$$\begin{aligned} x &= E_{\rm I}/(e\varepsilon\lambda), \qquad\qquad \xi = E_{\rm T}/E_{\rm I}, \\ P_{\rm T} &= 1 - {\rm e}^{-2rx(x-3)}\Big|_{x\geq 3} \quad \text{and} \quad P_{\rm T} = 0 \ \ \text{otherwise.} \end{aligned} \tag{2}$$

ε = electric field, λ = mean free path, $E_{\rm T}$ is the average energy of the thermalized electron and $P_{\rm T}$ is the probability of thermalization rather arbitrary put to zero by Ridley for $x \leq 3$.

The lucky drift mechanism is possible only if the energy relaxation time is much longer than the momentum relaxation time. At high energies typical of the impact ionization, the deformation potential scattering dominates electron phonon interaction; in this case, the ratio of the two relaxation times takes the simple form

$$\frac{\tau_{\mathrm{E}}}{\tau_{\mathrm{m}}} = \frac{E}{rE_{\mathrm{I}}}. \tag{3}$$

The factor r in the above equation represents the ratio of the effective energy loss per collision to E_{I} i.e.

$$r = \frac{\hbar\omega/E_{\mathrm{I}}}{2n(\omega)+1}, \tag{4}$$

where ω is the angular frequency of the phonon and $n(\omega)$ is the Bose-Einstein number.

Equation (1) is an expression for α, the reciprocal length traveled by the electron in the field before making an ionizing collision. The electron is assumed to be thermalized after three mean free paths with probability P_{T}. The two terms within each set of large parentheses represent the contribution from lucky-ballistic and lucky-drift electrons, which start from energy initial energy E_0 in the unthermalized case, and from the average energy in the thermalized case. We obtain the value for the mean free path at any temperature λ using the following equation [19]

$$\lambda = \frac{\lambda(0)}{2n(\omega)+1}. \tag{5}$$

3. *The soft lucky-drift formula*

A major weakness in the hard lucky-drift approach due to Ridley and Burt [14 − 16] is the assumption that carriers impact-ionize immediately upon attaining the threshold energy. In this section a lucky drift model, which does not make this assumption, is described. This is the soft threshold energy lucky drift model.

In order to include soft threshold energy effect, we considered the lucky drift approach. If $P(E)$ is the probability of an electron starting from zero energy and reaching energy E then

$$P(E) = \exp\left\{-\int_0^t \frac{\mathrm{d}t}{\tau_i(E)}\right\} = \exp\left\{-\int_0^E \frac{\mathrm{d}E'}{e\varepsilon v\tau_i}\right\} \qquad (\varepsilon > 0), \tag{6}$$

where v is the velocity (group velocity for ballistic carriers and drift velocity for lucky-drift carriers) and $\tau_i = W_{\mathrm{I}}^{-1}$. Keldysh [20] defined the energy dependence of impact ionization scattering rate W_{I} in the following way

$$W_{\mathrm{I}} = W_{\mathrm{ph}}(E_{\mathrm{I}})P\left(\frac{E-E_{\mathrm{I}}}{E_{\mathrm{I}}}\right)^2, \quad E > E_{\mathrm{I}}. \tag{7}$$

Wood [24] defined the energy dependence of impact ionization scattering rate as follows

$$W_{\mathrm{I}} = W_{\mathrm{ph}}(E_{\mathrm{I}}) P \sqrt{\frac{E_{\mathrm{I}}}{E}} \left(\frac{E - E_{\mathrm{I}}}{E_{\mathrm{I}}} \right), \quad E > E_{\mathrm{I}} \tag{8}$$

where $W_{\mathrm{ph}}(E_{\mathrm{I}})$ represents the scattering rate at threshold, usually determined by phonons, and in which P is a numerical factor. It measures the hardness of the threshold.

In this study, we use the reciprocal of the ionization relaxation rate, $\tau_i^{-1} = W_{\mathrm{I}}$,

$$W_{\mathrm{I}} = W_{\mathrm{ph}}(E_{\mathrm{I}}) \; P \left(\frac{E - E_{\mathrm{I}}}{E_{\mathrm{I}}} \right)^m, \quad E > E_{\mathrm{I}}. \tag{9}$$

Equation (9) reduces to the Keldysh formula (7) for $m = 2$, and to the Wood's formula (8) for $m = 1$ with $\sqrt{E_{\mathrm{I}}/E}$ approximated by 1. This approximation is valid only in the energy range $E > E_{\mathrm{I}}$. Thus, Eq. (9) is defined as a generalization of Keldysh formula and Wood's formula.

The probability for impact ionization becomes

$$P_{\mathrm{I}} = \int_{E_{\mathrm{I}}}^{\infty} P(E) \exp\left\{ - \int_{E_{\mathrm{I}}}^{E} \frac{\mathrm{d}E'}{e\varepsilon v \tau_i} \right\} \frac{\mathrm{d}E}{e\varepsilon v \tau_i}. \tag{10}$$

The ionization coefficient is then

$$\alpha = \int_{E_{\mathrm{I}}}^{\infty} \frac{e\varepsilon}{E} P(E) \exp\left\{ - \int_{E_{\mathrm{I}}}^{E} \frac{\mathrm{d}E'}{e\varepsilon v \tau_i} \right\} \frac{\mathrm{d}E}{e\varepsilon v \tau_i}. \tag{11}$$

The total ionization coefficient consists of a ballistic contribution and a lucky-drift contribution within the ballistic contribution,

$$e\varepsilon v \tau_i = \frac{e\varepsilon\lambda}{rp} \frac{E}{E_{\mathrm{I}}}. \tag{12}$$

On the other hand, in lucky drift, we have

$$e\varepsilon v \tau_i = \frac{\left(e\varepsilon\lambda\right)^2}{2rpE_{\mathrm{I}}}, \tag{13}$$

where

$$p = \frac{\tau_{\mathrm{E}}}{\tau_i}. \tag{14}$$

Equation (14) can be written as

$$p = p_0 \sqrt{\frac{E}{E_{\mathrm{I}}}} \left(\frac{E - E_{\mathrm{I}}}{E_{\mathrm{I}}} \right)^m. \tag{15}$$

A better parameter to measure the hardness p_0, known as the Keldysh factor, is defined by

$$p_0 \quad = \quad W_{\mathrm{ph}}\ (E_{\mathrm{I}})\ \tau_{\mathrm{E}}(E_{\mathrm{I}})\ P = \frac{E_{\mathrm{I}}(2n(\omega)+1)}{\hbar\omega}\ P, \tag{16}$$

where $\tau_{\mathrm{E}}(E_{\mathrm{I}})$ is equal to the energy-relaxation time at threshold energy (E_{I}).

4. *Theory fitted to experimental results*

Lucky drift ionization rate has the advantages of great simplicity and less extensive computations compared to the detailed Monte Carlo simulation. In a previous work of Ridley [17] and Marsland [18], best fit of lucky soft drift parameters has been derived by the Keldysh formula of Eq. (7). This equation depends on qualitative calculations of the first-order time dependent perturbation for a product of a single plane wave function and for Coulomb force among electrons in the conduction and valence bands. It yields $m = 2$ and it is well known as the Keldysh formula.

In his calculations of the average ionization rate in all momentum directions, Kane [25] used the first-order time dependent perturbation for Hartree–Fock type wave function, which consists of a single particle Bloch function for screened Coulomb force. The Bloch function and screened Coulomb force are numerically calculated for real complicated energy band of Si at very high electric field. The calculation, which yielded $m = 4$ for threshold energy $E_{\mathrm{I}} = 1.1$ eV, is considered exact.

Beattie [26] calculated the first-order time-dependent perturbation for the Hartree-type wave function with constant matrix element. Keldysh's result ($m = 2$) is obtained for the spherically symmetric conduction and valence bands. However, the higher powers ($m = 2.5$, $m = 3.5$) can be obtained by averaging over all momentum directions.

Theoretical prediction suggested that the exponent power in the Keldysh formula with $m = 4$ originates from realistic energy bands of Si at high electric field which reflects the density of states of energy bands [10, 22].

Ridley [17] suggested that the use of the Keldysh energy dependence is valid only near threshold energy, due to the existence of soft threshold, and it requires more accurate treatment.

A realistic impact ionization rate formula numerically extracted by Thoma et al. [11] used simple analytical bands reflecting realistic density of states. Analytical formulae with powers 3 and 2 in the energy range 1.1 to 3.0 eV were obtained.

The calculated impact ionization rate is well fitted to an analytical formula with the power exponent of 4.6 by Kamakura et al. [10], who used full-band Monte Carlo technique for Si.

The experimental data, in the cases of electron and hole in GaAs, Si, Ge, InP and $In_{0.47}Ga_{0.53}As$, have been used to fit the soft lucky drift according to the generalized Keldysh formulae described in the previous section. The phonon energy in each case has been chosen from the common phonon model of Ridley [14]. The published values of the threshold energy for ionization were used. The optimum Keldysh factor constant (p_0), the optimum mean free path and generalized Keldysh exponent power (m) are chosen to give the best least-squares fit to the experimental data.

The initial parameters and the corresponding data parameters for fitting program such as the mean free path at 0 K ($\lambda(0)$), the phonon energy ($\hbar\omega$), and the threshold energy (E_I) are selected according to the published values [17, 18].

4.1. GaAs

The experimental data of GaAs have been taken from the work of Bulman et al. [27]. These data were confirmed by photo multiplication and by noise measurements. Thus, they are considered highly accurate data. Anderson and Crowell [28] have calculated the threshold energies for ionization by electron and hole of GaAs to be 1.7 eV and 1.4 eV, respectively. Pearsall et al. [29] found the threshold energies to be 2.01 eV for an electron and 1.58 eV for a hole. Brennan and Hess [30] used the threshold energy as variable to fit the Monte Carlo calculation to experimental data. They found the ionizing energies of 1.7 eV and 1.42 eV for electron and hole, respectively.

In our calculations, the electron and the hole impact-ionization coefficients have been calculated with threshold energy of 1.7 eV for electron and 1.6 eV for hole based on Ridley calculation [17]. Ridley [14] raised Anderson and Crowell [28] threshold energy due to a small effect of spin–orbit splitting. The phonon energy was set to 29 meV as given by Ridley's common phonon model [14].

The deviation in the calculated soft lucky drift, according to the generalized Keldysh formula from the measured experimental data for the impact ionization coefficient rate (S_α), should be minimized to give the best fitting parameters such as p_0 and m.

The deviation S_α is defined as follows

$$S_\alpha \quad = \quad \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}\big(\ln\alpha_i(\varepsilon)^{\mathrm{calc}} - \ln\alpha_i(\varepsilon)^{\mathrm{exp}}\big)}, \tag{17}$$

where n is the number of the experimental points.

Plots of S_α, p_0 and $\lambda(0)$ versus m for electron in GaAs at 300 K are presented in Fig. 1, where the threshold electron energy was fixed at 1.7 eV. The following set of parameters corresponding to minimum of S_α in Fig. 1 is given by

$$p_0 = 3.2, \qquad \lambda(0) = 105.66\,\text{Å}, \qquad m = 1.86 \quad \text{and} \quad S_\alpha = 3.4\times10^{-3}.$$

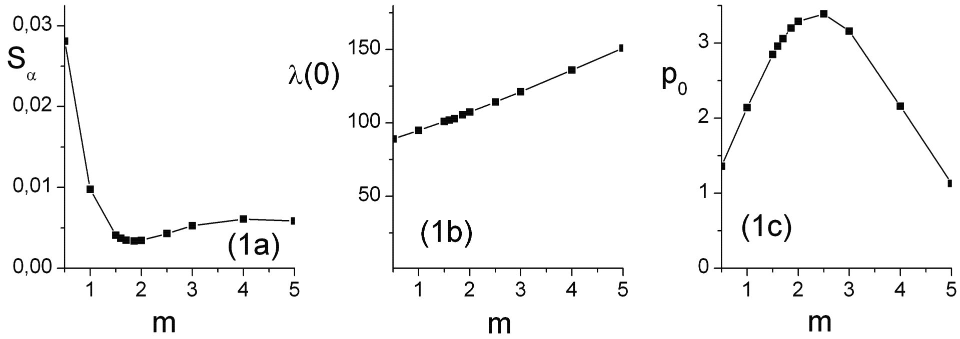


Fig. 1. Deviation function S_α, optimal mean free path $\lambda(0)$ and parameter p_0 versus parameter m in ionization relaxation of electron for GaAs.

The calculated impact-ionization coefficients that give the best fit to the experimental data of Bulman et al. [27] for both electron and hole are shown in Fig. 2(a) for GaAs at 300 K. The impact fitting parameters for both electron and hole are summarized and compared with both Ridley [17] and Marsland [18] calculations in Tables 1 and 2.

TABLE 1. Summary of our fitting results.

		E_I (eV)	m	$\lambda(0)$ (Å)	p_0	$P(0)$	S_α
GaAs	e	1.7	1.86	105.7	3.2	0.054	3.4×10^{-3}
	h	1.6	1.05	81.4	1.4	0.025	1.34×10^{-2}
InP	e	1.8	3.37	95.66	0.8	0.011	2.0×10^{-1}
	h	1.7	1.7	83.09	1.45	0.021	7.5×10^{-2}
Si	e	1.2	3.26	119.4	2.2	0.010	1.27×10^{-1}
	e	1.1	3.54	118.7	1.66	0.83	1.20×10^{-1}
	h	1.8	2.98	98.3	1.6	0.073	1.38×10^{-3}
Ge	e	0.8	5.1	185.27	4.7	0.17	1.8×10^{-2}
	h	0.9	8.54	174.7	213.21	6.87	2.28×10^{-3}
$In_{0.47}Ga_{0.53}As$	e	1.0	2.98	83.4	249.8	6.74	1.26×10^{-3}
Pearsall [42]	h	1.0	0.58	69.6	6.29	0.16	1.3×10^{-3}
$In_{0.47}Ga_{0.53}As$	e	1.0	3.24	160.2	0.085	0.001	6.0×10^{-4}
Osaka [43]	h	1.0	3.04	121.2	0.181	0.002	1.2×10^{-3}

TABLE 2. Comparsion of our results withs those of Ridley (R) and Marslan (M).

		E_I (eV)			$\lambda(300)$ (Å)			$p(300)$		
		ours	R	M	ours	R	M	ours	R	M
GaAs	e	1.7	1.7	1.7	53.7	54.9	50.4	0.027	0.027	0.007
	h	1.6	1.6	1.2	41.4	47	45	0.013	0.016	0.004
InP	e	1.8	1.8	1.47	43	33.1	34	0.005	0.007	0.002
	h	1.7	1.7	1.38	37.7	36.2	36	0.010	0.007	0.001
Si	e	1.2	1.2	1.1	94.0	68.2	77.4	0.086	0.38	0.011
	h	1.8	1.8	1.8	77.4	69.9	75.6	0.057	0.034	0.003
Ge	e	0.8	0.8	-	94.5	75	-	0.086	0.062	-
	h	0.9	0.9	-	88.9	65	-	3.49	0.531	-
$In_{0.47}Ga_{0.53}As$	e	1.0	-	1.0	40.0	-	35.9	3.23	-	∞
Pearsall [42]	h	1.0	-	1.0	33.4	-	33.4	0.081	-	0.46
$In_{0.47}Ga_{0.53}As$	e	1.0	-	-	76.78	-	-	0.0023	-	-
Osaka [43]	h	1.0	-	-	58.11	-	-	0.0049	-	-

4.2. InP

We have used the experimental data set from Umebu et al. [31], Kao and Crowell [32] and Cook et al. [33], which cover a large range of electric fields. Kao and Crowell [32] have calculated the threshold energies of electron and hole as 1.99 eV and 1.65 eV, respectively. However, Pearsall [34] has calculated to be 1.47 eV and 1.38 eV for electron and hole, respectively. Brennan and Hess [30] used the threshold energy as variable to fit the Monte Carlo calculation to experimental data and found the electron threshold to be 2.1 eV and hole to be 1.55 eV. The ionization coefficients of electron and hole presented in Fig. 2(b) using the soft lucky drift of threshold energy were reported by Ridley [17] (1.8 eV and 1.7 eV stand for electron and hole, respectively). The phonon energy of 35 meV has been set from Ridley's common phonon mode [14]. Note that the electron impact ionization is much lower in InP than in GaAs, as seen when comparing data in Figs. 2a and b.

The influence of variation of E_I on the optimal fitting parameters p_0, m and $\lambda(0)$ presented in Figs. 3a, b and c is studied due to widespread speculation about the electron threshold energy values reported in the published data, especially for GaAs and InP. The values of selected E_I are in the range of various reported data between 1.2 and 2.2 eV) [17−18,25−33]. The values of m and p_0 remained nearly constant in InP. However, p_0 increases slowly in GaAs. It is also interesting to note that the mean free path $\lambda(0)$ is enhanced by nearly 25 % at the highest electron threshold energy value for both GaAs and InP.

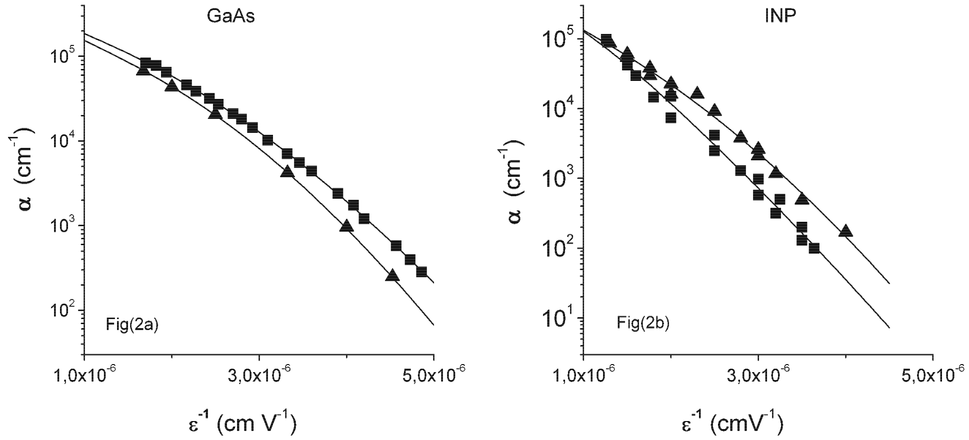


Fig. 2. Calculated (α, β) and experimental values for impact ionization coefficients of GaAs *and* InP *as function of reciprocal electric field (ε^{-1}). Experimental: electron – squares (■) and hole – triangles (▲). The solid curve represents our theoretical prediction.*

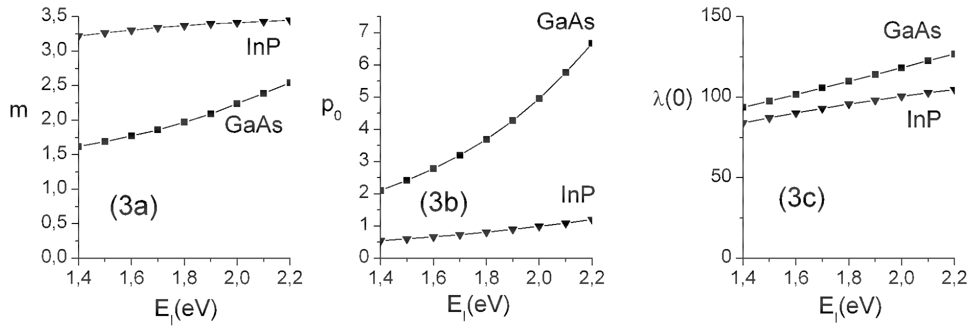


Fig. 3. Optimal parameters m, p_0 and mean free path versus threshold energy in ionization relaxation of electron in GaAs *and* InP*.*

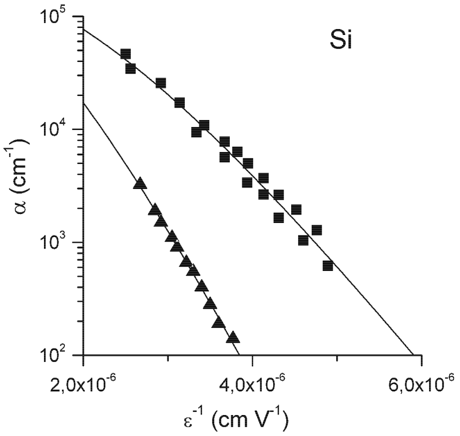
4.3. Si

The experimental data for the ionization coefficients of electron and hole in Si have been taken from the work reported by Lee et al. [35]. Threshold energy for ionization (E_I) has been calculated, utilizing energy and momentum conservation for carriers before and after ionization. Hauser [36] obtained a higher value for electron ionizing energy in Si ($E_{\mathrm{I}} = 1.4$ eV). Ahmad et al. [37] used a two-valley conduction band and minimized the energy associated with the final carriers. They found the electron threshold energies 1.115, 1.52, and 2.06 eV. Anderson et al. [28] generalized Ahmad's graphical method for realistic energy bands by minimizing the energy and momentum conservation. They obtained the threshold energy of 1.1 eV for electron and 1.8 eV for hole. Tang et al. [6] suggested the most probable ionizing energy to be 1.1 eV for electron. Figure 4 presents the electron and hole ionization coefficients calculated using the values of the threshold energy reported by Ridley [17] who raised the Anderson threshold energy value to correct for neglecting the spin–orbit splitting effect. The best fitting results for electron and hole are given in Table 1. The quality of the fit is excellent. It is also interesting to note that Tanaka [22] used a lucky drift model including soft threshold energy of Marsland and generalized Keldysh formula. The obtained optimal-fitting parameter set of electron with threshold energy 1.1 eV is

$$p(300) = 2.452 \times 10^{-2}, \quad \lambda(0) = 85.4\,\text{Å}, \quad m = 3.52 \quad \text{and} \quad S_\alpha = 3.642 \times 10^{-2}.$$

These values are in good agreement with our fitting parameters in Table 1, if we replace the threshold energy with 1.1 eV.

Figure 5 shows a comparison among impact ionization probability scattering rates for Si of our work and other reported data as function of energy. The reported data are either obtained by using realistic full band structure [9 – 10] or simplified band with the real density of state included [11]. The ionization proba-



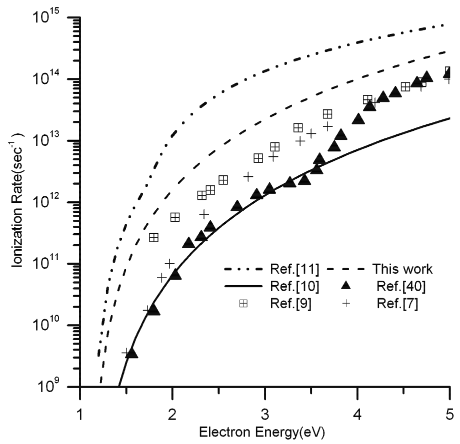


Fig. 4 (left). Calculated (α, β) and experimental values for impact ionization coefficients of Si *as function of reciprocal electric field (ε^{-1}). Experimental: electron – squares (■) and hole – triangles (▲). The solid curve represents our theoretical prediction.*

Fig. 5. Comparison between our calculated impact ionization rates with some other references presented in the literature for Si *as function of electron energy.*

bility scattering rates show that the well-known soft behavior increases slowly as the electron energy increases. It is important to note that most models cannot be fitted by the simple Keldysh formula.

The impact ionization probability scattering rates of our work are extracted from the optimal Keldysh factor and exponent power m in Table 1 for threshold energy 1.1 eV. In our calculations, $W_{\mathrm{ph}}(E_{\mathrm{I}})$ was taken from reported data [12, 38] to be approximately 0.5×10^{14} s^{-1} at threshold energy.

The Keldysh factor used by other authors seems to be much lower than our predicted value except that of Thoma et al. The large discrepancy can be explained as follows: the Keldysh formula contains two parameters that are usually fixed by the best fit of the experimental coefficient data as a function of inverse electric field. However, there is a large spread in the experimental data [35, 39] that may affect the possible choice of these parameters to give reasonable best fit for calculated impact ionization scattering rate. It is interesting to note that using the lucky drift model with generalized Keldysh formula does not lose physical effects significantly compared to the full band Monte Carlo model of electron ionization probability scattering rate.

4.4. Ge

The ionization coefficients have been calculated using the soft lucky drift for threshold energy of 0.9 eV and 0.8 eV for electron and hole, respectively. Anderson and Crowell [28] have calculated this threshold energy. In our calculations, we used the previous threshold energy and the phonon energy of 29 meV, which has been taken from Ridley's common phonon mode [14]. The impact ionization coefficients that gave the best fit to the experimental data [41] for both electron and hole are shown in Fig. 6.

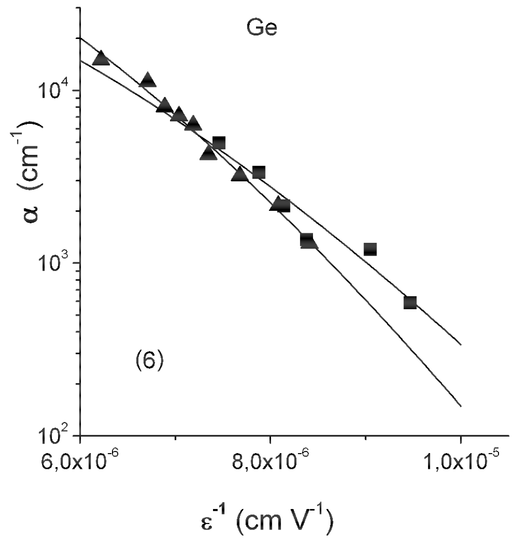


Fig. 6. Calculated (α, β) and experimental values of impact ionization coefficients of Ge *as functions of reciprocal electric field (ε^{-1}). Experimental: electron – ■ and hole – ▲. The solid curves represent our theoretical predictions.*

4.5. $In_{0.47}Ga_{0.53}As$

Increasing and widespread research on III–V alloys suitable for long-wavelength optoelectronic devices has stimulated investigation of the impact ionization coefficients of these alloys and their binaries. The experimental data for $In_{0.47}Ga_{0.53}As$ have been taken from the work of Pearsall et al. [42]. We have used both electron and hole threshold energy to be 1.0 eV as estimated by Pearsall [42]. The phonon energy of 27 meV has been taken from Ridley's common phonon mode [14].

The impact ionization coefficients that give the best fit to the experimental data of Pearsall [42] and Osaka al. [43] for both electron and hole, are shown in Figs. 7a and b, respectively. The values of α and β obtained by Osaka [43] are small compared with those of Pearesall [42]. Unfortunately, there is a great discrepancy in the reported ionization coefficients in $In_{0.47}Ga_{0.53}As$ for both electron and hole. The fitting parameter λ_0 obtained from the data of Osaka [43] is twice the value obtained from the data of Pearesall [42], while the value of p_0 obtained from the data of Osaka[43] is very small compared with that obtained from the data of Pearsall [42].

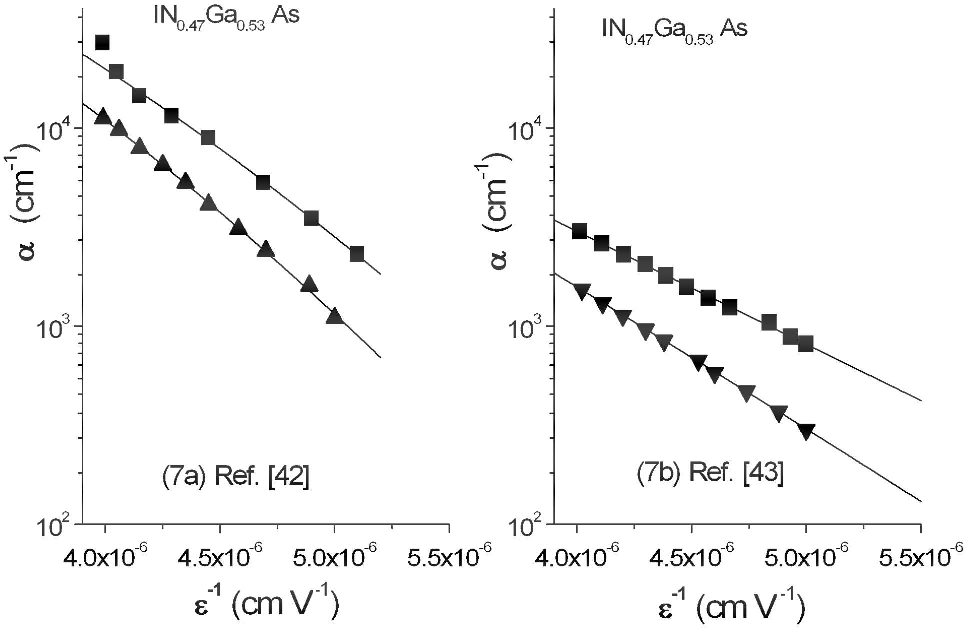


Fig. 7. Calculated (α, β) and experimental values of impact ionization coefficients of $In_{0.47}Ga_{0.53}As$ as functions of the reciprocal electric field (ε^{-1}). Experimental values: electron – ■ and hole – ▲. The solid curves represent our theoretical predictions (left) data of Pearsall [42] and (right) for data of Osaka [43] after Ref. [44].

5. *Summary and conclusion*

A fit of soft threshold lucky drift theory using a generalized Keldysh formula (8) to the experimental data has been made using the least-squares algorithm E04FDF (Nag library). It is useful to note that the generalized Keldysh formula becomes appropriate for realistic bands and reflects the density of states of energy bands.

The parameters p_0, $\lambda(0)$ and the optimum power exponent m have been obtained for both electron and hole in GaAs, Ge, InP, Si, and $In_{0.47}Ga_{0.53}As$. They are summarized according to the generalized Keldysh formula in Table 1.

It is important to note that the results (see Table 1) for the generalized Keldysh power, m, is equal to 1.0 for the hole impact ionization in GaAs. This is in agreement with Wood's classical theory [24]. The values of m for electrons impact ionization in InP, Ge, Si and $In_{0.47}Ga_{0.53}As$ are in the range 8.0 − 3.0, that is in agreement with results of other authors $[10-11, 22, 26]$. Meanwhile, for the hole ionization, the value of m varies between 1.6 and 0.58. In fact, the different values of m reflect the influence of real band structures and the density variation of energy band in the conduction and valence bands.

In each case, the fit is compared to a similar fit of the soft lucky drift formulated by Marsland and Ridley according to Keldysh formulae summarized in Table 2. The results have given good matching to the measured results. The quality of the fit is excellent, especially the mean free path at 300 K for the above semiconductors. On the contrary, the Keldysh factor at 300 K ($p(300)$) of our calculations and of Ridley differ from the Marsland calculation by approximately the factor of 10. We cannot explain the reason of this difference. All we can say is that the experimental data are well fitted with the values we have selected by using the least-squares fitting algorithm. Our investigation showed that neglecting the ballistic contribution in the impact ionization rates given by Marsland is not responsible for the above difference.

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KOEFICIJENTI UDARNE IONIZACIJE ELEKTRONOM I ŠUPLJINOM U POLUVODIČIMA S JAKIM POLJEM

Načinili smo prilagodbe Ridleyvog mekog sretno-posmičnog modela eksperimentalnim podacima za GaAs, InP, Si, Ge i $In_{0.47}Ga_{0.53}As$. Postigli smo odlično slaganje teorije i eksperimentalnih podataka primjenom metode najmanjih kvadrata. Primijenili smo poopćenu Keldyshevu formulu radi uvođenja mekog faktora praga. Poopćena Keldysheva formula proizlazi iz realnih razmatranja energijskih vrpci u poluvodičima s jakim električnim poljima što odražava gustoću stanja energijskih vrpci. Izračunali smo Keldyshev faktor i nove vrijednosti srednjih slobodnih puteva. Ridleyeve and Marslandove objavljene vrijednosti za srednje slobodne puteve u dobrom su skladu s našima, ali još uvijek nalazimo velike razlike među Keldyshevim faktorima.