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# SOME STUDIES ON THE VARIATION OF ELECTRON COLLISION FREQUENCY AND ABSORPTION COEFFICIENT WITHIN THE ANISOTROPIC ATMOSPHERE

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The analytical form of absorption index appropriate to a certain range of height of the upper atmosphere plasma medium during ordinary wave propagation under quasi-transverse approximations has been utilized to develop an integral equation through Wentzel-Kramers-Brillouin method of solution. The integral equation is useful for the determination of effective electron collision frequency. Numerical results are presented graphically along with the results of some earlier work.

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## 1. Introduction

The presence of the geomagnetic field in the upper atmosphere makes the medium magneto-active. Electromagnetic waves split into ordinary and extraordinary waves with different refractive indices and absorption coefficients.

The formula for the refractive indices of a cold magnetoplasma, as deduced by Appleton and Hartree, is applicable to the case where only electrons are effective and the electron-neutral-particle collision frequency is assumed to be independent of the electron velocity. Application of the Appleton and Hartree formula to the problems of propagation through the ionised medium is unsuitable. Therefore, quasi-longitudinal (QL) and quasi-transverse (QT) approximations are made.

The refractive indices depend on the angle between the normal to the wave front and the geomagnetic field. They also depend on electron density, electron-neutral particle collision frequency, electro gyro-frequency and some other parameters. Moreover, the effects of ions and the dependence of collision frequencies on their respective velocities are important. Based on these considerations, different formulae of refractive indices have been developed [1–4], which are valid under appropriate circumstances.

Some workers [5–8] generalized the Appleton-Hartree magnetoionic formula through the solution of the Boltzmann equation following the Chapman-Enskog method. In these articles, the effects of the velocity dependence of the collision frequency are derived in closed analytical forms. The results also yield the Appleton-Hartree form on elimination of the velocity dependence of collision frequency.

Budden [9] critically examined the Appleton-Hartree results of QL and QT approximations and pointed out certain limitations in their use. He presented some alternative angular approximations in place of traditional approximations. To justify different approaches, a more general method has been given for waves in a cold magnetoplasma [10]. In the work oh Heading [11], the propagation equations are generalized to $n$-dimensions from which any particular algebraic form can be obtained. In the method, the refractive indices of the upper atmospheric plasma and their different approximations can be justified for a specific dimension.

The solution of the wave equation for the ionospheric plasma having slowly varying indices with height may be chosen appropriately by the Wentzel-Kramers-Brillouin (WKB) method. In this work, a suitable expression for refractive indices of the ionospheric plasma in the QT mode of wave propagation is applied in the derivation of an integral equation through the use of the WKB solution. The effective collision frequency at the different height ranges can be determined from an integral equation. The results of some numerical computations are presented graphically.

## 2. *Mathematical formulation*

For an ordinary wave in the upper atmospheric plasma, assuming the QT approximation, the expression for the refractive indices can be written as (Eq. (29) of Ref. [9])

$$n^2 = \left(1 - \frac{X}{U}\right)\left(1 + \frac{X}{U}\cos^2\theta + \frac{X^2(2U^2+Y^2) - XU(U^2X^2)}{U^2Y^2}\cos^4\theta \dots\right), \quad (1)$$

where

$$n = (\mu - \mathrm{i}\chi), \quad X = \frac{Ne^2}{m\omega^2} = \frac{\omega_N^2}{\omega^2}, \quad U = 1 - \mathrm{i}\zeta, \quad \zeta = \frac{\nu_{\mathrm{eff}}}{\omega}, \quad Y = \frac{eB}{m\omega}.$$

$\theta$ is the angle between the normal to the wave front and the direction antiparallel to the geomagnetic field $B$, $n$ is the complex refractive index, $\mu$ the wave refractive index, $\chi$ the absorption index, $\omega_N$ the electron-plasma circular frequency, $\omega$ the circular signal frequency, $N$ the number of electrons per unit volume, $m$ the electron mass, $\nu_{\mathrm{eff}}$ the effective collision frequency and $e$ the electron charge. In Eq.(1), only electrons are treated, hence the suffix $e$ has been dropped.

Ionospheric plasma is a slowly varying medium, for which it is often appropriate to use the WKB approximations for the solution of the problems of wave propagation. The WKB solution for the $y$-component of the electric field of a wave propagating through the ionospheric plasma, traversed by the geomagnetic field, can be written as (Ref. [12], Eq. 9.11)

$$E_y(z) = A \exp\left[ \mathrm{i} \left( \omega t - k \int_{z_1}^{z} n(z)\mathrm{d}z \right) \right] , \tag{2}$$

$$E_y(z) = A \exp\left( -k \int_{z_1}^{z} \chi(z)\mathrm{d}z \right) \exp\left[ -\mathrm{i} \left( \omega t - k \int_{z_1}^{z} \mu(z)\mathrm{d}z \right) \right] , \tag{3}$$

where $k = \omega\sqrt{\mu_0 \epsilon_0} = \omega/c$, $\mu_0$ is the vacuum permeability, $\epsilon_0$ the vacuum permittivity, $z$ the height of the considered atmospheric layer and $z_1$ is the referential height, i.e. the bottom of the ionosphere.

The power reflection coefficient in the process of total reflection from the ionospheric layer at $z = z_{\max}$, where $X = 1$, i.e., $\omega_N = \omega$, is given by (Ref. [12], Eq. 9.37)

$$R(\omega) = \exp\left( -4k \int_{z_1}^{z_{\max}} \chi(z)\mathrm{d}z \right) . \tag{4}$$

After taking the logarithm, Eq. (4) may be expressed as

$$R_1(\omega) = 4\omega^2 \int^{\omega} \chi(\omega_N, \omega) \frac{\mathrm{d}z(\omega_N)}{\mathrm{d}\omega_N} \mathrm{d}\omega_N , \tag{5}$$

where $R_1(\omega) = C\omega \ln R(\omega)$.

$\chi(\omega_N, \omega)$ can be expressed from Eq. (1) as

$$\chi(\omega_N, \omega) = \frac{1}{\sqrt{2}} \sqrt{M - \sqrt{M^2 + 2N}} , \tag{6}$$

where

$$M = 1 - \frac{X}{1+\zeta^2} \sin^2\theta - \frac{X^2(1-\zeta^2)}{(1+\zeta^2)^2} \cos^2\theta + \frac{1}{(1+\zeta^2)^3 Y^2}$$

$$\times [\{A(1-\zeta^2) - 2B\zeta\}(1+\zeta^2 - X) + 2A\zeta^2 + B(1-\zeta^2)] \cos^4\theta ,$$

$$N = \frac{\zeta X}{1+\zeta^2} \left( \sin^2\theta + \frac{2X}{(1+\zeta^2)} \cos^2\theta \right)$$

$$-\frac{1}{(1+\zeta^2)^3 Y^2} [\{2A\zeta + B(1-\zeta^2)\}(1+\zeta^2 - X) - AX\zeta(1-\zeta^2) + 2BX\zeta^2] \cos^4\theta ,$$

$$A = X^2\{2(1-\zeta^2) + Y^2\} - X(1-\zeta^2 - X^2) - 2\zeta^2 X\,,$$

$$B = 2\zeta X - 4\zeta X^2 + X\zeta(1-\zeta^2-X^2)\,.$$

The other symbols have been explained earlier.

The expression in Eq. (6) can be simplified as

$$\chi = \frac{1}{\sqrt{2}}[\{b_1 + a_0^2(1+b_0)b_2\}\cos^2\theta + \{a_4(b_3-b_4) - 2a_0^4 b_2^2 - \frac{1}{2}b_0^2$$

$$-b_5 - 2a_{11}b_0b_6 + a_{12}\}\cos^4\theta - \frac{1}{2}\sqrt{2a_0 + b_0 + b_0^2}\,, \qquad (7)$$

where

$$\begin{array}{lll}
P = 1+\zeta^2, & Q = 1-\zeta^2, & \\
a_0 = \dfrac{X}{P}, & a_7 = 2A\zeta^2 + BQ, & a_8 = (A-B)Q - 2(A+B)\zeta, \\
a_1 = \zeta - 1, & a_9 = (A-2BX)\zeta^2, & a_{10} = (B + AX\zeta)Q, \\
a_2 = \dfrac{XQ}{P}, & a_{11} = \dfrac{X^4}{P^4Y^2}, & a_{12} = \dfrac{a^2Y^2}{2} - \dfrac{QPY^2}{X} + \dfrac{P^2Y^2}{2X^2}, \\
a_3 = Q - 2\zeta, & a_{13} = \dfrac{P}{X^4}, & b_0 = a_0^2\left(1 + \dfrac{2a_1}{a_0}\right), \\
a_4 = \dfrac{1}{P^3Y^2}, & b_1 = a_0(1-a_2), & b_2 = 1 + a_3 + \frac{a_1}{a_0} - a_2, \\
a_5 = AQ - 2B\zeta, & b_3 = a_5a_6 + a_7, & b_4 = a_6a_8 + 2a_9 + a_{10}, \\
a_6 = P - X, & b_5 = a_{11}a_{12}, & b_6 = a_{13}a_6a_8 + 2a_0 + a_{10}.
\end{array}$$

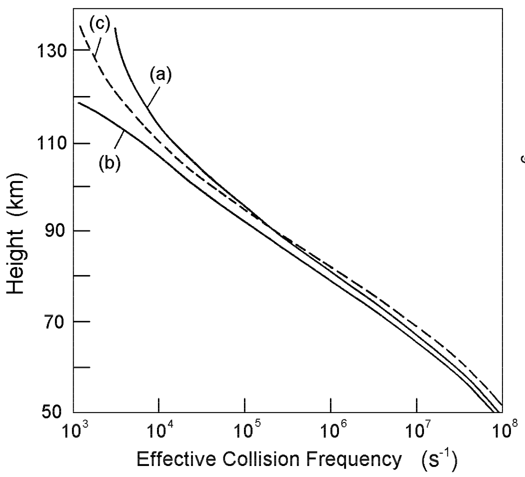
Substitution of (7) into (5) would result in the Volterra integral equation of the third kind.

## 3. *Results*

In the numerical analyses, the effective collision frequency has been taken as $\nu_{\text{eff}} = \nu_{en} + \nu_{ei}$, where $\nu_{en}$ and $\nu_{ei}$ are the electron–neutral and electron–ion collision frequencies, respectively. Figure 1 shows the curves of the effective collision frequency $\nu_{\text{eff}}$ versus height in the range 50 km to 140 km of the upper atmosphere. The full curve (a) of Fig. 1 shows the collision frequency values determined from the IRI [13] and CIRA [14] data under the assumptions of the daytime geophysical conditions for the stated range of height.

Equation (5) relates the reflection coefficient to $\nu_{\rm eff}$, and so in principle has been used to determine $\nu_{\rm eff}$ from signal strength measurements for the considered height range. This is presented by the full curve (b) in Fig. 1. These curves are superimposed on the results of an earlier work [15]., depicted by the dashed curve (c) in Fig. 1. The propagation frequencies for different heights are chosen in the range 0.01 to 2.5 MHz for numerical calculations.

Because of the weak density-dependent characteristics of the collision frequency at higher altitudes, some differences between results are observed, while at lower heights, the present result agrees well with the previous work.



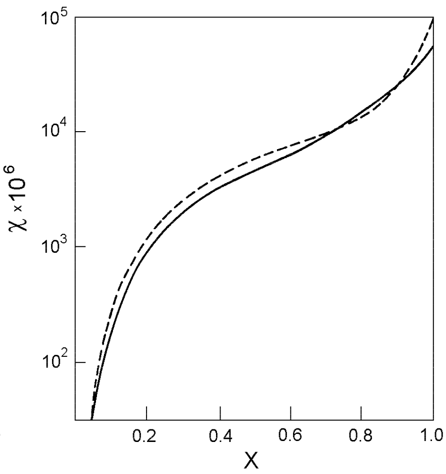




*Fig. 1. Variation of effective collision frequency with altitude in the range 50* km *to 140* km *of the upper atmosphere. The curve (a) shows the variation based on IRI [13] and CIRA [14] data, while (b) shows the calculated values of signal strength measurements. They are superimposed on the results of Thomas [15], represented by the curve (c).*

*Fig. 2 (right). Plot of the imaginary part of the refractive index in QT approximation for ordinary ray propagating in ionospheric plasma as a function of $X$ for $Y = 0.5$ and $\zeta = 0.01$. The full curve is the result of the present analysis, while the dashed curve is due to an earlier work of Chandrasekhar et al.*

The values of the ratio of square of the plasma frequency to signal frequency ($X$) are chosen for different heights. The ordinary ray absorption coefficient ($\chi$) was numerically computed for $Y = 0.5$ and $\zeta = 0.01$ from Eq. (7) as a function of $X$. The results are presented by the full curve of Fig. 2. These values are compared to the work of Chandrasekhar et al. [16], shown by the dashed curve in Fig. 2. $X$ was chosen to be larger than 0.035. The partial disagreement may be attributed to the local changes in the electron density profiles.

## *4. Conclusion*

The disagreement in the variation of the collision frequency with height represented by the three curves in Fig. 1 is probably due to the anisotropic effects at the corresponding range of height. Unlike other techniques, the present analysis can be taken as an alternative approach to study the variation of certain ionospheric parameters.

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## PROUČAVANJE PROMJENA FREKVENCIJE ELEKTRONSKIH SUDARA I APSORPCIJSKOG KOEFICIJENTA U NEIZOTROPNOJ ATMOSFERI

Primjenjujemo analitički izraz za apsorpcijski koeficijent prikladan za širenje običnih elektromagnetskih valova u razmatranom rasponu visina gornje atmosferske plazme. Primijenili smo kvazi-poprečno približenje za izvod integralne jednadžbe pomoću Wentzel-Kramers-Brillouinove metode. Integralna jednadžba je pogodna za određivanje efektivne frekvencije elektronskih sudara. Načinili smo numeričke analize i ishodi se uspoređuju grafički s nekim ranijim rezultatima.