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ELECTROSTATIC INTERACTION ENERGY AND FACTOR 1.23

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The factor $F \approx 1.23$ has originally been found in the redshift of quasars. Recently, it has been found in very different physical phenomena: the life-time of muonium, the masses of elementary particles (leptons, quarks,...), the correlation of atomic weight (A) and atomic number (Z) and the correlation of the sum of masses of all orbiting bodies with the mass of the central body in gravitational systems.

In this work, we describe further systems where the factor F appears, the distributions of electric charges. We consider the electrostatic interaction between various pairs of charge distributions: two uniformly charged spheres, a point charge and a uniformly charged sphere, two point charges, a point charge and an infinite uniform line charge, two parallel uniform infinite line charges and two infinite parallel plane charge distributions.

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1. *Introduction*

It has been shown [1] that the quantization period of the intrinsic redshift of quasars can well be presented by

$$(1 + z_2) = 1.23(1 + z_1) \quad \text{or} \quad z_n = (1 + z_0)1.23^n - 1, \quad n = 0, 1, 2... \tag{1}$$

where z_{n+1} is the next higher redshift to z_n. The resulting values of z are 0.061, 0.30, 0.60, 0.91, etc. Many details about observations of the redshift of quasars and galaxies and relevant references may be found in the book of H. Arp [2].

Equation (1) has been obtained by a fit to the observational data and does not have a definite theoretical explanation yet. Consequently, it is unknown whether the factor 1.23 (we denote it by F) is a fundamental constant or is given by some other constants.

The quantized redshift of quasars could be caused by smaller masses of elementary particles. An electron with a smaller mass within a given atom (also of a smaller mass) whould radiate the light of longer wavelength. Because the radiation is due to the interaction between elementary particles, one may expect that the factor $F = 1.23$ could be hidden in the masses of elemetary particles. It might also appear in some other peculiar physical phenomena.

By analyzing experimental and observational data of various systems, the authors of the present work have demonstrated [3] that the factor F seems to appear in various physical systems.

A brief summary of results in Ref. [3] will now be outlined in order to indicate a possible relation to energy of interaction for simple electrostatic systems.

a) Elementary particles and the factor *F*. An analysis of the masses of elementary particles, particularly of leptons: electron, muon and tauon has shown [3] that the factor F can be approximately estimated as

$$F = \sqrt{\frac{m_\mu}{m_\mathrm{e}}\alpha} = 1.2284\,, \tag{2}$$

where m_μ and m_e are the muon and electron masses, respectively, and α is the fine-structure constant (data from Ref. [4]). Factor F given by Eq. (2) may also be found in the life-time of the muon [5] in muonium (M, bound system $\mu^+ + \mathrm{e}^-$) as

$$\tau_\mathrm{M} = \tau_\mu^+ \left(1 + \frac{1}{2}\,\frac{m_\mathrm{e}}{m_\mu}\alpha^2\right)\,, \tag{3}$$

where τ_μ^+ is the life-time of a free muon. Equation (3), by taking into account Eq. (2), may be written as

$$\tau_\mathrm{M} = \tau_\mu^+ \left[1 + \frac{1}{2}\left(\frac{\alpha}{F}\right)^4\right]\,. \tag{4}$$

Extended formulae, including the mass of τ-lepton, are

$$\begin{aligned}
(m_\mathrm{e} + m_\mu)/m_\mathrm{e} &= F^2/\alpha & F &= 1.2313\\
(m_\mathrm{e} + m_\mu + m_\tau)/(m_\mathrm{e} + m_\mu) &= F^2/\alpha^{1/2} & F &= 1.2309\\
(m_\mathrm{e} + m_\mu + m_\tau)/m_\mathrm{e} &= F^4/\alpha^{3/2} & F &= 1.2311
\end{aligned}$$

The mean value of the three results for F is

$$F = 1.2311 \pm 0.0002\,. \tag{5}$$

We have also found that the masses of d, s and b quarks are satisfactorily given by

$$m_k = m_{\mathrm{d}} \left(\frac{F^3}{\sqrt{\alpha}} - 1 \right)^{k-1}, \quad F = 1.23 \tag{6}$$

where $k = 1, 2, 3$ stands for d, s and b quarks, respectively. The experimental values and the values from Eq. (6) are compared in the following table (assuming $m_{\mathrm{d}} = 9.9$ MeV/c^2)

	m_{d}	m_{s}	m_{b}
Exptl. values (MeV/c^2)	10 ± 5	200 ± 100	4300 ± 200
Eq. (6) (MeV/c^2)	9.9	206.3 ± 0.1	4301 ± 5

Similarly, a simple empirical equation for the masses of the quarks u, c and t ($k = 1, 2, 3$ for u, c and t quarks, respectively) is

$$m_k = m_{\mathrm{u}} \left(\frac{F}{\alpha} - 1 \right)^{k-1}, \qquad F = 1.23\,. \tag{7}$$

A comparison with the experimental values (taking $m_u = 6.4$ MeV/c^2) is given in the following table

	m_{u}	m_{c}	m_{t}
Exptl. values (MeV/c^2)	5 ± 3	1300 ± 300	180000 ± 12000
Eq. (7) (MeV/c^2)	6.4	1073.3 ± 0.3	180003 ± 58

The values of the quark masses from Eqs. (6) and (7) are well within the errors of the experimental values.

We have also considered the masses of mesons and baryons. We found that for all groups of elementary particles, it is possible to find simple functions of F and α which satisfactorily reproduce the masses of the particles. More details may be found in Ref. [3].

b) $\boldsymbol{A - Z}$ correlation and the factor $\boldsymbol{F}$. The next more complicated systems are the atomic nuclei. The $A - Z$ correlation of mean atomic weight A and atomic number Z has been examined. Instead of the well-known semi-empirical correlation based on the liquid drop model of nuclei [6] in the form

$$Z = \frac{A}{1.98 + 0.015 A^{2/3}}\,, \tag{8}$$

we rather use a simple correlation [3] given by

$$A = a Z^b\,. \tag{9}$$

The best fit for the parameters a and b results

$$A = (1.4991 \pm 0.0236) Z^{(1.1173 \pm 0.0036)} \,.$$

This power law extrapolated to very high Z is in a very good agreement with the recent theoretical prediction [7] for the nucleus $Z = 114$ and $A = 298$, and also with recent experiments [8].

One may take that, approximately, $a = F^2$ and $b = \sqrt{F}$. Therefore, with only one parameter, Eq. (9) transforms to

$$A = F^2 Z^{\sqrt{F}}, \qquad F = 1.2375 \pm 0.0003 \,. \tag{10}$$

In previous functions for elementary particles, it was shown that expressions for F include α (Eqs.(4), (6) and (7)). Therefore, also here one may try a small correction as

$$A = (F + \alpha)^2 Z^{\sqrt{F+\alpha}} \,, \tag{11}$$

and the best fit is obtained with

$$F = 1.2302 \pm 0.0003 \,.$$

However, one cannot justify such an arbitrary introduction of the fine-structure constant into Eq. (11).

c) Gravitational systems and the factor *F*. We consider the solar system. The mass of the central body (the Sun or a planet) is denoted by M_C. The sum of masses of all bodies (planets or satellites) moving around the central body is denoted by m_s. There are five such systems: the Sun with the planets and four systems of a planet and with its satellites. The last four systems are those of Jupiter, Saturn, Uranus and Neptune.

The correlation of m_s with M_C in a log-log coordinate system [3] gives a straight line in the form

$$\log(m_s) = (1.220 \pm 0.040) \log(M_C) + (-9.558 \pm 1.094) \,, \tag{12}$$

or $m_s = 2.76610^{-10} M_C^{1.220}$. Equation (12) may be written in another form

$$\frac{m_s}{M_{\rm LG}} = \left(\frac{M_C}{M_{\rm LG}} \right)^{1.22} \,, \tag{13}$$

where $M_{\rm LG} = 2.8 \times 10^{43}$ kg and it can be understood as the mass of the Local Group. The correlation is extended with the stars similar to the Sun, for which are known the extra-solar planets and also to the globular clusters [3]. The exponent (1.220 ± 0.040) is supposed to be the factor F, although the error is large.

All systems considered above suggest that it could be suitable to introduce a factor F close to 1.23, which could be a constant which is important in diverse systems, from elementary particles to large gravitational systems. The aim of this work is to present the available empirical evidence, with a hope that it will stimulate theoretical investigations.

In the next section we describe another system in which the factor 1.23 appears.

2. *Interaction energy of two charge distributions and the factor F*

2.1. *Two spherical charge distributions*

We consider the interaction energy between two positive spherical charge distributions of equal radii R, with charges q and q', at the centre-to-centre distance b. The charges on the spheres are supposed to be homogeneously distributed and fixed in order to avoid polarization effects. Parameters necessary for the calculation are written in Fig. 1. At a given point, the addition of the electric field vectors $\boldsymbol{E}$ and $\boldsymbol{E}'$ gives the resulting vector $\boldsymbol{E}_r$ whose value is given by

$$E_r^2 = E^2 + E'^2 + 2EE' \cos\theta_0 \,. \tag{14}$$

The energy W_V of the electric field in a volume V ([9,10] is given by

$$W_V = \frac{\epsilon_0}{2} \int E_r^2 \, \mathrm{d}V \,, \tag{15}$$

where ϵ_0 is the electrical permitivity of vacuum. Equations (14) and (15) give a total energy W_t of two charged spheres

$$W_t = \frac{\epsilon_0}{2} \int E^2 \, \mathrm{d}V + \frac{\epsilon_0}{2} \int E'^2 \, \mathrm{d}V + \epsilon_0 \int EE' \cos\theta_0 \mathrm{d}V \,. \tag{16}$$

The first two terms at the right are the self-energies of the charges q and q', respectively, while the third term is the interaction energy which, integrated over all space, generates the Coulomb potential energy $W = qq'/(4\pi\epsilon_0 b)$.

Thus, the interaction energy is

$$W = \epsilon_0 \int EE' \cos\theta_0 \mathrm{d}V \,. \tag{17}$$

We will calculate this energy in the two parts of space in order to determine its negative and positive contributions. From Fig. 1 and Eq. (17), one can see that inside the sphere of radius $b/2$ which touches the centres of the two spherical charge distributions at the oposite points, $\cos\theta_0$ is negative, because $\theta_0 > 90°$, and, therefore, the interaction energy is negative. On the surface of that sphere $\theta_0 = 90°$, the

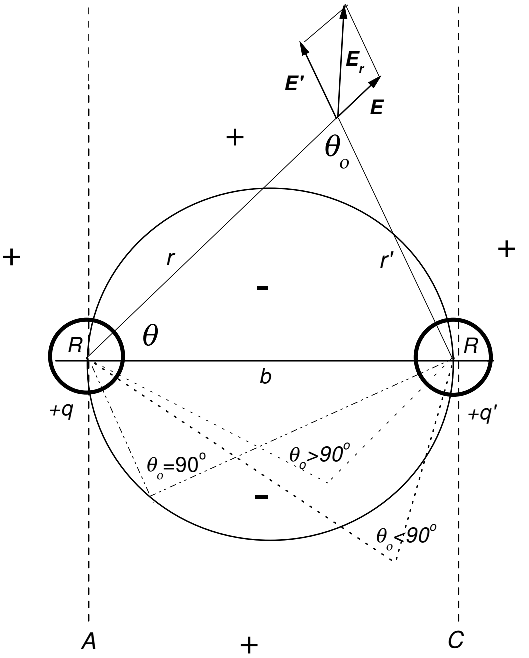


Fig. 1. Regions of positive (+) *and negative* (−) *interaction energy of two uniform spherical charged disributions of radii R, at the centre-to-centre distance b. Positive charges q and q' produce electric field vectors $\boldsymbol{E}$ and $\boldsymbol{E}'$, at distances r and r', respectively. A and C denote vertical planes orthogonal to b. The negative energy contribution is confined to the sphere of radius $b/2$, with the centre midway between the charges.*

vectors $\boldsymbol{E}$ and $\boldsymbol{E}'$ are mutually orthogonal and, consequently, the interaction energy is zero. Outside the sphere, the interaction energy is positive because $\theta_0 < 90°$ and $\cos\theta_0 > 0$.

In order to perform the integration, one must determine $\cos\theta_0$ and volume element $\mathrm{d}V$. Because of the axial symmetry,

$$\mathrm{d}V = 2\pi r \sin\theta \mathrm{d}\theta \mathrm{d}r \,,$$

while $\cos\theta_0$ is given by

$$\cos\theta_0 = \frac{r^2 + r'^2 - b^2}{2rr'} \,.$$

An elimination of r' is performed by using the cosine law

$$r'^2 = r^2 + b^2 - 2rb\cos\theta \,.$$

The interaction energy (Eq. (17)) is then

$$W = \epsilon_0 \int \frac{q}{4\pi\epsilon_0 r^2} \frac{q'}{4\pi\epsilon_0 r'^2} (\cos\theta_0) 2\pi r^2 (\sin\theta) \mathrm{d}r \mathrm{d}\theta \,,$$

and in the final form

$$\begin{aligned} W &= \frac{1}{2}\frac{qq'}{4\pi\epsilon_0} \int \sin\theta \mathrm{d}\theta \\ &\times \left[\int \frac{r\mathrm{d}r}{(b^2+r^2-2rb\cos\theta)^{3/2}} - b\cos\theta \int \frac{\mathrm{d}r}{(b^2+r^2-2rb\cos\theta)^{3/2}} \right] . \end{aligned} \tag{18}$$

The integration of Eq. (18) has to be performed in several steps.

a) The interaction energy between the planes A and C (see Fig. 1) inside the sphere of radius $b/2$, excluding the appropriate parts of volumes of charged spheres of radii R, is given by the following range of integration

$$R \le r \le (b/2)\cos\theta \quad \text{and} \quad 0 \le \theta \le \pi/4$$

and

$$R \le r \le b\cos\theta \quad \text{and} \quad \pi/4 \le \theta \le \arcsin\sqrt{1-R^2/b^2} \,.$$

This part of the interaction energy (twice the integral defined by the upper limits) reads

$$W_{\mathrm{A-C,inside\,sphere}} = \frac{qq'}{4\pi\epsilon_0 b} \left[\frac{1}{2} - \frac{b}{R} + \frac{\pi}{4} - \arcsin\sqrt{1+\frac{r^2}{b^2}} + \sqrt{\frac{b^2}{R^2}-1} \right] . \tag{19}$$

b) The interaction energy between the planes A and C outside the sphere of radius $b/2$, excluding again the appropriate parts of volumes of charged spheres of radii R, is obtained as twice the integral confined within the following limits

$$b\cos\theta \le r \le (b/2)\cos\theta \quad \text{and} \quad \pi/4 \le \theta \le \arcsin\sqrt{1-R^2/b^2}$$

and

$$R \le r \le (b/2)\cos\theta \quad \text{and} \quad \arcsin\sqrt{1-R^2/b^2} \le \theta \le \pi/2 \,.$$

The result is

$$W_{\mathrm{A-C,outside\,sphere}} = \frac{qq'}{4\pi\epsilon_0 b} \left[-\frac{1}{2} - \frac{\pi}{4} + \sqrt{1+\frac{b^2}{R^2}} + \arcsin\sqrt{1-\frac{R^2}{b^2}} - \sqrt{\frac{b^2}{R^2}-1} \right] . \tag{20}$$

c) The interaction energy between the plane A and $-\infty$ and between the plane C and $+\infty$ follows from the integration over the range

$$R \le r \le \infty \quad \text{and} \quad \pi/2 \le \theta \le \pi$$

resulting in

$$W_{\mathrm{A-left}} = W_{\mathrm{C-right}} = \frac{1}{2}\frac{qq'}{4\pi\epsilon_0 b}\left(1 + \frac{b}{R} - \sqrt{1 + \frac{b^2}{R^2}}\right). \tag{21}$$

Hence, the positive $W(+)$ and the negative $W(-)$ contributions to the interaction energy are given by

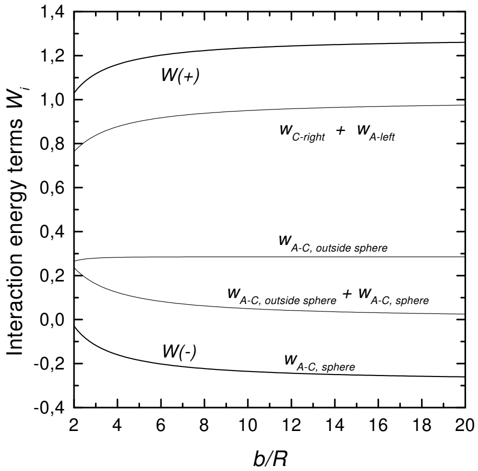
$$W(+) = \frac{qq'}{4\pi\epsilon_0 b}\left[1 + \frac{b}{R} - \frac{1}{2} - \frac{\pi}{4} + \arcsin\sqrt{1 - \frac{R^2}{b^2}} - \sqrt{\frac{b^2}{R^2} - 1}\right] \tag{22}$$

and

$$W(-) = \frac{qq'}{4\pi\epsilon_0 b}\left[-\frac{b}{R} + \frac{1}{2} + \frac{\pi}{4} - \arcsin\sqrt{1 - \frac{R^2}{b^2}} + \sqrt{\frac{b^2}{R^2} - 1}\right]. \tag{23}$$

The addition of $W(+)$ and $W(-)$ is equal to the Coulomb potential energy of the two charge distributions, $W = qq'/(4\pi\epsilon_0 b)$, as is well known.

The interaction energy terms W_i, calculated by Eqs.(19) to (23), as functions of b/R are plotted in Fig. 2, beginning with $b/R = 2$. Namely, when $b = 2R$, the charged spheres are in contact.



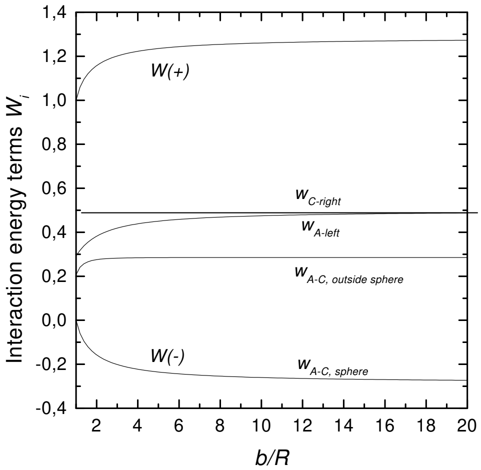


Fig. 2. Positive and negative contributions of the interaction energy $W(+)$ and $W(-)$, in units $W = qq'/(4\pi\epsilon_0 b)$, for two uniformly charged spheres, as functions of b/R. The particular energy terms W_i of definite parts of space are also presented. For $b/R >> 2$, the field becomes like that of two point charges.

Fig. 3 (at right). Interaction energy terms W_i as functions of b/R for a uniform spherical charge distribution (q) and a point charge (q'). The sphere acts like a point charge for $b/R >> 1$.

If the charges q and q' are negative, the resulting equations are unchanged, and the interaction energy is again negative inside the sphere of radius $b/2$, while the force between charges is repulsive. However, if the charges are of the opposite sign, then the interaction energy inside the sphere is positive and outside the sphere it is negative, and the force is attractive. Formally, negative interaction energy tends to increase its space.

2.2. A charged sphere and a point charge

Interaction energy terms for a uniform spherical charge distribution of radius R and a point charge (assuming both positive) show a similar behaviour as those of two uniform spherical charges. The calculations are straightforward and the results are given grafically in Fig. 3, beginning with $b/R = 1$. Then the point charge is at distance $b = R$ (on the surface of the sphere) and the Coulomb interaction energy is simply $qq'/(4\pi\epsilon_0 R)$.

For $b < R$, the point charge is inside the charged sphere, and the potential energy is constant. The negative interaction energy is equal to zero. In this case, all remaining interaction energy is of the same sign, and an absence of the force is a necessary consequence. This is, of course, a well-known result in electrostatics that within a charged spherical distribution, the electric field is zero and the force is zero, too. For the sake of completness, the plot of corresponding energies, analogous to the previous ones in Figs. 2 and 3, are shown in Fig. 4. For $b/R = 1$, the values

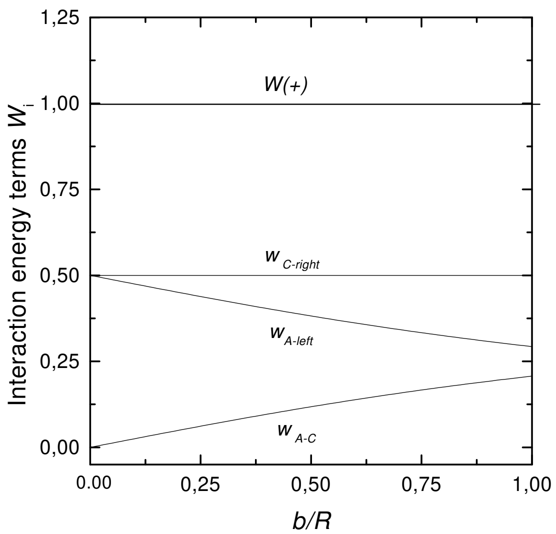


Fig. 4. Interaction energy terms W_i, for a shallow charged sphere (q) and a point charge (q') within the sphere, as functions of b/R. The center of the sphere of radius R is in the plane A, while a distance b between parallel planes A and C changes from 0 to R.

of related interaction energies are, of course, the same as in Fig. 3 for the model of a point charge just outside the sphere.

2.3. Two point charges

The interaction energy of two point charges is the most important, because such a model neglects completely the geometry of electric charges.

The result for the interaction energies obtained from previous equations (see Sect. 2.1), when b/R tends to infinity (see also Figs. 2 and 3), are listed in the following relationships:

$$W_{\mathrm{C-right}} = \frac{qq'}{4\pi\epsilon_0 b}\left(\frac{1}{2}\right), \tag{24}$$

$$W_{\mathrm{A-left}} = \frac{qq'}{4\pi\epsilon_0 b}\left(\frac{1}{2}\right), \tag{25}$$

$$W_{\mathrm{A-C,inside\,sphere}} = \frac{qq'}{4\pi\epsilon_0 b}\left(\frac{1}{2}-\frac{\pi}{4}\right), \tag{26}$$

$$W_{\mathrm{A-C,outside\,sphere}} = \frac{qq'}{4\pi\epsilon_0 b}\left(\frac{\pi}{4}-\frac{1}{2}\right). \tag{27}$$

It follows that the total positive energy is

$$W(+) = \frac{qq'}{4\pi\epsilon_0 b}(1.28539816) \tag{28}$$

and the negative one is

$$W(-) = -\frac{qq'}{4\pi\epsilon_0 b}(0.28539816). \tag{29}$$

The ratio $W(+)/W(-) = -4.50387679$ is constant, of course, independent of the mutual distance of the two charges.

The calculation has been exact until now. However, the relation with the factor $F = 1.23$, mentioned in the Introduction, may be obtained by a simple consideration: assume that the factor F is multiplied by a constant a, so that

$$Fa = 0.28539816$$

Then, with $F = 1.2311$, according to Eq. (5), $a = 0.2318$ is very close to $F - 1$. Therefore, we put

$$F(F-1) = 0.28539816, \tag{30}$$

resulting in $F = 1.2317$. Using Eqs.(26) or (27) and (30), F may be defined as follows

$$F(F-1) = \frac{\pi}{4} - \frac{1}{2}, \tag{31}$$

or explicitly

$$F = \frac{1}{2}(1 + \sqrt{\pi - 1}) = 1.23170907\,. \tag{32}$$

In order to reach the value of F in Eq. (5), one may do a slight change of Eq. (30). As described in the Introduction, one may also include here the fine-structure constant α using the factor $(1+\alpha/2) = 1.00363868$. Hence, Eq. (30) may be changed to

$$F(F-1)(1+\alpha/2) = \frac{\pi}{4} - \frac{1}{2}\,. \tag{33}$$

The resulting value $F = 1.2310$ is very close to that of Eq. (5). If $(1+\alpha/2)$ in Eq. (33) is replaced by $(1+\alpha)$, then the factor F becomes 1.2303, which is very close to that in Eq. (1).

Of course, Eq. (33) is not derived and it may be considered only as a numerical trial to reach the value of F in Eq. (5) or Eq. (1). Besides, this equation demonstrates how simple intervention may lead to expected value of $F = 1.23$, which is encountered in Eq. (1). However, only a proper theoretical approach, such as that of Eqs. (3) and (4), can justify, change or deny such an arbitrary procedure.

It should be pointed out that the calculations of the electrical interaction energy for two electric charges and of gravitational interaction of two masses m and m' are almost identical. This is a consequence of the inverse square law and vector properties of electric and gravitational fields. One needs only to replace $qq'/(4\pi\epsilon_0)$ in the resulting equations above by Gmm'. The only difference is that a gravitational force is always attractive, while the electric force is repulsive for two charges of the same sign. Because of the general tendency of decrease of potential energy, in the case of gravitation, the space with the negative interaction energy tends to decrease, while for electric charges, regardless of the sign, the tendency is the oposite, i.e., the space with the negative interaction energy tends to increase (see Sect. 2.1).

The total energy W_t, given by Eq. (16), remains positive in each part of space due to the prevailing self-energy terms. The interaction energy term, given by Eq. (17), changes significantly the energy contents in particular parts of space. Obviously, this redistribution of energy is responsible for the Coulomb interaction.

2.4. A point charge and an infinite line charge

We consider an infinite uniformly-charged straight line placed, for example, along the X-axis and a point charge located on the Y-axis at a distance b, as shown in Fig. 5. Charge on the line is fixed in order to avoid a redistribution of charge due to the presence of the point charge q'. Linear charge density is denoted by λ. The electric field vector of the line charge is given by $\boldsymbol{E} = [\lambda/(4\pi\epsilon_0 r)]\boldsymbol{r}_0$, where the unit vector $\boldsymbol{r}_0$ is in the plane of the line charge, oriented away from the line and orthogonal to the line charge. Electric field of the point charge is $\boldsymbol{E}' = [q'/(4\pi\epsilon_0 r')]\boldsymbol{r}'_0$, where $\boldsymbol{r}'_0$ is the radial unit vector oriented from the point charge.

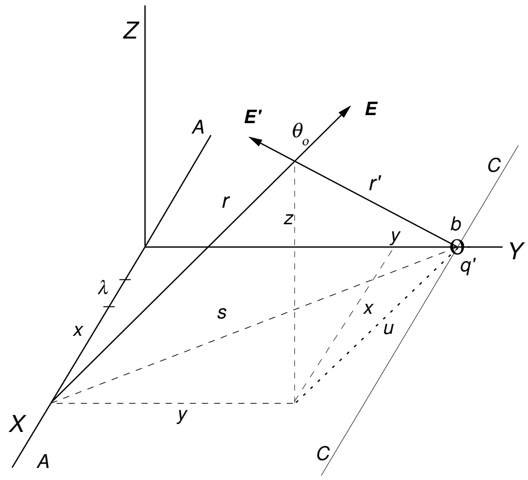


Fig. 5. System of an infinite line charge (linear charge density λ) located at the X-axis and a point charge q' at $y = b$ with $x = z = 0$. Symbols A and C denote the planes orthogonal to b. The negative energy contribution $W(-)$ is confined to the infinite cylinder with its axis at $y = b/2$, parallel to the X-axis. Parameters in the figure correspond to those in Eqs. (34).

For the calculation of the interaction energy defined by Eq. (17), we use a rectangular Cartesian coordinate system. The following relations are valid (see Fig. 5):

$$\begin{gathered} r^2 = y^2 + z^2, \quad s^2 = x^2 + b^2, \quad u^2 = x^2 + (b-y)^2\,, \\ r'^2 = x^2 + (b-y)^2 + z^2, \quad s^2 = r^2 + r'^2 - 2rr'\cos\theta_0\,, \end{gathered} \tag{34}$$

$$\cos\theta_0 = \frac{r^2 + r'^2 - s^2}{2rr'} = \frac{y^2 - by + z^2}{\sqrt{y^2+z^2}\,\sqrt{x^2+(b-y)^2+z^2}}\,. \tag{35}$$

For $\cos\theta_0 = 0$, i.e. $\theta_0 = 90°$, Eq. (35) reduces to

$$y^2 - by + z^2 = 0\,,$$

which represents the circle $(y - b/2)^2 + z^2 = (b/2)^2$, passing through the origin of the coordinate system and the point $y = b$ (located in the $Y - Z$ plane) with the center at $y = b/2$. Within the circle $\theta_0 > 90°$, on the circle $\theta_0 = 90°$ and outside the circle $\theta_0 < 90°$ (see Fig. 1). Due to the infinite line charge, $\theta_0 > 90°$ remains inside an infinite cylinder with its axis parallel to the X-axis and at the distance $y = b/2$. Outside that cylinder $\theta_0 < 90°$. Consequently, the interaction energy is negative within this cylinder and positive outside it (charges of equal sign are assumed).

The interaction energy, Eq. (17), using a volume element $\mathrm{d}V = \mathrm{d}x\mathrm{d}y\mathrm{d}z$, is

$$W = \frac{\lambda q'}{8\pi^2\epsilon_0}\int \frac{1}{rr'}\cos\theta_0 \mathrm{d}x\mathrm{d}y\mathrm{d}z\,. \tag{36}$$

or

$$W = \frac{\lambda q'}{8\pi^2 \epsilon_0} \int \frac{y^2 - by + z^2}{(y^2+z^2)(x^2+(b-y)^2+z^2)^{3/2}} \mathrm{d}x\mathrm{d}y\mathrm{d}z\,. \tag{37}$$

The integration of Eq. (37) has to be done in several steps. The interaction energy contribution within the cylinder located between the planes A and C is given by

$$W_{\mathrm{A-C,cyl.}} = \frac{\lambda q'}{8\pi^2 \epsilon_0} \int\limits_0^b \mathrm{d}y \int\limits_0^{\sqrt{yb-y^2}} 2\mathrm{d}z \frac{y^2 - by + z^2}{(y^2+z^2)} \int\limits_0^\infty 2\mathrm{d}x \frac{1}{(x^2+(b-y)^2+z^2)^{3/2}}\,. \tag{38}$$

The factor 2 in the last two integrals in Eq. (38) is a consequence of the chosen range of integration. After a tedious, but straightforward calculation, the result is given by

$$W_{\mathrm{A-C,cyl.}} = \frac{\lambda q'}{2\pi^2 \epsilon_0} \left(-\frac{\pi}{2}\ln 2\right)\,. \tag{39}$$

It is still necessary to determine the constant ahead of the bracket. The force of the line charge on the point charge is $\phi = \lambda q'/(2\pi\epsilon_0 b)$, and a change of energy by moving away the point charge from b to b_1 is

$$\Delta W = \frac{\lambda q'}{2\pi\epsilon_0} \int\limits_b^{b_1} \frac{\mathrm{d}b}{b} = \frac{\lambda q'}{2\pi\epsilon_0} \ln\frac{b_1}{b}\,.$$

Thus, Eq. (39) may be written as

$$W_{\mathrm{A-C,cyl.}} = \frac{\lambda q'}{2\pi\epsilon_0} \left(-\frac{\ln 2}{2}\right)\,. \tag{40}$$

In a similar way, one may calculate the interaction energy between the A and C planes but outside the cylinder. One needs only to change the range of integration over z in the second integral of Eq.(38): from $\sqrt{yb-y^2}$ to ∞. The result is

$$W_{\mathrm{A-C,outside\,cyl.}} = \frac{\lambda q'}{2\pi\epsilon_0} \left(+\frac{\ln 2}{2}\right)\,. \tag{41}$$

As in the model with point charges, here also the interaction energy between the planes A and C is zero, which is evident by summing Eqs. (40) and (41).

The integration from plane A to $-\infty$, as well as from plane C to $+\infty$, gives a well-known result, the interaction energy is infinite.

The coefficient $\ln 2/2 = 0.34657359$ in Eqs. (40) or (41) may be expressed by

$$F^2(F-1) = 0.34657359\,, \tag{42}$$

from which the factor F takes the value

$$F = 1.2293291\,. \tag{43}$$

In the case of two point charges, the factor $F = 1.23170907$ (Eq. (32)), what is greater than the one in Eq. (43) by 0.2%.

2.5. Two parallel infinite line charges

$\boldsymbol{r}_0$ is in the plane of the line charge, oriented away from the line and orthogonal to the line charge. Electric field of the point charge is $\boldsymbol{E}' = [q'/(4\pi\epsilon_0 r')]\boldsymbol{r}_0'$,

The interaction energy given by Eq. (17) may also be calculated for two infinite parallel line charges. This problem is very similar to the previous calculation, so only some remarks and the final results will be given here. Two line charges located at $y = 0$ (along the X-axis) and at $y = b$ (see Fig. 5) define an infinite cylinder of radius $b/2$ with its axis at $y = b/2$. If charges of the same sign are assumed, the interaction energy inside the cylinder is negative, while everywhere outside it is positive. The electric field vectors $\boldsymbol{E} = [q/(4\pi\epsilon_0 r)]\boldsymbol{r}_0$ and $\boldsymbol{E}' = [q'/(4\pi\epsilon_0 r')]\boldsymbol{r}_0'$ of both line charges are located in the planes parallel to the $Y-Z$ plane. The vectors $\boldsymbol{r}_0$ and $\boldsymbol{r}_0'$ are unit vectors, while λ and λ' are linear charge densities. Due to the infinite line charges, the interaction energy should be calculated per an arbitrary length L.

Namely, the energy distribution in each cross-section orthogonal to the lines is the same, which is an advantage of this system. Thus, inside the cylinder of length L and of volume $\Delta V_{\mathrm{cyl.}}$, the interaction energy is given by

$$\begin{aligned} W_{\mathrm{A-C},\Delta \mathrm{V_{cyl.}}} &= L\frac{\lambda\lambda'}{2\pi^2\epsilon_0}\int\limits_0^{\pi/2}\mathrm{d}\theta\left[\int\limits_R^{b\cos\theta}\frac{(r-b\cos\theta)\mathrm{d}r}{b^2-2br\cos\theta+r^2}\right] \\ &= L\frac{\lambda\lambda'}{2\pi^2\epsilon_0}\int\limits_0^{\pi/2}\ln\sin\theta\mathrm{d}\theta = L\frac{\lambda\lambda'}{2\pi^2\epsilon_0}\left(-\frac{\pi}{2}\ln 2\right)\,, \end{aligned} \tag{44}$$

and with the same reasoning as in Eqs.(39) and (40), the final form is

$$W_{\mathrm{A-C},\Delta \mathrm{V_{cyl.}}} = L\frac{\lambda\lambda'}{2\pi\epsilon_0}\left(-\frac{\ln 2}{2}\right)\,. \tag{45}$$

It can also be shown that the interaction energy outside that cylinder, but between the planes A and C, is equal to that given in Eq. (45) with opposite sign, i.e., $W_{\mathrm{A-C,outside}\,\Delta\mathrm{V_{cyl.}}} = -W_{\mathrm{A-C},\,\Delta\mathrm{V_{cyl.}}}$. Therefore, the interaction energy between planes A and C is equal to zero.

The interaction energy between the plane A and $-\infty$ and between the plane C and $+\infty$ is infinite. It may easily be proved by calculation of the force of one charged line on the segment of the length L of the other line. The force is $\Phi = L\lambda\lambda'/(2\pi\epsilon_0 b)$, and the energy $W_\Phi = \int_b^\infty L\lambda\lambda'/(2\pi\epsilon_0 b)(\mathrm{d}b/b) = \infty$. The same result is obtained by a direct integration of this part of interaction energy ($\Delta W_{\mathrm{C-right}}$).

The value of the parameter L in Eq. (45) is arbitrary. For the determination of an adequate value of L, in order to deduce the coefficient of interaction energy between the two line charges, an energy per a specific volume should be used. In the system of two point charges, the finite negative interaction energy inside the sphere, is stored in the volume $(4\pi/3)(b/2)^3$ (see Fig. 1). The interaction energy stored in the same volume, but now in the cylinder of length L and of the cross-section $\pi(b/2)^2$ should be in some correlation with the energy contained inside the sphere. From the equality $(4\pi/3)(b/2)^3 = \pi L(b/2)^2$, it follows that $L = 2b/3$. According to that, Eq. (45) can be replaced by

$$W_{\mathrm{A-C},\Delta\mathrm{V_{cyl.}}} = b\frac{\lambda\lambda'}{2\pi\epsilon_0}\left(-\frac{\ln 2}{3}\right) . \tag{46}$$

The constant term in the brackets (regadless the sign), as the coefficient of the electrostatic interaction, may be assumed equal to

$$\frac{1}{3}\ln 2 = 0.23104906 = F - 1 . \tag{47}$$

The factor F is, therefore,

$$F = 1 + \frac{1}{3}\ln 2 = 1.23104906 . \tag{48}$$

One may also note that the system of two infinite parallel or antiparallel direct currents is analogous to the system of two parallel infinite line charges. The intensities of the magnetic field are (in standard symbols) $B = \mu_0 I/(2\pi r)$ and $B' = \mu_0' I/(2\pi r')$, which have the same dependence on r and r' as the intensities of the electric fields in the system of two parallel line charges. The calculation gives the same final result in that the characteristic coefficient is ln 2/3.

2.6. *Two parallel infinite plane charges*

Two parallel infinite and uniformly charged planes at a finite distance represent one of the simplest electrostatic systems. In reality, it corresponds to well known parallel-plate capacitor of very large plates compared to their mutual distance.

A calculation of the interaction energy for such a system is very similar to those for previous ones. The first plane charge may be located in the plane A, the $X-Y$ plane of the coordinate system (see Fig. 5), with another plane charge (of the same sign) in the plane C located at $y = b'$. The surface charge densities are σ and σ' in the planes A and C, respectively. At a given point between the two planes, the electric field vectors are $\boldsymbol{E} = (\sigma/2\epsilon_0)\boldsymbol{j}$ and $\boldsymbol{E}' = -(\sigma'/2\epsilon_0)\boldsymbol{j}$, where $\boldsymbol{j}$ is the unit vector in the direction of the Y-axis. The energy of the interaction between the planes A and C, in the volume $\Delta V = \Delta x \Delta y \Delta z$ (using Eq. (17)), is simply

$$W_{\mathrm{A-C},\Delta \mathrm{V}} = -\epsilon_0 \frac{\sigma}{2\epsilon_0}\frac{\sigma'}{2\epsilon_0}\Delta V\,. \tag{49}$$

The distribution of the interaction energy is uniform and obviously, this energy in all space between the planes A and C will be infinite, if relevant coordinates are permited to take all values in the range from $-\infty$ to $+\infty$. The energy is negative (if the both charges are positive, or negative), i.e. $W_{\mathrm{A-C}} < 0$, while for all three previous models $W_{\mathrm{A-C}} = 0$.

The above model may be treated as a boundary case of two charged spheres (see Sect. 2.1 and Fig. 1) for $R \to \infty$ at $b/R \sim 2$. But real distance between the planes A and C is now $b' = b - 2R << R$. Thus only negative and infinite contribution of interaction energy $W_{\mathrm{A-C}}$ "inside sphere" remains, without compensation of the positive one, "outside sphere", which disappears, as it is confirmed here with Eq. (49).

Using an analogous approach to a force between two charges, as in Sect. 2.4, but here between infinite plane charge and a charge $\sigma'\Delta P'$, where $\Delta P'$ is an arbitrary area of the other plane, one obtains the force $\Phi = \sigma\sigma'\Delta P'/(2\epsilon_0)$. Then, the change of energy is $W = \frac{\sigma\sigma'}{2\epsilon_0}\Delta P' \int_{b'}^{b'_1} \mathrm{d}b' = \frac{\sigma\sigma'}{2\epsilon_0}\Delta P'(b'_1 - b')$. Thus, Eq.(49) should be transformed into

$$W_{\mathrm{A-C},\Delta \mathrm{V}'} = -\frac{\sigma\sigma'}{2\epsilon_0}\Delta V'\,. \tag{50}$$

However, an amount of energy stored in the finite volume of the sphere $(4\pi/3)(b/2)^3$ (defined in Sect. 2.1 (see Fig. 1), and analogously as in Sect. 2.5) should be extracted, in order to find an appropriate coefficient of the interaction energy, comparable with those for the previous systems. A specific volume in Eq. (50), chosen in such a way, may be written as $(4\pi/3)(b/2)^3 = b'\Delta P' = \Delta V'$, where $\Delta P' = b'^2\pi/6$ is its boundary surface at A and C plane, respectively. Then, it follows

$$W_{\mathrm{A-C},\Delta \mathrm{V}'} = \frac{\sigma\sigma'}{2\epsilon_0}b'^{\,3}\left(\frac{\pi}{6}\right). \tag{51}$$

The coefficient $\pi/6$ may be connected with the factor F as

$$F^4(F-1) = \frac{\pi}{6} = 0.52359878\,, \tag{52}$$

resulting in

$$F = 1.22928882\,. \tag{53}$$

This value of F is in a very good agreement with the value $F = 1.2293291$, obtained in Sect. 2.4 for the model of an infinite line charge and a point charge (Eq. (43)).

3. Discussion

So far, we have reached four characteristic coefficients N_k of the interaction energy for various electrostatic systems. These coefficients may be expressed as functions of F as

$$N_k = F^k(F-1)\,, \tag{54}$$

where k is an integer. The results and parameters are summarized in the Table 1.

TABLE 1. Characteristic coefficients N_k of the interaction energy, corresponding to four various electrostatic systems, as functions of the factor F (Eq. (54)), and their associated values dependent on the integer k.

The system	Coefficient N_k	Function	F	k
Two parallel infinite line charges	$\frac{\ln 2}{3} = 0.23104906$	$F-1$	1.23104906	0
Two point charges	$\frac{\pi}{4} - \frac{1}{2} = 0.28539816$	$F(F-1)$	1.23170907	1
Infinite line charge and point charge	$\frac{\ln 2}{2} = 0.34657359$	$F^2(F-1)$	1.22932909	2
Two parallel infinite plane charges	$\frac{\pi}{6} = 0.52359878$	$F^4(F-1)$	1.22928882	4

According to the values of F in Table 1, the mean value is

$$< F >= 1.2303 \pm 0.0010\,, \tag{55}$$

with the mean error of 0.08%.

Obviously, the functions of F, which define particular values N_k, are not "true functions". However, they are good approximations, i.e., introducing the mean value of F given by Eq. (55) into Eq. (54), approximate values of N_k are 0.2307, 0.2839, 0.3494 and 0.5276 for numbers $k = 0, 1, 2$ and 4. Thus, Eq. (54) demonstrates a natural appearance of factor F. Its more convincing value is obtained by the application of the least squares (l.s.) method to the calculated coefficients N_k as a function of k, although based on only four systems (see Fig. 6). The resulting value of F is

$$F = 1.2296 \pm 0.0004\,, \tag{56}$$

with the standard deviation of only 0.03%.

Small deviations of F are still present. It suggests the necessity for a fine tuning, which should be founded by a proper theoretical approach. We may extend the fitting procedure with a two parameter function $N_k = a^k b$, which is of the same form as Eq. (1) in the Introduction, i.e., as the relationship for redshift of quasars.

The l.s. method gives the following values for the parameters: $a = 1.2263 \pm 0.0017$, slightly smaller than F in Eq. (56), and $b = 0.2314 \pm 0.0010$, slightly larger than $(F - 1)$. Therefore, a new function, including an additional correction c, could be used in the following form

$$N_k = (F - c)^k (F - 1 + c)\,. \tag{57}$$

Again, the l.s. method gives

$$F = 1.2289 \pm 0.0005 \quad (0.04\%) \quad \text{and} \quad c = 0.0026 \pm 0.0014\,. \tag{58}$$

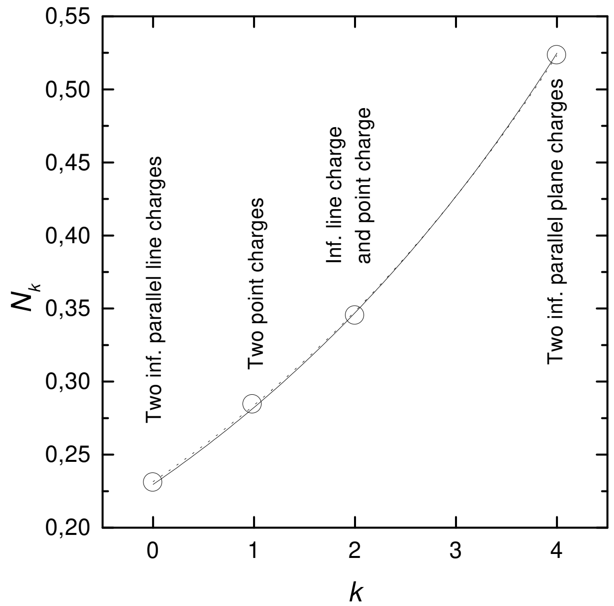


Fig. 6. Coefficients N_k of the interaction energy, for electrostatic systems, as functions of the integer k (open circles). Each point has an attached vertical notation of the corresponding system. The best fit of Eq. (54) is shown by solid line, while that of Eq. (60) by the dotted line.

The value of the correction term c is close to $\alpha/3 = 0.0024..$, where α is the fine-structure constant.

The corrections may be taken into account also in a different form:

$$N_k = [F(1 - c')]^k (F - 1)(1 + c')\,, \tag{59}$$

resulting, by means of the l.s. method, in

$$F = 1.2306 \pm 0.0006 \quad (0.05\%) \quad \text{and} \quad c' = 0.0035 \pm 0.0019\,,$$

where c' is nearly equal to $\alpha/2$. An appearance of the fine-structure constant α in the correction terms is similar as in Sect. 1 (Eq. (7)) and Sect. 1 (Eq. (11)). Thus, Eq.(59) may be rewritten as

$$N_k = [F(1-\alpha/2)]^k (F-1)(1+\alpha/2)\,, \tag{60}$$

with

$$F = 1.2307 \pm 0.0003 \quad (0.02\%)\,, \tag{61}$$

resulting in the best fit (dotted line) in Fig. 6.

Although the introduced corrections are empirical, they are simple and offer satisfactory accuracy, so that presented functions could be stimulating in the search for hopefully similar functions based on physical arguments.

The results for the factor F in all systems considered are given in Table 2.

TABLE 2. Deduced values of the factor F for various physical systems.

System	Value of factor F	
Redshift of quasars	1.23	Eq.(1)
Masses of electron and muon	1.2284	Eq.(2)
Muonium	1.2284	Eq.(4)
Leptons	1.2311 ± 0.0002	Eq.(5)
Quarks u,c,t and d,s,b	1.23	Eqs.(6,7)
$A-Z$ correlation	1.2375 ± 0.0003	Eq.(10)
$A-Z$ correlation with correction	1.2302 ± 0.0003	Eq.(11)
Gravitational systems	1.220 ± 0.040	Eqs.(12,13)
Electrostatic systems:		
The mean value of F		
by the l.s. method	1.2303 ± 0.0010	Eq.(55)
by the l.s. method	1.2296 ± 0.0004	Eq.(56)
a) with correction c	1.2289 ± 0.0005	Eq.(58)
b) with correction $c' = \alpha/2$	1.2307 ± 0.0003	Eq.(61)

The data in the above list suggest that factors F for various systems considered, pile up between the values 1.2284 and 1.2311. The factor F, deduced from coefficients of interaction energy for presented electrostatic systems, appear to be at a medium of these limits. Namely, using correlation of Eq. (54) without any correction, Eq. (56) follows (by means of the l.s. method), i.e.

$$F = 1.2296 \pm 0.0004 \quad (0.03\%)\,.$$

On the basis of all listed values of the factor F, the deviation is larger,

$$F = 1.2300 \pm 0.0015 \qquad (0.12\%)\,.$$

4. Conclusion

It has been shown that the factor $F \sim 1.23$ appeared in diverse systems, which are, at first sight, completely uncorrelated. The aim of this work is to find a new support for an existence of the factor F. There is a hope that sufficient experimental and observational data, and some exact approaches, could encourage research which would find a proper theoretical interpretation.

The main objection is that a rather arbitrary procedure is dominant in particular segments of the present work. But, it is inevitably, due to a lack of a theoretical basis.

In this work, electrostatic systems are considered. The calculations of the interaction energy between two point charges, a point charge and an infinite line charge, two parallel infinite line charges, and two parallel infinite plane charges, are exact. For example, for two positive point charges, the positive part of the interaction energy is larger than the Coulomb potential energy by the term $(\pi/4+1/2) = 1.2853..$, while the negative part of this energy is $(1/2-\pi/4) = -0.2853...$, giving the correct result. The question is now: is there any sense to tie 1.2853.. with the factor F? If yes, then one may suppose that $F = 1.2853..$ is larger than F in other systems by about 4.5%, but it could be specific for an electrostatic interaction. Another possibility is to find the simplest function of F, or F and α (the fine-structure constant), in order to reach the value about 1.23 for F. We have shown that such a simple function could be $F(F-1) = 0.2853...$, resulting in $F = 1.2317..$, or $F(F-1)(1+\alpha) = 0.2853..$, resulting in $F = 1.2303$ (for the system of two point charges).

Calculations for another three electrostatic systems lead to analogous functions of F (see the Table 1), enclosed by Eq. (54). Thus, the coefficients of the interaction energy for four elaborated systems, indicate the existence of F, which appears as a consequence of a different geometry of electrostatic systems. It has also been shown that exact calculations give slightly different values for the factor F (Eqs. (32), (43), (48) and (53)), suggesting that a fine tuning should be necessary. In a lack of theoretical arguments, some very simple empirical corrections (including the fine-structure constant α) were introduced in order to reach a smaller dissipation of values of F (Eqs.(60) and (61)).

Obviously, the new functions need not still be the "true functions", and they remain approximate. The values of F, obtained numerically, although close to 1.23, need not to be the same when compared with other systems investigated as, for example, elementary particles or gravitational systems. For an additional support to the factor F, more sofisticated electrostatic models should be included. But, a definite conclusion, based on the four systems considered, is that the factor F has a purely geometrical origin. Nevertheless, the most important result reached in this paper is that both relationships, Eq. (1) for the redshift of quasars and Eq. (54) for coefficients of the electrostatic interaction, are of the same form $y = const F^x$. A deep physical reason for this behaviour remains still to be discovered. We believe that if some values close to the factor F appear in various physical systems, and if it is possible by simple functions to adjust these values very close to a single value, which may be claimed to be the definite factor F, then it could be a good basis for future investigation.

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ELEKTROSTATSKA ENERGIJA MEĐUDJELOVANJA I FAKTOR 1.23

Crveni su pomaci kvazara prvotno ukazali na faktor $F \approx 1.23$. Nedavno se je našao i u vrlo različitim sustavima, uključujući poluživot muonija, mase elementarnih čestica (leptoni, kvarkovi, ...), odnos atomskih težina (A i atomskih brojeva (Z) i odnos zbroja svih kružećih masa s centralnom masom u gravitacijskim sustavima.

U ovom radu opisujemo još neke sustave u kojima se također pojavljuje faktor F, a to su elektrostatski sustavi niza raspodjela električnih naboja: dvije jednoliko površinski nabijene kugle, točkast naboj i jednoliko nabijena kugla, dva točkasta naboja, točkast naboj i jednolika beskonačna pravocrtna raspodjela naboja, dvije usporedne jednolike beskonačne pravocrtne raspodjele naboja i dvije usporedne jednoliko nabijene ravnine.