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ON THE PROBLEM OF SELF-FOCUSSING AND TRANSVERSAL STABILITY IN A MAGNETISED PLASMA WITH STREAMING IONS

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Self-focusing in a magnetised plasma consisting of cold electrons and ions has been studied in a situation when ions have a streaming velocity and the external magnetic field is at an angle θ with the streaming direction. By the use of a modified scaling, we have deduced an analogue of a non-linear Schrödinger equation involving only the space variables, which actually controlls the transversal stability of the system, thus signalling the possibility of self-focusing. The various situations arising due to the variation of the propagation angle θ and the streaming velocity u_0 are discussed in detail for the specific cases of D_2 and N_2 ions.

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1. Introduction

In last few years, extensive research has been done both theoretically and experimentally to understand the nonlinear behaviour of the plasmas [1-13]. In a nonlinear medium, a high-power electromagnetic beam creates a refractive-index profile across its cross-section corresponding to its own intensity profile. Consequently, the beam focuses itself in the apparent lens-like medium created by itself; in other words, the beam self-focuses [14,15]. Self-focusing of electromagnetic waves in plasmas has been studied by various authors as this has significant impact on fusion research [16,17]. In the ionosphere, high-power radio waves are also selffocussed, as a consequence the medium may be heated [18-20]. Self-modulation of electromagnetic waves helps to understand some of the physical processes in pulsar magnetospheres [21–24]. Taniuti and Washimi [25], Washimi [26], Litvak [27,28], Max et al. [29], Cardinali [30] and other authors showed that some physical conditions are necessary for having self-focusing of an electromagnetic wave due to the various changes of the refractive index of the medium that take place due to striction, heating and non-linear motion of the particles. In all references cited above, only the simple case when the magnetic field is in the direction of the wave propagation was considered. The event of self-focusing can be ascertained by the analysis of the transversal stability of a wave inside the plasma. It was observed by Sato et al. [31] that by adopting a modified form of stretched variables, one can deduce a type of nonlinear Schrödinger equation which does involve only space variables. The dispersive and nonlinear coefficients of this equation are to be analysed as functionals of plasma parameters for searching the region of stability. On the other hand, relativistic and ponderomotive self-focusing of laser beam in an inhomogeneous plasma was studied by Bonnaud et al. [32] by a two-dimensional simulation technique. Kates and Kaup [33] studied nonlinear interactions including modulational instability of a plane electromagnetic pulse propagating through a magnetised cold plasma at an arbitrary oblique angle of propagation in a very strong ambient magnetic field that gives rise to several regimes of modulational instability both for an electron – ion plasma and for an electron – positron plasma which are effective in the context of pulsar micropulses. Moreover, due to oblique propagation of the wave, solitons would be produced in pulsar magnetosphere. Geogieva et al. [34] investigated the self-focusing of surface waves in a cylindrical plasma waveguide. They obtained the linear dispersion law and the nonlinear changes in the wave field distribution power flow from which the effect of self-focusing of the wave under the conditions of weak nonlinearity was described.

In this communication, we have generalised the previous treatment of the longitudinal magnetised plasma to the situation when there is a streaming of ions along the x-axis and also the magnetic field is at an angle θ with this direction. Our approach relies on the space-time scaling and we deduce the explicit expressions for the coefficients of the dispersive (p) and nonlinear terms (q) as functions of θ, the streaming velocity and other plasma parameters. A similar approach was used by Mukherjee and Roychowdhury [35] to study self-focusing in an unmagnetised relativistic plasma with both positive and negative ions. The analysis of the behaviour of these coefficients p and q gives us explicitly range and condition of self-focusing and how they vary with the streaming velocity and angle θ. There is another important aspect for studying p and q. It is the fact that the power at the point of self-focusing is really proportional to p/q.

2. Formulation

We consider a fully ionised, collisionless plasma consisting of electrons and ions placed in a uniform magnetic field. We assume that the two-fluid model of hydrodynamic description for the dynamics of the plasma is applicable [36].

In our present investigation, our aim is to derive the nonlinear Schrödinger equation and to find the role of streaming ions and propagation angle on the stability of the waves in the plasma. For our analysis, we eliminate the electron component in the Maxwell equations. So, the basic equations in terms of ion components can be written as [2]

$$\frac{\partial n_i}{\partial t} + \vec{\nabla} \cdot (n_i \vec{V}_i) = 0\,, \tag{1}$$

$$M_A^2 \left(\frac{\partial}{\partial t} + \vec{V}_i \cdot \vec{\nabla} \right) \vec{V}_i = \frac{1}{n} \left(\vec{\nabla} \times \vec{B} \right) \times \vec{B} + \frac{1}{R_e} \left[\left(\frac{1}{n} \vec{\nabla} \times \vec{B} \cdot \vec{\nabla} \right) \vec{V}_i \right. \tag{2}$$

$$\left. + \left(\frac{\partial}{\partial t} + \vec{V}_i \cdot \vec{\nabla} \right) \left(\frac{1}{n} \vec{\nabla} \times \vec{B} \right) \right] - \frac{1}{{M_A}^2 R_i R_e} \left(1 + \frac{m_e}{m_i} \right) \left(\frac{1}{n} \vec{\nabla} \times \vec{B} \cdot \vec{\nabla} \right) \left(\frac{1}{n} \vec{\nabla} \times \vec{B} \right) ,$$

$$\frac{\partial \vec{B}}{\partial t} = \vec{\nabla} \times (\vec{V}_i \times \vec{B}) - \frac{1}{R_i} \vec{\nabla} \times \left(\frac{\partial}{\partial t} + \vec{V}_i \cdot \vec{\nabla} \right) \vec{V}_i\,, \tag{3}$$

where n_i and V_i, respectively, denote the density and velocity of the ions, m_e and m_i being the masses of electrons and ions, respectively. $\vec{B}$ denotes the magnetic field. All quantities have been normalised with respect to the characteristic number density n_0^*, characteristic speed u_0^*, characteristic length L_0^* and characteristic magnetic field B_0^*. $\omega_0^* = u_0^*/L_0^*$ is the characteristic frequency. V_A is the Alfven speed and M_A is the Alfven Mach number. We may note also

$$R_e = \Omega_e/\omega_0^*;\; R_i = \Omega_i/\omega_0^*;\; R_{pe} = \omega_{pe}/\omega_0^*;\; \Omega_i = eB_0^*/m_i c;\; \Omega_e = eB_0^*/m_e c;$$

$$\omega_{pe}^2 = 4\pi n_0^* e^2/m_e; M_A = u_0^*/V_A, E_0 = u_0^* B_0^* \text{ and } V_A = B_0^*/[4\pi n_0^*(m_i + m_e)]^{1/2}\,.$$

We now assume that the magnetic field components are $\vec{B}(B_x, B_y, B_z)$ and that the initial constant magnetic field is in the (x, y) plane making an angle θ with the x-axis (the direction of propagation) that is its components are $(B_0 \cos\theta, B_0 \sin\theta, 0)$. The ion velocity components are (u, v, w). Over and above, we assume that the physical quantities such as magnetic field, velocity, density etc. do not vary with the z-coordinate.

Following Sato et al. [31], we use the reductive perturbation method by stretching the coordinates x and y. The amount of stretching is given by the factor ϵ whereas all the dependent variables are expanded in Fourier series whose coefficients are also scaled by ϵ. Such an approach has been adopted by many authors in the past to deduce the NLS equation. Now we define new space coordinates (ξ, η) as

$$\xi = \epsilon^2 x, \eta = \epsilon y \tag{4}$$

and expand the field variables as

$$\begin{pmatrix} n \\ u \\ v \\ w \\ B_x \\ B_y \\ B_z \end{pmatrix} = \begin{pmatrix} 1 \\ u_0 \\ 0 \\ 0 \\ B_0\cos\theta \\ B_0\sin\theta \\ 0 \end{pmatrix} + \sum_{m=1}^{\infty} \epsilon^m \sum_{l=-\infty}^{l=\infty} U_l^{(m)}(\xi,\eta)\ \exp[\mathrm{i}(kx-\omega t)]\,, \qquad (5)$$

where $U_l^{(m)}$ denotes the vector $[n_l^{(m)}, u_l^{(m)}, v_l^{(m)}, w_l^{(m)}, B_{xl}^{(m)}, B_{yl}^{(m)}, B_{zl}^{(m)}]^T$ where T stands for transpose, and ϵ is the expansion parameter.

Since the computation using the forms (4) and (5) are routine and elaborate, we just quote the important results. Using (4) and (5) in (1)-(3), we obtain the field variables from the equations containing the first order of ϵ,

$$\begin{aligned} n_l^{(1)} &= \frac{k}{\omega_1} u_l^{(1)}\,, \\ u_l^{(1)} &= \frac{kB_0\sin\theta}{\omega_1 M_A^2} B_{yl}^{(1)}\,, \\ \mathrm{i}\left[\omega_1 v_l^{(1)} + \frac{kB_0\cos\theta}{M_A^2} B_{yl}^{(1)}\right] &= \frac{k\omega_1 l}{R_e M_A^2} B_{zl}^{(1)}\,, \\ B_{xl}^{(1)} &= 0\,, \\ \mathrm{i}\left[\omega_1 w_l^{(1)} + \frac{kB_0\cos\theta}{M_A^2} B_{zl}^{(1)}\right] + \frac{k\omega_1 l}{R_e M_A^2} B_{yl}^{(1)} &= 0\,, \\ \mathrm{i}\omega_1 B_{yl}^{(1)} - \mathrm{i}kB_0\sin\theta u_l^{(1)} + \mathrm{i}kB_0\cos\theta v_l^{(1)} + \frac{k\omega_1 l}{R_i} w_l^{(1)} &= 0\,, \\ -\mathrm{i}\left[\omega_1 B_{zl}^{(1)} + kB_0\cos\theta w_l^{(1)}\right] + \frac{k\omega_1 l}{R_i} v_l^{(1)} &= 0\,, \end{aligned} \qquad (6)$$

which is a set of linear homogeneous equations from which dispersion relation is obtained as

$$\begin{aligned} 1 + \frac{k^2\sin^2\theta}{R_i R_e M_A^2} + \cos^2\theta\left[1 + \frac{k^2}{M_A^2}\left(\frac{1}{R_i^2} + \frac{1}{R_e^2} - \frac{B_0^2}{\omega_1^2}\right)\right] & \qquad (7) \\ -\frac{\omega_1^2 M_A^2}{k^2 B_0^2}\left[1 + \frac{2k^2}{R_i R_e M_A^2} + \frac{k^4}{R_i^2 R_e^2 M_A^4}\right] = 0\,, & \end{aligned}$$

where $\omega_1 = \omega - u_0 k$, u_0 is the stream velocity along the x-direction. On simplification dispersion relation (7) can be written as

$$A\omega^4 + B\omega^3 + C\omega^2 + D\omega + E = 0\,, \qquad (8)$$

where

$$
\begin{aligned}
A &= -\left[\frac{M_A^2}{B_0^2} + \frac{2k^2}{B_0^2 R_i R_e} + \frac{k^4}{B_0^2 R_i^2 R_e^2 M_A^2}\right], \\
B &= \frac{4u_0 k M_A^2}{B_0^2} + \frac{8k^3 u_0}{B_0^2 R_i R_e} + \frac{4u_0 k^5}{B_0^2 R_i^2 R_e^2 M_A^2}, \\
C &= k^2 + \frac{\sin^2\theta k^4}{R_i R_e M_A^2} + k^2\cos^2\theta + \frac{k^4\cos^2\theta}{M_A^2}\left(\frac{1}{R_i^2}+\frac{1}{R_e^2}\right) - \frac{6M_A^2 u_0 k^2}{B_0^2} \\
&\quad - \frac{12k^4 u_0^2}{B_0^2 R_i R_e} + \frac{6u_0^2 k^6}{B_0^2 R_i^2 R_e^2 M_A^2}, \qquad (9) \\
D &= -2u_0 k^3 - \frac{2u_0\sin^2\theta k^5}{R_i R_e M_A^2} - 2u_0 k^3\cos^2\theta - \frac{2u_0\cos^2\theta k^5}{M_A^2}\left(\frac{1}{R_i^2}+\frac{1}{R_e^2}\right), \\
E &= u_0^2 k^4 + \frac{u_0^2 k^6\sin^2\theta}{R_i R_e M_A^2} + u_0^2 k^4\cos^2\theta + \frac{u_0^2\cos^2\theta}{M_A^2}\left(\frac{1}{R_i^2}+\frac{1}{R_e^2}\right)k^6 - \frac{B_0^2 k^4\cos^2\theta}{M_A^2} \\
&\quad - \frac{M_A^2 k^4 u_0^4}{B_0^2} - \frac{2u_0^4 k^6}{R_i R_e B_0^2} - \frac{u_0^4 k^8}{R_i^2 R_e^2 M_A^2 B_0^2} + \frac{4M_A^2 u_0^3 k^3}{B_0^2} + \frac{8u_0^3 k^5}{B_0^2 R_i R_e} + \frac{4u_0^3 k^7}{B_0^2 R_i^2 R_e^2 M_A^2}.
\end{aligned}
$$

Solving (8), one would find that ω may be real or complex depending upon the values of A, B, C, D and E which are mainly controlled by the strength of the magnetic field B_0, stream velocity u_0 and propagation angle θ. The complex values of ω indicate the instability of the wave propagating through the plasma. In our following analysis, we have numerically solved the dispersion relation (8) explicitly and have used the real values of ω to find the nonlinear instability of the wave.

From the second order terms in ϵ for $l = 1$, the following equations are obtained.

$$
\begin{aligned}
\mathrm{i}\omega_1 n_1^{(2)} - \mathrm{i}k u_1^{(2)} &= \frac{\partial}{\partial\eta} v_1^{(1)}, \\
\omega_1 u_1^{(2)} - kB_0\sin\theta B_{y1}^{(2)} &= \frac{\omega_1}{R_e M_A^2}\frac{\partial}{\partial\eta}B_{z1}^{(1)}, \\
\omega_1 v_1^{(2)} + \frac{kB_0\cos\theta}{M_A^2}B_{y1}^{(2)} + \frac{\mathrm{i}k\omega_1}{R_e M_A^2}B_{z1}^{(2)} &= 0, \\
\omega_1 w_1^{(2)} + \frac{kB_0\cos\theta}{M_A^2}B_{z1}^{(2)} - \frac{\mathrm{i}k\omega_1}{R_e M_A^2}B_{y1}^{(2)} &= \frac{\mathrm{i}B_0\sin\theta}{M_A^2}\frac{\partial}{\partial\eta}B_{z1}^{(1)}, \qquad (10) \\
B_{x1}^{(2)} &= \frac{\mathrm{i}\omega_1^2 M_A^2}{k^3 B_0\sin\theta}\frac{\partial}{\partial\eta}n_1^{(1)}, \\
-\mathrm{i}\omega_1 B_{y1}^{(2)} + \mathrm{i}B_0\sin\theta k u_1^{(2)} - \mathrm{i}kB_0\cos\theta v_1^{(2)} - \frac{k\omega_1}{R_i}w_1^{(2)} &= 0,
\end{aligned}
$$

$$-\mathrm{i}\omega_1 B_{z1}^{(2)} - \mathrm{i}kB_0\cos\theta w_1^{(2)} + \frac{k\omega_1}{R_i}v_1^{(2)} = B_0\sin\theta\frac{\partial w_1^{(1)}}{\partial\eta} - \frac{\mathrm{i}\omega_1}{R_i}\frac{\partial u_1^{(1)}}{\partial\eta}\,.$$

Similarly, for $l = 2$ we get

$$\begin{aligned}
\omega_1 n_2^{(2)} - ku_2^{(2)} &= \omega_1[n_1^{(1)}]^2\,,\\
-2\omega_1 u_2^{(2)} + 2kB_0\sin\theta B_{y2}^{(2)} &= A_1[n_1^{(1)}]^2\,,\\
-\mathrm{i}\omega_1 v_2^{(2)} + \frac{2k\omega_1}{R_e M_A^2}B_{z2}^{(2)} - \frac{\mathrm{i}kB_0\cos\theta}{M_A^2}B_{y2}^{(2)} &= \mathrm{i}A_2[n_1^{(1)}]^2\,,\\
-\mathrm{i}\omega_1 w_2^{(2)} - \frac{\mathrm{i}kB_0\cos\theta}{M_A^2}B_{z2}^{(2)} - \frac{2k\omega_1}{R_e M_A^2}B_{y2}^{(2)} &= A_3[n_1^{(1)}]^2\,,\\
B_{x2}^{(2)} &= 0\,,\\
-\mathrm{i}\omega_1 B_{y2}^{(2)} + \mathrm{i}kB_0\sin\theta u_2^{(2)} - \frac{2k\omega_1}{R_i}w_2^{(2)} - \mathrm{i}kB_0\cos\theta v_2^{(2)} &= \mathrm{i}A_4[n_1^{(1)}]^2\,,\\
-\mathrm{i}\omega_1 B_{z2}^{(2)} - \mathrm{i}kB_0\cos\theta w_2^{(2)} + \frac{2k\omega_1}{R_i}v_2^{(2)} &= A_5[n_1^{(1)}]^2\,.
\end{aligned} \tag{11}$$

In our analysis, we will also require those equations at the ϵ^3 order for $l = 1$. But due to their elaborate nature, we do not quote those here, but give their most important implications. In conjunction with those third-order set for $l = 0$, one can obtain

$$u_0^{(2)} = B_2|n_1^{(1)}|^2, \qquad v_0^{(2)} = -B_1|n_1^{(1)}|^2, \qquad w_0^{(2)} = 0, \qquad B_{xo}^{(2)} = B_3|n_1^{(1)}|^2\,. \tag{12}$$

It is to be noted that $B_{yo}^{(2)}, B_{zo}^{(2)}$ and $n_0^{(2)}$ remain undetermined, and they are assumed to be zero.

Eliminating the higher-order quantities in favour of $n_1^{(1)}$, we get the following nonlinear Schrödinger (NLS) equation

$$\mathrm{i}\frac{\partial n_1^{(1)}}{\partial\xi} + p\frac{\partial^2 n_1^{(1)}}{\partial\eta^2} + q|n_1^{(1)}|^2\; n_1^{(1)} = 0\,, \tag{13}$$

where p and q are complicated functions involving the plasma parameters. However, when the wave propagates perpendicularly to the direction of the ambient magnetic field, i.e., $\theta = 90°$, p and q take the following form

$$p = \frac{-D_1\lambda_1 + \lambda_3 D_5}{-\frac{\omega_1^2}{k^2}\lambda_1 + D_4\lambda_3 + b_7 D_7} \qquad \text{and} \qquad q = \frac{-\lambda_1 D_2 + \lambda_3 D_6 + b_7 D_8}{-\frac{\omega_1^2}{k^2}\lambda_1 + D_4\lambda_3 + b_7 D_7}\,, \tag{14}$$

where

$$\lambda_1 = \frac{\omega_1^4 R_e M_A^4}{kB_0}\left(1+\frac{k^2}{R_i R_e M_A^2}\right)^2,$$

$$\lambda_3 = -k\omega_1^3 \frac{R_e}{R_i} M_A^2 \left(1+\frac{k^2}{R_i R_e M_A^2}\right),$$

$$D_1 = \frac{\omega_1^2}{k^3} - \frac{\omega_1}{R_e M_A^2}\left[\frac{\omega_1}{kR_i} - \frac{R_i^2 B_0^2}{k\omega_1}\left(1-\frac{\omega_1^2 M_A^2}{k^2 B_0^2}\right)\right] \Big/ \left(1+\frac{k^2}{R_i R_e M_A^2}\right),$$

$$D_2 = -\left[\frac{2\omega_1^2}{k} + \frac{2\omega_1^2}{kB_0} + \left(\frac{2}{M_A^2} + \frac{\omega_1^2}{2k^2 B_0^2}\right)\left(\frac{2\omega_1^2}{k} - \frac{2\omega_1^2 M_A^2}{k} - \frac{\omega_1^4 M_A^4}{k^3 B_0^2}\right)\right], \quad (15)$$

$$D_4 = -\frac{\omega_1^3}{R_e k^2 B_0},$$

$$D_5 = \frac{B_0}{R_i k\omega_1 M_A^2}\left[\omega_1^2 - R_i^3 B_0^2\left(1-\frac{\omega_1^2 M_A^2}{k^2 B_0^2}\right)\right] \Big/ \left(1+\frac{k^2}{R_i R_e M_A^2}\right) - \frac{\omega_1^3}{R_e B_0 k^3},$$

$$D_6 = \frac{3\omega_1^3}{R_e k B_0} - \frac{6\omega_1 R_i B_0}{k} - \frac{6k\omega_1 B_0}{R_e M_A^2} + \frac{4\omega_1^3 R_i M_A^2}{k^3 B_0},$$

$$-\left(\frac{\omega_1^4 M_A^4}{k^2 B_0^2} + 2\omega_1^2 M_A^2 - 2\omega_1^2\right)\left(\frac{R_i\omega_1}{B_0 k^3} + \frac{3\omega_1}{R_e k B_0 M_A^2}\right),$$

$$D_7 = -\omega_1^3 M_A^2/k^3 B_0, \qquad D_8 = \omega_1 B_0 - \frac{\omega_1^3 M_A^2}{k^2 B_0}$$

$$\text{and} \qquad b_7 = R_e \omega_1^3 M_A^2 \left(1+\frac{k^2}{R_e R_i M_A^2}\right).$$

It is to be noted that when the streaming ions are absent, i.e. $u_0 = 0$, $\omega_1 = \omega$. So, in this case, the values of the above parameters will remain the same replacing ω_1 by ω.

3. *Analysis of stability*

It is important to note that Eq. (13) does not involve any time variable and as such both ξ and η are space variables, one along the direction of propagation and the other being transverse to it. Our motivation is to study self-focusing which effectively boils down to the analysis of transversal stability. Similar type of NLS equation was also obtained by Lontano et al. [37] and Taniuti et al. [25]. The coefficient q is a measure of nonlinearity which is actually coming from the ponderomotive force type term, while p is the coefficient of dispersion coming from the induced charge fluctuation. In the equation of this type, waves become steeper due to the nonlinearity, that is, rays optically converge, but the dispersion acts to smear out the steepening to give rise to a steady profile. The solitary wave solution of equation (13) is

$$n_1^{(1)} = \epsilon \sqrt{\frac{-2k'}{q}} \operatorname{sech}\left(\epsilon \sqrt{\frac{-k'}{p}}\, y\right) \exp(-\mathrm{i}k'\epsilon^2 x)\,, \tag{16}$$

which exhibits the focusing to the slab shape. So we observe that for any solitary wave of Eq. (13),

$$(Amplitude) \times (width) = \sqrt{2p/q} \tag{17}$$

is constant and is independent of ϵ. The ratio p/q is proportional to the critical power for the self-focusing. On the other hand, the stability of the system depends on the sign of the quantity $p\,q$. It may be kept in mind that due to the generation of inhomogeneous electromagnetic field inside the plasma, the electron motion becomes non-uniform and the refractive index of the medium becomes distorted. As a result, the self-focusing of a beam takes place. We have graphically depicted the various situations for the N_2 ion in Figs. 1 to 6, where the variation of the functions p and q are shown. Figure 1 shows that for a fixed value of θ, the functions p and q both remain positive, signalling to the possibility of self-focusing of the ion-wave. It is also found that at first p decreases with u_0 (the streaming velocity) and then

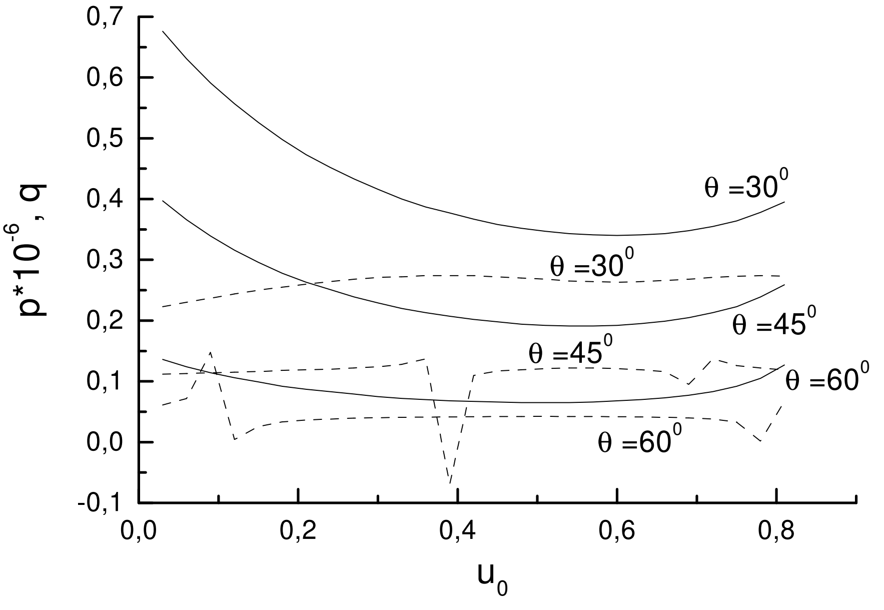

Fig. 1. Plot of variation of p (solid curve) and q (dashed curve) with u_0, the streaming velocity of the ions in the case of space-plasma corresponding to N_2-ions for $R_i = 0.016$, $R_e = 880$, $M_A = 0.9$, $B_0 = 1$ and $k = 0.01$ with θ as parameter.

increases, while θ remains unchanged, but variation of q with u_0 even for the same value of θ is somewhat different. In Fig. 2, variation of p and q with θ for various

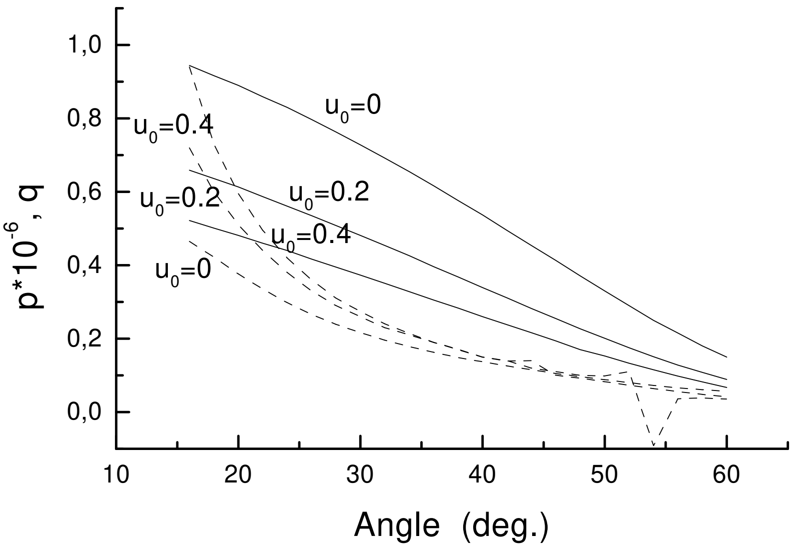

Fig. 2. Change of p (solid curve) and q (dashed curve) with θ, angle between the wave normal and the applied uniform magnetic field in the case of space-plasma corresponding to the set of values $R_i = 0.016$, $R_e = 880$, $M_A = 0.9$, $B_0 = 1$ and $k = 0.01$ with u_0 as parameter.

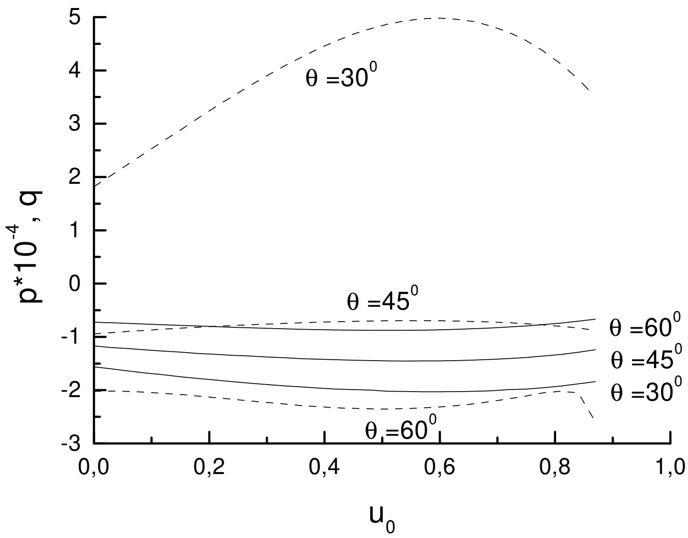

Fig. 3. Dependence of p and q on u_0 for $R_i = 0.016$, $R_e = 880$, $M_A = 0.9$, $B_0 = 1$ and $k = 0.1$ with θ as parameter.

fixed values of u_0 is displayed. In this case also, both p and q are positive, so self-focusing of the ion-wave is likely to take place. It is to be mentioned that in

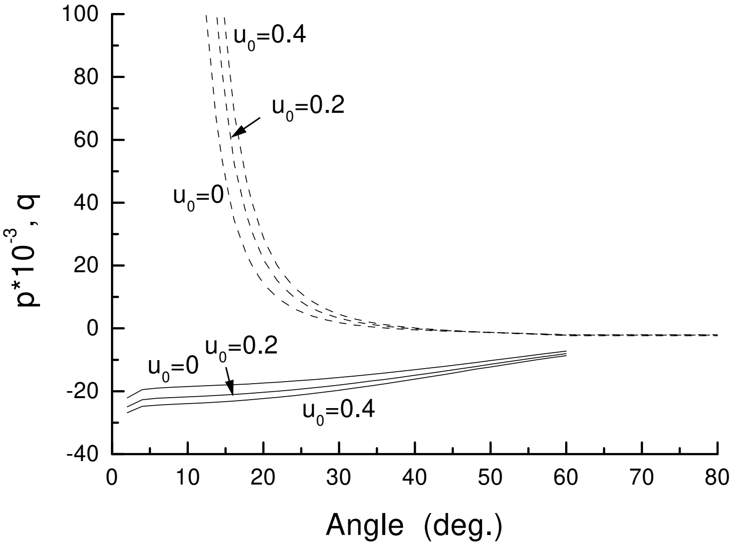

Fig. 4. Variation of p and q with θ for the set of values $R_i = 0.016$, $R_e = 880$, $M_A = 0.9$, $B_0 = 1$ and $k = 0.1$ with u_0 as parameter.

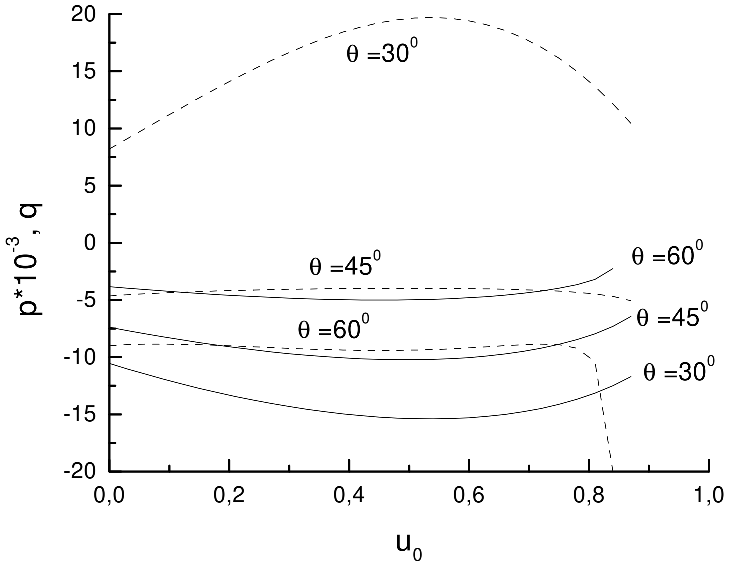

Fig. 5. Plot of change of p and q with u_0 for $R_i = 0.016$, $R_e = 880$, $M_A = 0.9$, $B_0 = 1$ and $k = 0.5$ with θ as parameter.

both of these cases the other parameter values are kept fixed. We have obtained ω for small values of k (less than unity). On the other hand, for higher values

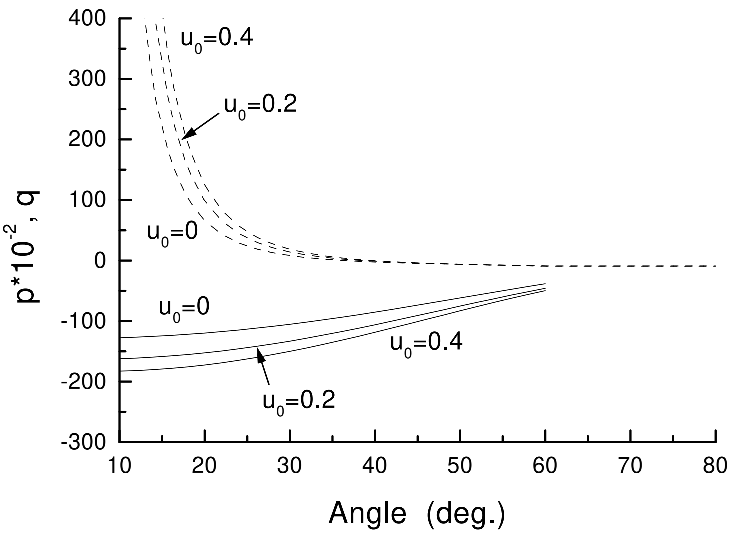

Fig. 6. Dependence of p *and* q *on* θ *for the set of values* $R_i = 0.016$, $R_e = 880$, $M_A = 0.9$, $B_0 = 1$ *and* $k = 0.5$ *with* u_0 *as parameter.*

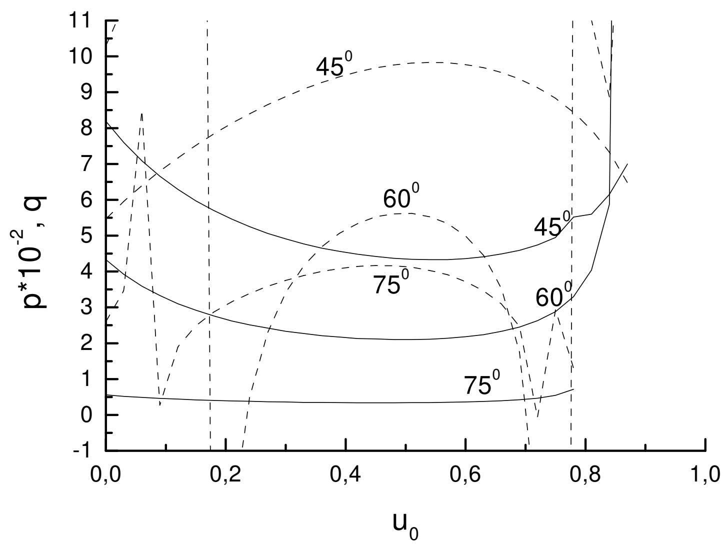

Fig. 7. Variation of p *and* q *with* u_0 *for* D_2*-ions (in the case of thermonuclear phenomena), corresponding to the values* $R_i = 0.48$, $R_e = 175.6$, $M_A = 0.9$, $B_0 = 1$ *and* $k = 0.5$ *with* θ *as parameter.*

of k (as obtained from the basic dispersion relation) the situation changes. This can be visualised from Figs. 3 to 6. As k is increases from 0.01 to 0.1, p becomes

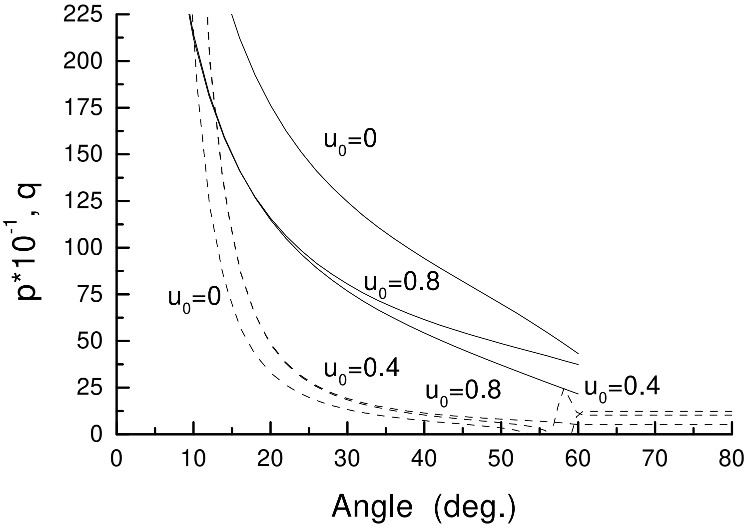

Fig. 8 (right). Plot of change of p and q with θ for $R_i = 0.48$, $R_e = 175.6$, $M_A = 0.9$, $B_0 = 1$ and $k = 0.5$ with u_0 as parameter.

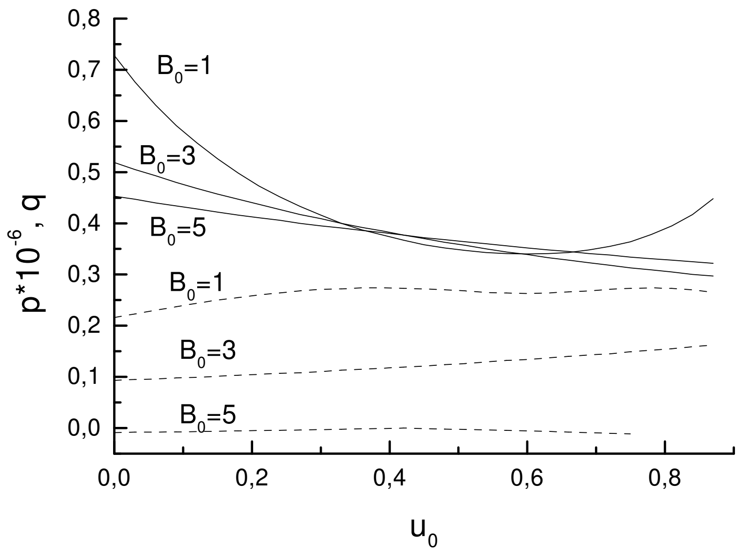

Fig. 9. Variation of p and q with u_0 for $R_i = 0.016$, $R_e = 880$, $M_A = 0.9$, $k = 0.01$ and $\theta = 30°$ with B_0 as parameter.

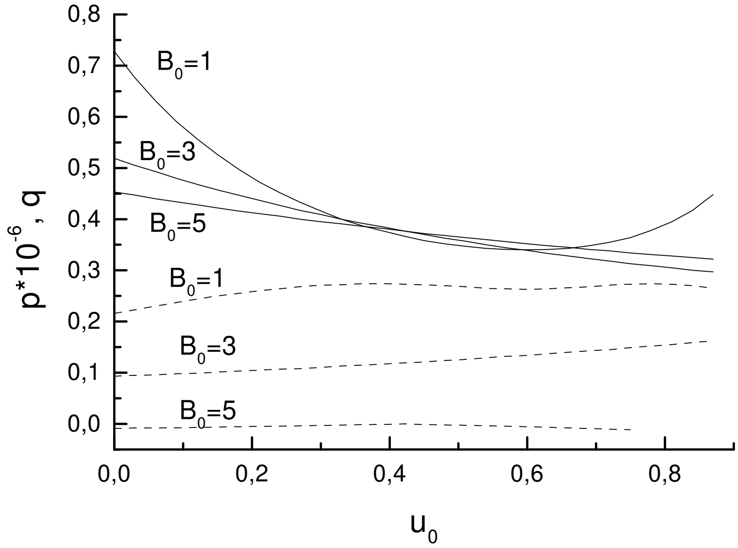

Fig. 10 (right). Plot of variation of p and q with θ for $R_i = 0.016$, $R_e = 880$, $M_A = 0.9$, $k = 0.01$ and $u_0 = 0.2$ with B_0 as parameter.

negative for the full range of values of u_0 and θ, whereas q remains positive only for $\theta = 30°$. So, we may infer that the ion-wave becomes transversely unstable and there will be self-focusing for $\theta \geq 45°$ for both $k = 0.1$ and 0.5. But below this value of θ, the system retains transversal stability and self-focusing is not likely to occur. So, for high values of θ, there may be a change from stable to unstable situation, and θ, the angle between the direction of the magnetic field and that of the wave propagation, plays a very significant role in causing such a transition. In Figs. 7 and 8, we have depicted the situation for D_2 ions but only for high value of k (= 0.5). Lastly, in Figs. 9 and 10, the mode of dependence of the coefficient of dispersion and nonlinearity on u_o and θ for various values of the magnetic field strength (B_0) have been displayed.

4. Summary and concluding remarks

In our present investigation, we have derived and then analysed the nonlinear Schrödinger equation for the study of self-focusing of an electromagnetic wave in a magnetised plasma with streaming ions. We have seen that the strength of the magnetic field, the angle of propagation of the wave and the streaming velocity of the ions play dominant role in the self focusing process. It is important to note that Kates and Kaup's analysis [33] for the modulational instability is applicable only in the presence of a strong magnetic field, but our present theory is applicable for both low and high values of magnetic field in the plasma. Namely, their method depends on a perturbation procedure related to the strength of the magnetic field, but ours is valid for a general strength of B. Also, our ions are nonrelativistic, but in their case ions are relativistic. It is to be mentioned that self-focusing mechanism requires a threshold power depending mainly on plasma density, pulse duration and laser frequency below which it is insignificant. The threshold power for self-focusing is larger for picosecond pulses than for nanosecond pulses. For the Nd-glass laser ($\lambda \approx 1.06\ \mu$m, $\omega = 17.8 \times 10^{15}$ s^{-1}), the threshold power according to the estimation of Kaw [38] and Kaw and Dawson [39] is about 10^{19} W/cm^2. It is seen that for the above mentioned threshold power of the wave, the motion of electrons and ions becomes relativistic. It is to be noted that below the threshold power for the occurrence of self-focusing, the other nonlinear effects, i.e., the frequency shift and wave precession will be present.

During the propagation of a high-power electromagnetic wave through a dense plasma, one important phenomenon takes place, the inverse Faraday effect (magnetic-moment field) [40]. Stamper et al. [41], Briand et al. [42], Stamper [43] and many others experimentally observed the self-generated magnetic-moment field that is of the order of 0.1 T (kiloGauss) to 100 T (megaGauss). Chakraborty et al. [44,45] and his coworkers (Srivastava et al. [46] and Das [47]) theoretically showed that zero-harmonic magnetic-moment field of the order of 100 T may be generated in an underdense plasma due to the propagation of high-power laser beam. In our present study, we have not considered the magnetic-moment field generated due to the self-action effect in the plasma. Moreover, both electrons and ions are assumed to be non-relativistic. In our numerical estimation, the static magnetic field in the plasma is taken to be of the order of 1 T (10^4 Gauss), from which it is seen that both the applied magnetic field and the self-generated magnetic-moment field are of the same order of magnitude and so consideration of magnetic-moment field would not cause any considerable change in our physical analysis. However, for better understanding of the self-focusing of a high-power electromagnetic wave, we have to consider electrons and ions to be relativistic. Considering all these problems, we plan to investigate self-focusing of an electromagnetic wave in a more generalised way in the near future.

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PROBLEM SAMOFOKUSIRANJA I POPREČNE STABILNOSTI U MAGNETIZIRANOJ PLAZMI S IONSKOM STRUJOM

Proučavamo samofokusiranje u magnetiziranoj plazmi koja se sastoji of hladnih elektrona i iona u uvjetima kada ioni struje a magnetsko je polje pod nekim kutom prema smjeru njihovog strujanja. Primjenom modificirane promjene mjera, izveli smo jednadžbu koja je analogna Schrödingerovoj jednadžbi s prostornim varijablama koja opisuje poprečnu stabilnost sustava i tako najavljuje mogućnost samofokusiranja. Raspravljaju se podrobno razni uvjeti koji nastaju zbog promjena smjera strujanja iona (kuta θ) i brzine strujanja (u_0) za posebne slučajeve D_2 i N_2 iona.