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LINEAR AND NONLINEAR PROPAGATION OF ION-ACOUSTIC WAVE IN A BOUNDED PLASMA CONTAINING NEGATIVE IONS

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Linear and nonlinear propagation of ion-acoustic waves are theoretically investigated in a multicomponent plasma consisting of electrons, positive ions and negative ions bounded in a cylindrical waveguide. The stability of the ion-acoustic wave is discussed taking into account the role of finite geometry and the concentration of negative ions of the plasma. The effect of nonlinearity on the ion-acoustic wave is investigated through the derivation of the effective potential (Sagdeev potential) and the results are discussed graphically with the variation of ion-streaming and the geometry of the bounded plasma.

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1. Introduction

Linear and nonlinear propagation of waves in multicomponent plasma consisting of electrons, positive ions and negative ions have been studied theoretically and experimentally by many authors as they are very important in different contexts of laboratory experiments and space plasmas. Smith [1] showed that the negative ions affect the grouptravel time of whistlers at the mid-latitude and at the equator and cyclotron damping of whistlers at the mid-latitude of the ionosphere. D'Angelo et al. [2] showed that ion waves have two kinds of modes of propagation in plasma having negative ions, one of these is 'a slow-ion-mode' and other mode is 'a fast-ion-mode'. One should mention here that 'fast-ion-mode' has been observed by Wong et al. [3] and 'slow-ion-mode' by Sato and Ameniya [4]. Propagation of electrostatic wave, particularly of ion-acoustic waves (IAW) through ionospheric plasma consisting of negative ions in addition to electrons and positive ions, has been found to be more interesting, specially for the formation of solitary waves and double-layers in the plasma. A detailed study of ion-acoustic solitary wave (IASW) in the presence of negative ions has been done by Das [5] and the remarkable conclusion is that negative ions may take an effective role in order to prevent the breaking of solitary waves into many more solitons. The credit for the pioneering work in this regard also goes to Das and Tagare [6]. They showed that even a small quantity of negative ions may be important for the formation of IASW. Subsequently, a large number of researchers investigated the effects of negative ions on solitary waves and obtained interesting results from which it has been found that both compressive and rarefactive soliton may exist in a negative-ion plasma. At a particular density of negative ions, the nonlinear coefficient vanishes and this density is known as critical density. The behaviour of solitons at critical density is described by the modified Korteweg-de Vries (K-dV) equation which takes into account the cubic nonlinearity [7]. Nakamura et al. [8] showed that when the concentration of negative ions is larger than a critical value, a small compressive pulse evolved into subsonic wave trains and a large pulse develops into a solitary wave. They also measured the threshold amplitude and the velocity of the solitary waves and compared with the predictions using the pseudopotential method. Verheest [9], Singh and Das [10] and other authors have also studied the propagation of solitary waves at critical density of negative-ions in multicomponent plasma. In an inhomogeneous negative-ion plasma, considering the plasma density and ion drift to be spatially varying, Chauhan and Dahia [11] derived the modified K-dV equation and showed that the amplitude of compressive and rarefactive mK-dV solitons depends on the relation between the zero-order velocity and density.

Recently, Chattopadhyaya et al. [12] found that the streaming ions have significant contribution to the excitation of IASW and double-layers in a negative-ion plasma. When the drift velocity of ions is small, the amplitude of the solitary waves becomes large. If the drifting velocity of ions is very close to the phase velocity of the wave, the amplitude of the solitary wave becomes very small and solitons may not exist in the plasma. They also showed that with the increase of negative-ion concentration, the amplitude of the solitary waves increases and the potential difference of the double layers decreases. Regarding instability of ion-acoustic wave, the work of Paul et al. [13] has some new findings. They showed that the concentration of negative ions and stream velocity have major role on the instability of IAW. From the dispersion relation, they showed that IAW has six modes of propagation and some modes are unstable. To study the instability of IAW, they considered (H^+,O^-) ions, (H^+,O_2^-) ions, (H^+, SF_5^-) ions, (He^+,Cl^-) ions and (Ar^+,O^-) ions. In a plasma having (He^+,Cl^-) ions, the first and second modes of IAW will be highly unstable if the concentration of negative ions is large. Moreover, for heavy negative ions, the instability of IAW is lower than that of lighter negative ions. On the other hand, for the third and fourth modes of IAW, the instability decreases with the increase of negative-ion concentration. In a plasma consisting of (Ar^+,O^-) ions, there exists a limiting value of the relativistic velocity ($u_0/c \neq 0.6$) for an unstable wave. However, the works of the earlier authors mentioned above were done in an infinite or unbounded plasma. But, the finite geometry of the bounded plasma is important for the stability of the wave and also for the excitation of solitary waves and double-layers in negative-ions plasma. Little work has been done by the researches on the propagation of waves in a bounded plasma system, though it is important and relevant to experimental set up, as shown by Ghosh and Das [14]. Mondal et al. [15] studied the propagation of ion-acoustic wave in a bounded plasma system having electrons and positive-ions only. They showed that the trapping of particles is favoured when the effect of boundary is taken into account keeping other parameters within certain range of values. Recently, Bhattacharya et al. [16] theoretically investigated the instability of ion-acoustic wave in relativistic plasma consisting of positive ions, negative ions and two temperature electrons and compared the results of unbounded and bounded plasma systems. It is seen that the phase velocity of the wave in unbounded plasma is greater than that of bounded plasma. Moreover, the phase velocity falls off with the increase of negative-ion concentration in the same fashion for bounded plasma. But for the unbounded plasma, the phase velocity decreases with the increase of negative ions up to some smaller value and then remains unchanged.

As the findings of the propagation of IAW in plasma in the presence of negative ions have been found to be more interesting in bounded plasma than in unbounded plasma, we are interested in the present paper to study both the linear and nonlinear propagation of waves in a plasma bounded in finite geometry, considering the effect of electron inertia. In the first part, study the instability of IAW in the bounded negative-ion plasma through the derivation of the first-order dispersion relation. In the second part, nonlinear propagation is studied using the pseudopotential method. The profiles of effective potential have been depicted to see the effect of concentration of negative ions and stream velocity and also the geometry of the plasma.

2. Formulation

We consider the plasma to be collisionless, unmagnetised, non-relativistic and confined to a perfectly conducting cylinder of radius R. The plasma consists of electrons, positive ions and negative ions. It is assumed that the ions have streaming velocities along the axis of the cylinder taken to be the x-axis. The effect of electron-inertia has also been taken into consideration. The set of dimensionless equations governing the dynamics of such a plasma system are:

For positive ions and negative ions

$$\frac{\partial n_s}{\partial t} + \frac{\partial}{\partial x}(n_s u_s) = 0, \tag{1}$$

$$\frac{\partial u_s}{\partial t} + u_s \frac{\partial u_s}{\partial x} = -\frac{\psi_s}{Q_s}\frac{\partial \phi}{\partial x}, \tag{2}$$

where the subscript s is given by $s = i$ for positive ions, $s = j$ for negative ions; $\psi_i = 1$, $\psi_j = -1$; $Q_i = 1$ and $Q_j = Q$.

For electrons

$$\frac{\partial n_e}{\partial t} + \frac{\partial}{\partial x}(n_e u_e) = 0, \tag{3}$$

$$q\left(\frac{\partial u_e}{\partial t} + u_e \frac{\partial u_e}{\partial x}\right) + \frac{1}{n_e}\frac{\partial n_e}{\partial x} = \frac{\partial \phi}{\partial x}. \tag{4}$$

We use the Poisson's equation

$$\nabla_\perp^2 \phi + \frac{\partial^2 \phi}{\partial x^2} = n_e - \sum \psi_s n_s,$$

where

$$\nabla_\perp^2 = \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2} \tag{5}$$

is the tranverse Laplacian, $Q = m_j/m_i$ and $q = m_e/m_i$. In the above equations, m_e, m_i and m_j denote the masses of electrons, positive ions and negative ions, respectively. n_e, n_i, n_j and u_e, u_i, u_j are the densities and velocities of the corresponding species. In the above equations, the velocities are normalised by $\sqrt{k_\mathrm{B} T_e/m_i}$, the densities by n_0, the equilibrium electron density and all lengths by the Debye length $\sqrt{k_\mathrm{B} T_e/(4\pi n_0 e^2)}$, whereas the potential ϕ by $k_\mathrm{B} T_e/e$, k_B being the Boltzmann constant and time by ω_{pi}^{-1}, i.e., $m_i/(4\pi n_0 e^2)$.

3. Instability of ion-acoustic wave

For the linear analysis, we consider the variables in Eqs. (1)–(5) are perturbed as deviations of the quantities

$$\begin{aligned} &n_s = n_{so} + n_{s1}, \qquad n_e = 1 + n_{e1}, \\ &u_s = u_{s0} + u_{s1}, \qquad u_e = u_{e1}, \qquad \phi = \phi_1. \end{aligned} \tag{6}$$

We use Eqs. (6) in Eqs. (1) – (5) and assume the spatial and temporal dependence of the perturbed part to be of the form $f(r)\exp\{\mathrm{i}(kx - \omega t)\}$, where k is the wave number and ω is the frequency of the wave. Following Sayal and Sharma [17], we impose the condition that the electric potential on the surface of the cylinder is zero, i.e., $\phi(r) = 0$ for $r = R$, and obtain

$$p_{0n}^2 = (kR)^2 \left(\sum \frac{n_{s0}}{Q_s(\omega - k u_{s0})^2} + \frac{n_{e0}}{(\omega^2 q - k^2)^2} - 1 \right), \tag{7}$$

where p_{0n} are the roots of $J_0(x) = 0$ ($J_0(x)$ is Bessel function). On simplification, relation (7) gets transformed into

$$a_6 k^6 + a_5 k^5 + a_4 k^4 + a_3 k^3 + a_2 k^2 + a_1 k + a_0 = 0, \tag{8}$$

where

$$
\begin{aligned}
a_0 &= qQp_{0n}^2\omega^4, \\
a_1 &= -2qQu_0^2p_{0n}^2\omega^3, \\
a_2 &= qQp_{0n}^2\omega^2u_0^2 - \omega^2p_{0n}^2Q - R^2\big(qQ\omega^4 + n_{i0}\omega^2qQ + n_{j0}q\omega^2 + Q\omega^2\big), \\
a_3 &= 2Qp_{0n}^2\omega u_0 + R^2\big(2u_0\omega Q + 2qQu_0\omega^3\big), \\
a_4 &= -u_0^2p_{0n}^2Q + R^2\big(n_{i0}Q + n_{j0} + \omega^2Q - qQu_0^2\omega^2 - Qu_0^2\big), \\
a_5 &= -2Qu_0\omega R^2, \\
a_6 &= QR^2u_0^2,
\end{aligned}
$$

$n_{i0} = 1 + n_{j0}$ (charge neutrality condition) and $u_{i0} = u_{j0} = u_0$. Eq. (8) has been solved numerically and the solutions represent six modes of propagation of the IAW in the bounded plasma. Every root of Eq. (8) corresponds to a particular mode of propagation. On solving Eq. (8) for different sets of plasma parameters, it has been found that the first root is always real and the fifth and sixth roots are complex, one being conjugate to another. From the real parts of fifth and sixth roots, the phase velocities of IAW have been calculated. The imaginary parts of these complex roots have been used to find the decay rate (k_{im}). One should mention that the positive and negative values of k_{im} represent the decay and the growth rate of the wave, respectively. In this regard, it is important to note that Eq. (8) is reduced to the fourth order of k, when the negative ions are not present in the plasma. Moreover, Eq. (8) will be reduced to the equation of Mondal et al. [15], when the presence of negative ions and streaming electrons are neglected. Figs. 1a−1c show the variation of the phase velocity ($v_{\rm ph}$) with negative-ion concentration (n_{j0}) of the plasma having (Cl^-,H^+) and (O^-,Ar^+) ions, for various values of the radius of cylinder (R) as well as the mass ratio of negative ions and positive ions (Q) and the streaming velocity (u_0). From Fig. 1a, it is observed that for the forward-going wave, the phase velocity increases with the increase of the concentration of negative ions. Moreover, for a large value of the radius of the cylinder, the phase velocity is large. The rate of increase of the phase velocity for small radius of the cylinder is higher than that for the large radius. The phase velocities of the reflected mode of the wave are shown in Figs. 1b and 1c. It is seen that the phase velocity in (Cl^-,H^+) plasma is lower than that in (O^-,Ar^+) plasma. Due to the increase of stream velocity, the phase velocity also decreases for the reflected wave. Figure 1d shows the effect of electron inertia on the variation of phase velocity with n_{j0}. It is observed that for the values of $q(1/10, 1/36, 1/1836)$ phase velocity increases linearly with n_{j0} almost at the same rate with the only exception in the region $0.27 < n_{j0} < 0.35$ when $q = 1/1836$. In this region, a hump appears indicating sudden change in the rate of increase of the phase velocity. In Fig. 1e, the combined effects of R, u_0 and q on the variation of $v_{\rm ph}$ with n_{j0} are displayed. A close study of the plot reveals some interesting features. Firstly, the IAWs propagate with greater phase velocity when the wave guide is of smaller radius, the ions have smaller streaming velocities and

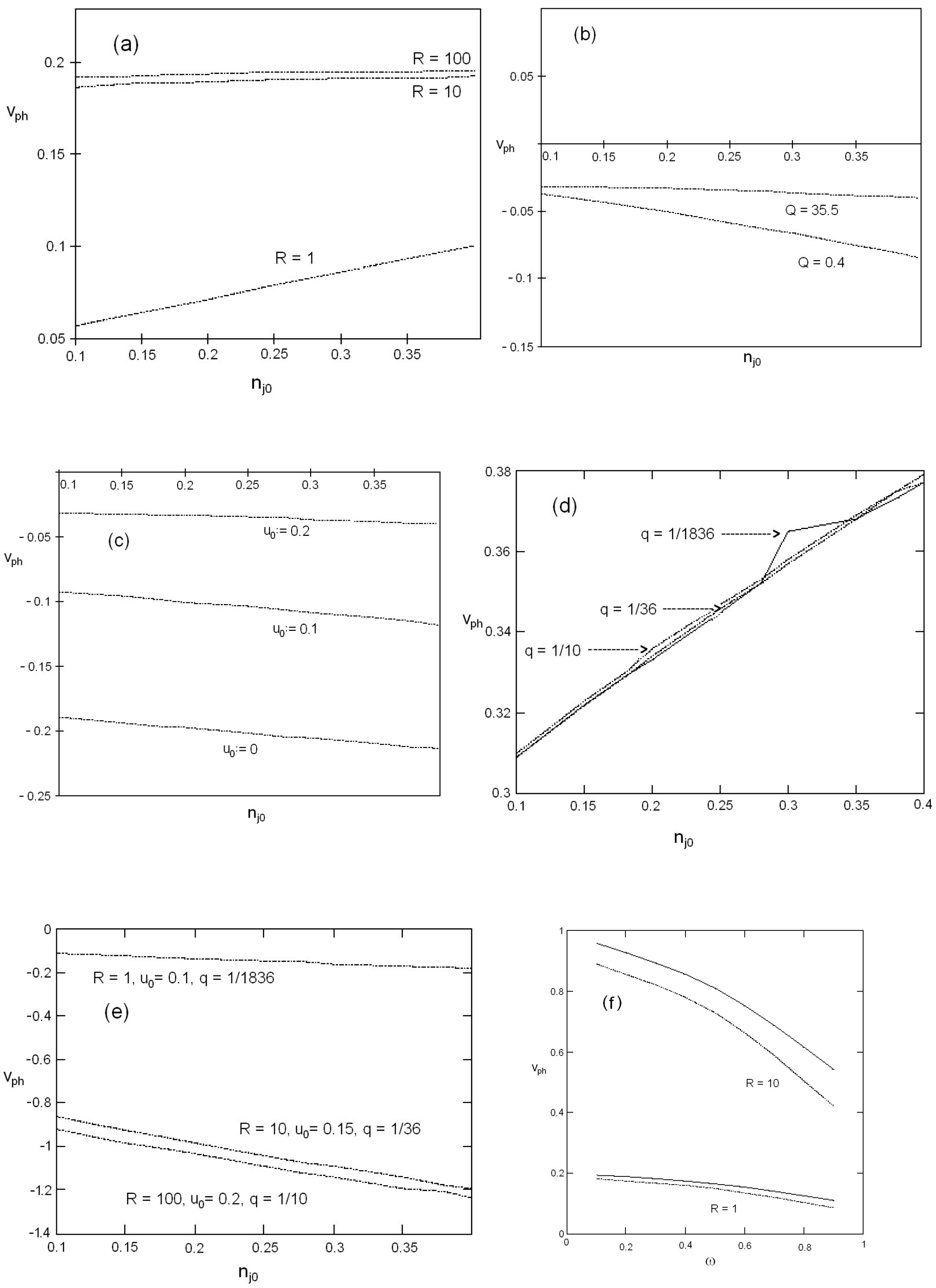
(a)
R = 100
R = 10
R = 1
v_{ph}
n_{j0}
(b)
Q = 35.5
Q = 0.4
v_{ph}
n_{j0}
(c)
u_0:= 0.2
u_0:= 0.1
u_0:= 0
v_{ph}
n_{j0}
(d)
q = 1/1836
q = 1/36
q = 1/10
v_{ph}
n_{j0}
(e)
R = 1, u_0= 0.1, q = 1/1836
R = 10, u_0= 0.15, q = 1/36
R = 100, u_0= 0.2, q = 1/10
v_{ph}
n_{j0}
(f)
R = 10
R = 1
v_{ph}
ω

Fig. 1 (figures at left). (a) Variation of $v_{\rm ph}$ with n_{j0} for various values of R where other parameters are $Q = 0.4$, $u_0 = 0.2$, $p_{0n} = 5.5$, $\omega = 0.1$, $q = 1/1836$ (Mode 5). (b) Dependence of $v_{\rm ph}$ (for the reflected mode) on n_{j0} for different values of Q where other parameters are $R = 1$, $u_0 = 0.2$, $p_{0n} = 5.5$, $\omega = 0.1$, $q = 1/1836$ (Mode 1). (c) Variation of $v_{\rm ph}$ (for the reflected mode) with n_{j0} for different values of u_0 where other parameters are $Q = 35.5$, $R = 1$, $p_{0n} = 5.5$, $\omega = 0.1$, $q = 1/1836$ (Mode 1). (d) Variation of $v_{\rm ph}$ with n_{j0} for different values of q and other parameters $u_0 = 0.2$, $\omega = 0.1$, $R = 1$, $Q = 0.4$, $p_{0n} = 5.5$ (Mode 4). (e) Plot of variation of $v_{\rm ph}$ (for the reflected mode) with n_{j0} for different values of the parameters R, u_0 and q when other parameters are $\omega = 0.1$, $Q = 0.4$, $p_{0n} = 5.5$ (Mode 1). (f) Variation of $v_{\rm ph}$ with ω for various values of R in the case of Ref. [15] (dotted line) ($n_{j0} = 0$) and present results (solid line) ($n_{j0} = 0.15$, $Q = 35.5$) where other parameters are $u_0 = 0$, $p_{0n} = 5.5$, $q = 1/1836$.

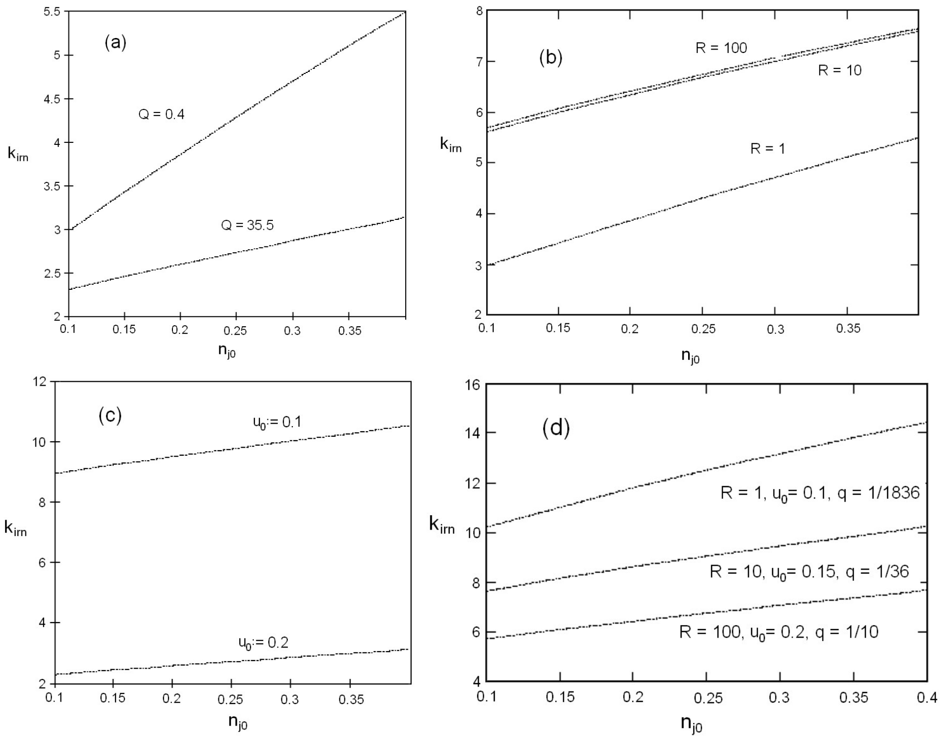

Fig. 2. (a) Change of k_{im} the instability factor with n_{j0} for different values of Q where other parameters are $u_0 = 0.2$, $R = 1$, $p_{0n} = 5.5$, $\omega = 0.1$, $q = 1/1836$ (Mode 5). (b) Dependence of k_{im} on n_{j0} with R as parameter where other parameters are $Q = 0.4$, $u_0 = 0.2$, $p_{0n} = 5.5$, $\omega = 0.1$, $q = 1/1836$ (Mode 5). (c) Variation of k_{im} with n_{j0} for different streaming velocities u_0 where other parameters are $Q = 35.5$, $R = 1$, $p_{0n} = 5.5$, $\omega = 0.1$, $q = 1/1836$ (Mode 5). (d) Plot of variation of k_{im} on n_{j0} for different values of the parameters R, u_0 and q when other parameters are $\omega = 0.1$, $Q = 0.4$, $p_{0n} = 5.5$ (Mode 5).

the electron inertia is smaller simultaneously. Secondly, the phase velocity of the IAW come close as the values of the parameters mentioned above become large. Thirdly, the phase velocity decreases with n_{j0} for any set of parameter values. The rate of decrease of the phase velocity with the increase of in n_{j0} is found to be slower for smaller parameter values, whereas the phase velocity falls off at faster rate for larger parameter values. The dependence of the phase velocity on the frequency of the wave is observed in Fig. 1f from which it is seen that phase velocity increases in the presence of negative ions. Furthermore, the phase velocities in the absence of negative ions are greater than in (Cl^-, H^+) plasma. Again, we see that the phase velocity decreases with the increase in the wave frequency (ω). When the radius of the cylinder is large, the velocity decreases rapidly. Figs. 2a–2c show the dependence of the decay rate (k_{im}) on the negative ion concentration for different values of Q, R and streaming velocity (u_0). In Fig. 2a, we observed that the decay rate increases almost linearly with negative ion concentration and the rate of increase is greater for the plasma having smaller value of Q. Figure 2b shows that the decay rate increases with n_{j0} nearly at the same rate for smaller and larger values of the radius. When the stream velocity is low, the growth rate is high which is shown in Fig. 2c. It is observed from Fig. 2d that the instability of the IAWs increases linearly with n_{j0} for any set of the parameter values. For a particular value of the negative ion concentration, the IAW become more unstable if the values of radius of the wave guide, ion streaming velocities and the electron inertia are smaller simultaneously.

4. The effective potential

In order to derive the expression for the effective potential, we use the perturbation expansions as adopted in the linear part, but we do not linearise the basic equations. Consequently, we obtain a set of nonlinear equations. At this stage, we assume that the radial behaviour of the perturbations is described by the lowest-order Bessel function. Thus, following Mondal et al. [15], we take the perturbed quantities to be of the form $J_0(k_\perp r)f(x,t)$, where $f(x,t) = N_s(x,t), N_e(x,t), U_s(x,t), \ldots$ and $k_\perp = P_{0n}/R$. Integrating the former nonlinear equations over r from 0 to R, after multiplying with $rJ_0(k_\perp r)$, we have

$$\frac{\partial N_s}{\partial t} + n_{s0}\frac{\partial U_s}{\partial x} + u_{s0}\frac{\partial N_s}{\partial x} + \alpha N_s \frac{\partial U_s}{\partial x} + \alpha U_s \frac{\partial N_s}{\partial x} = 0, \tag{9}$$

$$\frac{\partial N_e}{\partial t} + \frac{\partial U_e}{\partial x} + \alpha N_e \frac{\partial U_e}{\partial x} + \alpha U_e \frac{\partial N_e}{\partial x} = 0, \tag{10}$$

$$\frac{\partial U_s}{\partial t} + u_{s0}\frac{\partial U_s}{\partial x} + \alpha U_s \frac{\partial U_s}{\partial x} = -\frac{\psi_s}{Q_s}\frac{\partial \phi}{\partial x}, \tag{11}$$

$$q\left(\frac{\partial U_e}{\partial t} + \alpha U_e \frac{\partial U_e}{\partial x} + \alpha N_e \frac{\partial U_e}{\partial t} + \beta N_e U_e \frac{\partial U_e}{\partial x}\right) + \frac{\partial N_e}{\partial x} = \left(1 + \alpha N_e\right)\frac{\partial \phi}{\partial x}, \tag{12}$$

and

$$\frac{\partial^2 \phi}{\partial x^2} - \phi \quad = \quad N_e - \sum \psi_s N_s \tag{13}$$

where

$$\alpha = \int_0^R J_0^3 r \mathrm{d}r \Big[\int_0^R J_0^2 r \mathrm{d}r \Big]^{-1}, \qquad \beta = \int_0^R J_0^4 r \mathrm{d}r \Big[\int_0^R J_0^2 r \mathrm{d}r \Big]^{-1}.$$

In order to have the stationary solutions of Eqs. (9) − (13), we have to use the transformation $\xi = x - Mt$ where M is the velocity of the nonlinear structure in units of speed of sound (Mach number). As a result, Eqs. (9)–(13) yield

$$N_s \quad = \quad \frac{n_{s0}/\alpha}{-1 + \sqrt{1 - 2\alpha\phi/(Q_s(M - u_{s0})^2)}} \tag{14}$$

$$N_e \quad = \quad \frac{U_e}{M - \alpha U_e} \tag{15}$$

and

$$\big(\beta - \alpha^2\big)\frac{q}{3}U_e^3 + \frac{\alpha q M}{2}U_e^2 + \big(1 - qM^2 q\big)U_e - \big(\frac{M \ln M}{\alpha} + M\phi\big) = 0. \tag{16}$$

The real roots of (16) are given by

$$\big(U_e\big)_{\mathrm{real}} = \frac{Z - A_1}{A_0}, \tag{17}$$

where, $Z = X - Y/X$, $X^3 = (-G + \sqrt{G^2 + 4Y^3})/2$, $G^2 + 4Y^3 > 0$, $G = A_0^2\delta - A_0 A_1 \gamma + 2A_1^3$, $Y = A_0\gamma/3 - A_1^2$, $A_0 = q(\beta - \alpha^2)/3$, $A_1 = qM\alpha/6$, $\gamma = -1 - qM^2$ and $\delta = -M(\phi + \ln M/\alpha)$. So, Poisson's equation (13) takes the form

$$\frac{\partial^2 \phi}{\partial \xi^2} = \phi + \frac{U_e}{M - \alpha U_e} - \sum \frac{\psi_s n_{s0}/\alpha}{-1 + \sqrt{1 - \alpha\phi/(Q_s(M - u_{s0})^2)}} = -\frac{\partial V}{\partial \phi}, \tag{18}$$

where $V(\phi)$ is the effective potential. Equation (18) is highly complicated and it is difficult to get the explicit expression for $V(\phi)$. So, we have solved (18) numerically and calculated the values of V for a model plasma for various values of Q, n_{j0}, p_{0n} and M. The positive value of ϕ and negative value of V indicates that the nature of the excited soliton is compressive. From Figs. 3a − 3d we see how the effective potential V is influenced by various parameters of the plasma. Moreover, we find that the profiles, as shown in Figs. 3a and b, are very close to each other, whereas those shown in Figs. 3a and d are widely different. From Fig. 3a, it is seen that for (O^-, Ar^+) plasma, V is more negative than for (SF_5^- ,H^+) plasma, i.e. the trapping of particles is more likely for $Q = 0.4$ (O^-, Ar^+) where $n_{j0} = 0.1$, $u_0 = 0.2$, and Mach number 1.5. It is seen that for (O^-, Ar^+) plasma, $V(\phi)$ is negative when

ϕ runs from 0.23 to 0.55, which indicates that compressive solitary wave will be

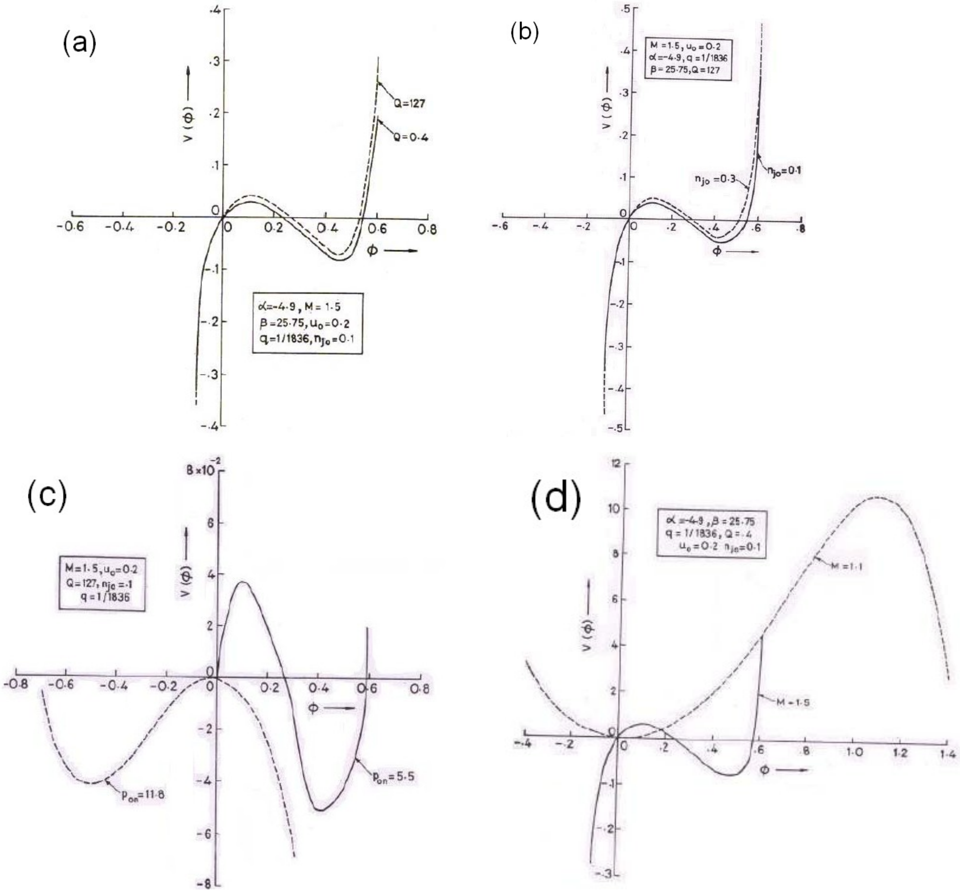

Fig. 3. (a) Variation of the effective potential $V(\phi)$ with ϕ for different plasmas. (b) Dependence of the effective potential $V(\phi)$ on ϕ for different negative-ion concentrations n_{j0}. (c) Plot of the effective potential $V(\phi)$ against ϕ for $p_{0n} = 5.5$, 11.8. (d) Dependence of the effective potential $V(\phi)$ on ϕ for the mach numbers $M = 1.1$ and 1.5.

formed. But when $\phi < 0.23$, $V(\phi)$ is positive, i.e. solitary wave will not exist in this plasma. In (SF_5^-, H^+) plasma, $V(\phi)$ is negative when $\phi = 0.23$ to 0.53, i.e. compressive solitary wave will be formed, but when $\phi < 0$, $V(\phi)$ is negative and no potential well is formed. In Fig. 3b, trapping of ions is more probable when $n_{j0} = 0.1$ than when $n_{j0} = 0.3$, and in this case also compressive solitons are excited. In Fig. 3c, the nature of change of $V(\phi)$ with p_{0n} is found to be peculiar. It is observed that for $p_{0n} = 11.8$, ϕ is negative, i.e., rarefactive soliton exists, but for $p_{0n} = 5.5$, ϕ is positive, i.e. compressive soliton exists. Lastly, from Fig. 3d we see that when $M = 1.1$, $V(\phi)$ is positive and this implies non-existence of soliton, whereas for $M = 1.5$, $V(\phi)$ turns out to be negative when $\phi = 0.23$ to 0.55, and this indicates excitation of compressive soliton. It is to be noted that Figs. 3a–3d, represented by the energy integral equation (18), correspond to solitary waves excited in the plasma under specific condition, not for all the values of the plasma parameters.

5. *Concluding remarks*

We have theoretically investigated the propagation of IAW in a multi-component plasma consisting of electrons, positive ions and negative ions bounded in finite geometry. Comparing our results with those of Mondal et al. [15], we here find that the presence of negative ions will create some new features in the linear and non-linear propagation of IAW. In the first-order approximation, it is observed that the phase velocity of the wave increases with the concentration of negative ions and the increase of the radius of the cylinder. When the mass ratio of the negative ion to the positive ions is large, the phase velocity is lower. In the analysis of the stability of the wave, it is found that the decay rate is higher for the (O^-, Ar^+) plasma than for the (Cl^-, H^+) plasma and with the increase of the concentration of negative ions the decay rate increases. In non-linear analysis, we find that the trapping of charged particles is higher when the negative-ion concentration is lower. The most interesting results are seen for the variation of the value of p_{0n}. When $p_{0n} = 11.8$, the rarefactive soliton will occur. But the compressive soliton exists when $p_{0n} = 5.5$. So, with the variation of p_{0n}, the excitation of solitons may be controlled. Moreover, we find that for low value of Mach number ($M = 1.1$), the soliton can't be excited in bounded plasma. But when $M = 1.5$, the soliton of compressive type may be excited.

Experimental results obtained by Lonngren [18] and others [19,20] in a negative-ion plasma for linear and nonlinear propagation of waves may be compared if the conditions imposed on our theoretical analysis are matched with the experimental configuration. It is to be noted that we have theoretically discussed the possibility of excitation of solitary waves. The potential structure of the solitary wave and its amplitude and width have not yet been calculated. In some cases, we can compare our results with those obtained by Mondal et al. [15]. If Fig. 1d of our paper is compared with Fig. 1c of Mondal et al. [15], then some important differences become evident. Firstly, the rate of decrease of the phase velocity with ω gets slowed down in the presence of negative ions, irrespective of the radius of the wave guide, compared with that in the absence of negative ions. Secondly, for particular values of ω and R, the phase velocity is greater in the presence of negative ions. Again, we can make a rough comparism between Fig. 7 of Mondal et al. [15] and Fig. 3c of our work. There exists a potential well only for $\phi > 0$ (as shown in Fig. 7 of Mondal et al. [15]) which corresponds to the excitation of a compressive soliton, whereas in our case, i. e. in the presence of negative ions in the plasma, there exists an inverted profile for certain range of values of $\phi (0 \leq \phi \leq 0.25)$ which indicates nonexistence of soliton in the plasma, and for another range of values of

$\phi(0.25 \leq \phi \leq 0.6)$, there is the possibility of trapping of particles, implying the existence of compressive soliton as found in Fig. 7 of Mondal et al. [15]. In both cases rarefactive solitons are likely to occur in the plasma and trapping of particles is less favoured for larger values of p_{0n}, the root of $J_0(x) = 0$. In other figures for $V(\phi)$ we find the presence of inverted profile for certain range of values, signaling to reflection of particles that shows nonexistence of soliton.

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LINEARNO I NELINEARNO ŠIRENJE IONSKO-ZVUČNIH VALOVA U PLAZMI U POSUDI KOJA SADRŽI I NEGATIVNE IONE

Teorijski istražujemo linearno i nelinearno širenje ionsko-zvučnih valova u višekomponentnoj plazmi koja se sastoji od elektrona, te pozitivnih i negativnih iona i nalazi se u valjkastom valovodu. Raspravljamo stabilnost ionsko-zvučnih valova uzimajući u obzir utjecaj ograničenog prostora i koncentracije negativnih iona u plazmi. Istražujemo učinke nelinearnosti na ionsko-zvučni val putem izvođenja efektivnog (Sagdeevog) potencijala a ishodi numeričkih računa predočuju se grafički za različita strujanja iona i oblike valovoda.