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ON THE EXISTENCE OF ION-ACOUSTIC SOLITON IN A WEAKLY RELATIVISTIC PLASMA HAVING COLD IONS AND TWO-TEMPERATURE ELECTRONS

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Propagation of a finite-amplitude ion-acoustic solitary wave in a weakly-relativistic plasma consisting of cold ions and warm electrons of two different temperatures have been studied analytically. Sufficient and necessary conditions for the existence of ion-acoustic solitons in such a plasma are obtained from which it is observed that both the relativistic effect and the two-temperature electrons have important role for the formation of the soliton. Critical values for the soliton amplitude and its velocity are numerically estimated for different values of the concentration of two-temperature electrons and relativistic stream velocity.

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1. Introduction

In last few years, propagation of ion-acoustic solitary waves in relativistic plasmas have been theoretically investigated by various authors. Das and Paul [1] first considered a weakly-relativistic plasma for the study of ion-acoustic soliton using the reductive perturbation method. They showed that the relativistic effect for the formation of solitons in the plasma is important only in the presence of streaming of ions. Later, Nejoh [2] introduced the ionic temperature to study the ion-acoustic solitons in relativistic plasma. Subsequently, the effect of non-isothermality [3], beam ions [4], magnetic field [5], electron-inertia [5], negative ions [7–9], density gradient [10] etc., have been considered by other authors in theoretical investigations of the ion-acoustic solitons in relativistic plasmas. However, the presence of two-temperature electrons, i.e. two categories of electrons having lower and higher temperatures, contributes significantly to the formation of ion-acoustic soliton in both non-relativistic [11–14] and relativistic plasma [15,16].

It is important to note that most authors used the reductive perturbation method [17]. They studied the propagation of ion-acoustic solitary waves starting from the K-dV equation and derived the widths and amplitudes of the solitons. But Roychowdhury and Bhattacharyya [18] used the pseudo-potential method for the study of ion-acoustic soliton in relativistic plasma. On the other hand Ghosh and Roy [19] studied the propagation of ion-acoustic solitary wave in relativistic plasma following a new analytical method. They obtained some necessary and sufficient conditions for the existence of solitons in the plasma.

In this paper, we have followed the work of Ghosh and Roy [19] for theoretical investigation on the ion-acoustic solitons in a weakly relativistic plasma consisting of cold ions and two-temperature electrons. We have found that the necessary conditions are greatly modified when we assume the existence of solitons in the presence of two-temperature electrons and we derive a relation between the concentration of lower and higher temperature electrons which satisfies the necessary condition for the soliton in the plasma.

2. Formulation

We consider a collisionless plasma consisting of cold ions and two-temperature electrons. The ions are weakly relativistic and have streaming motion. So, the basic equations in normalized form for the dynamics of such plasma in unidirectional propagation are

$$\frac{\partial n_i}{\partial t} + \frac{\partial}{\partial x}(n_i v_{ir}) = 0\,, \tag{1}$$

$$\frac{\partial v_{ir}}{\partial t} + v_i \frac{\partial v_{ir}}{\partial x} = -\frac{\partial \phi}{\partial x}\,, \tag{2}$$

$$\frac{\partial^2 \phi}{\partial x^2} = n_{el} + n_{eh} - n_i\,, \tag{3}$$

where $n_{el} = \mu \exp[\phi/(\mu + \nu\beta)]$, $n_{eh} = \nu \exp[\phi/(\mu + \nu\beta)]$, $\beta = T_{el}/T_{eh}$,

$$v_{ir} = v_i/\sqrt{1 - v_i^2/c^2} \approx v_i(1 + (1/2)v_i^2/c^2)\,,$$

n_i and v_i are the density and velocity of ions, respectively, n_{el} and n_{eh} are the densities of cold electron and hot electrons, respectively, μ and ν are, respectively, the equilibrium densities of low- and high-temperature electron components satisfying the charge neutrality condition $\mu + \nu = 1$, T_{el} and T_{eh} are the temperatures of cold and hot electrons, respectively, and ϕ is the electrostatic potential. c is the normalized velocity of light, $c = c_0(kT_{\rm eff}/m_i)^{-1/2}$, where c_0 is the velocity of light in vacuum, k the Boltzmann constant, $T_{\rm eff}$ the effective temperature of the plasma and m_i the mass of ions.

To obtain the solitary wave solution from Eqs. (1) to (3), we change the two dependent variables to a single independent variable $\eta = x - Vt$, where V is the velocity of the soliton. Moreover, we assume the boundary condition at $|x| \to \infty$

$$n_i \longrightarrow 1, \qquad v_i \longrightarrow v_0, \qquad \phi \longrightarrow 0\,.$$

Therefore, from Eqs. (1) and (2), we obtain

$$n_i = \frac{V - v_0}{V - v_i}\,, \tag{4}$$

$$\phi(v_i) = a_0 + v_i V \left(1 + \frac{v_i^2}{2c^2}\right) - \frac{v_i^2}{2} - \frac{3v_i^4}{8c^2}\,, \tag{5}$$

where

$$a_0 = -v_0 V \left(1 + \frac{v_0^2}{2c^2}\right) + \frac{v_0^2}{2} + \frac{3v_0^4}{8c^2}\,. \tag{6}$$

Now, using Eqs. (4) and (5) in Eq. (3), we get

$$\frac{\mathrm{d}^2\phi}{\mathrm{d}\eta^2} = G(v_i)\,, \tag{7}$$

where

$$G(v_i) = \mu \exp\left(\frac{\phi}{\mu + \nu\beta}\right) + \nu \exp\left(\frac{\beta\phi}{\mu + \nu\beta}\right) - \frac{V - v_0}{V - v_i}\,. \tag{8}$$

Consequently, Eq. (5) leads to

$$\frac{\mathrm{d}\phi}{\mathrm{d}v_i} = (V - v_i)\left(1 + \frac{3v_i^2}{2c^2}\right)\,. \tag{9}$$

Moreover, integrating Eq. (7), we obtain

$$\left(\frac{\mathrm{d}\phi}{\mathrm{d}v_i}\right)^2 \left(\frac{\mathrm{d}v_i}{\mathrm{d}\eta}\right)^2 = H(v_i) - K\,, \qquad 10)$$

where

$$H(v_i) = 2\int G \frac{\mathrm{d}\phi}{\mathrm{d}v_i} \mathrm{d}v_i\,, \tag{11}$$

and K is an arbitrary constant.

Now, using Eqs. (6), (8) and (9), we get from Eq. (11)

$$H(v_i) = \tag{12}$$

$$2\left\{\mu(\mu+\nu\beta)\exp\left[\frac{\phi}{\mu+\nu\beta}\right]+\frac{\nu(\mu+\nu\beta)}{\beta}\exp\left[\frac{\beta\phi}{\mu+\nu\beta}\right]-v_i(V-v_0)\left(1+\frac{v_i^2}{2c^2}\right)\right\}.$$

3. Analytical study

3.1. Necessary and sufficient condition

From Eq. (10), one may obtain the physically admissible solution for the ion-acoustic solitary wave in a relativistic plasma having two-temparature electrons. In order to get the physically admissible solution of Eq. (10), it is observed that

(i) $\left(\frac{\mathrm{d}\phi}{\mathrm{d}v_i}\right)^2\left(\frac{\mathrm{d}v_i}{\mathrm{d}\eta}\right)^2$ must be positive

(ii) v_i and $\frac{\mathrm{d}v_i}{\mathrm{d}\eta}$ must be bounded.

Also, it is observed that $G \longrightarrow \pm\infty$ as $v_i \longrightarrow V$, which follows from $v_{i_{\max}} < V$ or $v_{i_{\min}} > V$.

Consequently, one may take the following requirements:

I. Between v_0 and V there exists $v_{i_{\max}}$ or $v_{i_{\min}}$ such that

$$H(v_0) = H(v_{i_{\max}}) = K \quad \text{for} \quad v_0 < v_{i_{\max}} < V\,,$$

$$H(v_0) = H(v_{i_{\min}}) = K \quad \text{for} \quad V < v_{i_{\min}} < v_0\,.$$

II. $H(v_i) \geq K$ under the condition either a) $v_0 < v_i < v_{i_{\max}}$, or b) $V < v_1 < v_2 < v_0$.

The function $G(v_i)$ vanishes at most once between two values of v_i, viz. v_0 and V. (13)

To prove this assertion, let us assume that G vanishes at two different values of v_i, say v_1 and v_2 where either a) $v_0 < v_1 < v_2 < V$, or b) $V < v_1 < v_2 < v_0$.

From Eq. (8), one can obtain

$$G(v_0) = \mu+\nu-1 = 1-1 = 0\,.$$

Now, by Rolle's theorem, there exist values of v_i, say v_3 and v_4, such that

$$G'(v_3) = G'(v_4) = 0\,, \tag{14}$$

where $G'(v_i) = \mathrm{d}G/\mathrm{d}v_i$, and either a) $v_0 < v_1 < v_3 < v_2 < v_4 < V$, or b) $V < v_1 < v_3 < v_2 < v_4 < v_0$.

From Eqs. (8) and (14), we get

$$F(v_i) = 0 \quad \text{at} \quad v_i = v_3,\ v_4\,,$$

where

$$F(v_i) = \tag{15}$$

$$(V - v_i)\left(1 + \frac{3v_i^2}{2c^2}\right) - \frac{1}{V - v_0}\left\{\frac{\mu}{\mu+\nu\beta}\exp\left(\frac{\phi}{\mu+\nu\beta}\right) + \frac{\nu\beta}{\mu+\nu\beta}\exp\left(\frac{\beta\phi}{\mu+\nu\beta}\right)\right\}.$$

Since $F(v_3) = F(v_4) = 0$, we have again (by Rolle's theorem) that there exist a value of v_i between v_3 and v_4 such that $\mathrm{d}F/\mathrm{d}v_i = 0$ at $v_i = v_5$, so that we obtain

$$\frac{1}{2c^2}(6v_iV - 2c^2 - 9v_i^2) \tag{16}$$

$$= \frac{(V - v_i)(1 + 3v_i^2/(2c^2))}{(V - v_0)(\mu+\nu\beta)^2}\left[\mu\exp\left(\frac{\phi}{\mu+\nu\beta}\right) + \nu\beta^2\exp\left(\frac{\beta\phi}{\mu+\nu\beta}\right)\right],$$

for $v_i = v_5$.

Regarding Eq. (16), one can say that the right-hand side is definitely positive for either the case $v_0 < v_5 < V$, or $V < v_5 < v_0$. Hence, for the left-hand side to be positive, the only possibility is

$$\frac{1}{2c^2}(6v_iV - 2c^2 - 9v_i^2) > 0 \quad \text{for} \quad v_i = v_5\,,$$

$$\text{or} \quad 6v_iV - 9v_i^2 > 2c^2\,,$$

$$\text{or} \quad 6v_ic - 9v_i^2 > 2c^2 \quad \text{for} \quad c > V\,,$$

$$\text{or} \quad 3v_i(2c - 3v_i) > 2c^2\,,$$

$$\text{or} \quad c^2 + (c - 3v_i)^2 < 0 \quad \text{for} \quad v_i = v_5\,,$$

which is impossible. Hence, $G(v_i)$ vanishes at most once between v_0 and V for all values of μ and ν (> 0).

We now establish two new results.

Result A) For a physically bounded admissible solution of Eq. (10), $v_{i_{\max}}$ or $v_{i_{\min}}$ are determined uniquely by either a) $H(v_0) = H(v_{i_{\max}})$ and $v_0 < v_i < V$, or b) $H(v_0) = H(v_{i_{\min}})$ and $V < v_i < v_0$.

This result can be easily proved following the article of Ghosh and Ray [19].

Result B) A real and bounded solution of Eq. (10) will be admitted if the following conditions (i) and (ii) are satisfied:

(i) $V > v_0 + (1+\frac{3v_0^2}{2c^2})^{-1/2}$ for $V > v_0$ or $v_0 > V + (1+\frac{3v_0^2}{2c^2})^{-1/2}$ for $V < v_0$

(ii) $H(v_i) - H(v_0) < 0$. The condition (i) is necessary. If the requirement (ii) is fulfilled, then one may obtain $H(v_i) > H(v_0)$ for $v_i = v_0 + \epsilon$, where $\epsilon > 0$ is an arbitrarily small number. It immediately follows that

$$H''(v_0) = \frac{\mathrm{d}H}{\mathrm{d}v_i^2} > 0 \quad \text{or} \quad G'(v_0)\phi'(v_0) > 0\,, \tag{17}$$

where $\phi'(v_i) = \mathrm{d}\phi/\mathrm{d}v_i$ (from Eq. (8), $G(v_0) = 0$). From Eq. (9), we get $\phi'(v_i) > 0$ for $v_0 < v_i < V$ and $\phi'(v_i) < 0$ for $V < v_i < v_0$, and using Eq. (17), we obtain

$$G'(v_0) > 0 \quad \text{for} \quad v_0 < v_i < V\,,$$

$$\text{or} \quad \left(\frac{\mu}{\mu+\nu\beta} + \frac{\nu\beta}{\mu+\nu\beta}\right)\left(1+\frac{3v_0^2}{2c^2}\right) - \frac{1}{(V-v_0)^2} > 0\,,$$

$$\text{or} \quad (V-v_0)^2 > \left(1+\frac{3v_0^2}{2c^2}\right)^{-1}\,,$$

$$\text{or} \quad V > v_0 + \left(1+\frac{3v_0^2}{2c^2}\right)^{-1/2} \quad \text{for} \quad V > v_0\,. \tag{18}$$

Similarly, we get from Eq. (17)

$$v_0 > V + \left(1+\frac{3v_0^2}{2c^2}\right)^{-1/2} \quad \text{or} \quad V < v_0 - \left(1+\frac{3v_0^2}{2c^2}\right)^{-1/2} \quad \text{for} \quad V < v_0\,. \tag{19}$$

The condition (ii) is necessary.

Let us assume $H(V) \geq H(v_0)$ for $v_0 < v_{\max} < V$; then we have from the the requirement (I)

$$H(v_0) = H(v_{i_{\max}}) = K$$

so that $H'(v_i)$ vanishes between v_0 and $v_{i_{\max}}$ and also between $v_{i_{\max}}$ and V.

Similarly, for $V < v_{i_{\min}} < v_0$, we have from the requirement I

$$H(v_0) = H(v_{i_{\min}}) = K$$

from which we say that $H'(v_i)$ must vanish between v_0 and $v_{i_{\min}}$ and also between $v_{i_{\min}}$ and V.

These considerations mean that $H'(v_i))$ vanishes twice between v_0 and V. We also have

$$H'(v_i) = 2G(v_i)\,\frac{\mathrm{d}\phi}{\mathrm{d}v_i}$$

i.e., $G(v_i)\mathrm{d}\phi/\mathrm{d}v_i$ vanishes twice between v_0 and V. That contradicts assertion (13), i.e. $G(v_i)$ vanishes at most once between v_0 and V. From

$$\frac{\mathrm{d}\phi}{\mathrm{d}v_i} = (V - v_i)\left(1 + \frac{3v_i^2}{2c^2}\right)$$

we see that $\frac{\mathrm{d}\phi}{\mathrm{d}v_i}$ does not vanish between v_i and V. Hence this condition is necessary.

Therefore,

$$H(V) < H(v_0)\,. \tag{20}$$

Remark: The conditions (i) and (ii) of the Result B) are necessary as well as sufficient to hold in the case of ion-acoustic solitary waves in a relativistic plasma composed of cold ions and one electron component as shown by Ghosh and Ray [19].

3.2. Simplification of conditions

From the necessary condition $H(v_i) < H(v_0)$, we can write

$$\mu\left[\exp\left(\frac{\phi(V)}{\mu+\nu\beta}\right) - 1\right] + \frac{\nu}{\beta}\left[\exp\left(\frac{\beta\phi(V)}{\mu+\nu\beta}\right) - 1\right] \tag{21}$$

$$< \frac{(V-v_0)^2}{\mu+\nu\beta}\left(1 + \frac{1}{2c^2}(v_0^2 + v_0V + V^2)\right)\,.$$

The condition (21) is the relation between μ and ν for which the necessary condition $H(V) < H(v_0)$ is satisfied. Since $\beta < 1$ and $\mu < 1$ because of $\mu+\nu = 1$, the condition (21) changes to

$$\frac{\exp(\beta m) - 1}{\beta m} << \frac{(V-v_0)^2}{\mu+\nu\beta}\left(1 + \frac{1}{2c^2}(v_0^2 + v_0V + V^2)\right) \qquad (22)\,.$$

where $m = \phi(V)/(\mu + \nu\beta)$.

4. Numerical study

Retaining terms up to ϕ^3, we obtain from (21)

$$\phi(V) + \frac{1}{2}\phi^2(V) + \frac{1}{6}\frac{\mu+\nu\beta^2}{(\mu+\nu\beta)^2}\phi^3(V) + \ldots < (V-v_0)^2\left(1 + \frac{1}{2c^2}(v_0^2 + v_0V + V^2)\right)\,. \tag{23}$$

Simplifying (23), we get

$$\phi(V) < \phi_c(V)\,,$$

where

$$\phi_c(V) = \tag{24}$$
$$-a_1 + a_1^{1/3}\left[\left(a_2+\sqrt{a_2^2-a_1(a_1-2)^3}\right)^{1/3}+\left(a_2-\sqrt{a_2^2-a_1(a_1-2)^3}\right)^{1/3}\right],$$

and

$$a_0 = (V-v_0)^2\left(1+\frac{1}{2c^2}(v_0^2+v_0V+V^2)\right),$$

$$a_1 = \frac{(\mu+\nu\beta)^2}{\mu+\nu\beta^2},$$

$$a_2 = 3a_0+3a_1-a_1^2\,.$$

To get the physical ideas about $\phi(V)$ in a weakly relativistic plasma having cold ions and two-temperature electrons, we have computed the results satisfying Eq. (24) for different values of μ, ν and β as well as v_0/c. Also, we have computed the critical values of V, i.e., V_c for two different cases, $V > v_0$ and $V < v_0$. All results are shown in Tables 1A to 5.

5. *Critical values of V*

From Eq. (18), the critical values of the soliton velocity (i.e. $V = V_c$) are given by

$$V_c = v_0 + \frac{1}{\sqrt{1+\frac{3v_0^2}{2c^2}}} \quad \text{when} \quad V > v_0\,.$$

The critical values of the soliton velocity have been numerically estimated for different values of the stream velocity and the relativistic parametar. They are shown in Tables 1A and 1B.

From Eq. (19), we have the critical values of the soliton velocity (i.e. $V = V_c$)

$$V_c = v_0 - (1+3v_0^2/(2c^2))^{-1/2} \quad \text{when} \quad V < v_0\,.$$

The critical values of the above soliton velocity numerically estimated for the different values of stream velocity and relativistic parameter are shown in Tables 2A and 2B.

TABLE 1. Critical values of the soliton velocity (V_c) for different values of the stream velocity (v_0) and relativistic parameter.

A) When $v_0/c = 0.01$					
v_0	0.10	0.15	0.20	0.25	0.30
V_c	1.0999	1.1499	1.1999	1.2499	1.2999
B) When $v_0 = 0.1$					
v_0/c	0.012	0.014	0.016	0.018	0.020
V_c	1.0999	1.0999	1.0998	1.0998	1.0997

TABLE 2. Critical values of the soliton velocity (V_c) for different values of the stream velocity (v_0) and relativistic parameter.

A) When $c = 10$					
v_0	2.0	2.1	2.2	2.3	2.4
V_c	1.0281	1.1315	1.2344	1.3375	1.4406
B) When $v_0 = 2.1$					
v_0/c	0.21	0.22	0.23	0.24	0.25
V_c	1.1315	1.1344	1.1375	1.1406	1.1438

Using the values of the critical velocity of Tables 1A, 1B, 2A and 2B, the critical values of the electrostatic potential ϕ_c for the existence of ion-acoustic solitary waves have been calculated which are given in Tables 3 to 5 for different values of the concentration of two-temperature electrons, stream velocity (v_0), relativistic parameter and β. It is observed from the Tables 3A and 3B (when $V > V_c$) that ϕ_c is at minimum when the density of the low-temperature electrons (μ) is 0.03 for $\beta = 0.025$. But from Table 3C, we observe that ϕ_c will have minimum values for $\mu = 0.05$ and $\mu = 0.06$ when $\beta = 0.05$ and 0.075, respectively.

From Tables 3A and 3C, we also see that the values of ϕ_c will be greater in every case when the soliton velocity and the values of β are increased gradually. But, the values of ϕ_c in Table 3B are gradually decreasing when the drift velocity (v_0) is increasing rapidly. When $V > V_c$, ϕ_c increases when increasing V and v_0/c.

Further, from Table 4C, we see that when $V < V_c$, ϕ_c will have minimum values when $\mu = 0.03$ for $\beta = 0.03$, and when $\mu = 0.05$ for $\beta = 0.05$. It is interesting to observe from Table 4B that when $V < V_c$, ϕ_c has no minimum values, and it increases with the increase of the drift velocity (v_0).

In a plasma having single-temperature electrons (i.e., $\mu = 1$, $\nu = 0$), ϕ_c has been calculated and the values are shown in Table 5, from which it is observed that ϕ_c will always be greater than its values given in Table 3D.

TABLE 3. Case I. Critical values of the electrostatic potential (ϕ_c) for $V > v_0$ and $V > V_c$.

A) When $v_0 = 0.1,\ v_0/c = 0.01,\ \beta = 0.025$				
		ϕ		
μ	ν	When $V = 1.67$	When $V = 1.72$	When $V = 1.78$
0.01	0.99	0.9220	0.9502	0.9913
0.02	0.98	0.8908	0.9177	0.9495
0.03	0.97	0.8861	0.9127	0.9443
0.04	0.96	0.8986	0.9261	0.9581
0.05	0.95	0.9101	0.9377	0.9706
B) When $V = 1.67,\ v_0/c = 0.01,\ \beta = 0.025$				
		ϕ		
μ	ν	When $v_0 = 0.10$	When $v_0 = 0.15$	When $v_0 = 0.20$
0.01	0.99	0.9220	0.8903	0.8605
0.02	0.98	0.8908	0.8608	0.8324
0.03	0.97	0.8861	0.8561	0.8281
0.04	0.96	0.8986	0.8684	0.8397
0.05	0.95	0.9101	0.8790	0.8498
C) When $V = 1.67,\ v_0 = 0.1,\ v_0/c = 0.01$				
		ϕ		
μ	ν	When $\beta = 0.025$	When $\beta = 0.050$	When $\beta = 0.075$
0.01	0.99	0.9220	1.1013	1.1864
0.02	0.98	0.8908	1.0491	1.1370
0.03	0.97	0.8861	1.0278	1.1129
0.04	0.96	0.8986	1.0174	1.0978
0.05	0.95	0.9101	1.0171	1.0913
0.06	0.94	0.9261	1.0185	0.9947
D) When $\nu = 0.99,\ v_0 = 0.1,\ \beta = 0.075$				
		ϕ		
μ	v_0/c	When $V = 1.67$	When $V = 1.72$	When $V = 1.78$
0.01	0.010	1.1864	1.2294	1.2807
0.01	0.012	1.1906	1.2340	1.2857
0.01	0.014	1.1957	1.2396	1.2917
0.01	0.016	1.2014	1.2458	1.2986
0.01	0.018	1.2081	1.2530	1.3063
0.01	0.020	1.2154	1.2607	1.3148

TABLE 4. Case II. Critical values of the electrostatic potential (ϕ_c) for $V < v_0$ and $V < V_c$.

A) When $v_0 = 2.55,\ v_0/c = 0.26,\ \beta = 0.01$				
		ϕ		
μ	ν	When $V = 1.15$	When $V = 1.35$	When $V = 1.55$
0.01	0.99	0.6570	0.5723	0.4809
0.02	0.98	0.6783	0.5900	0.4946
0.03	0.97	0.7072	0.6134	0.5122
0.04	0.96	0.7353	0.6363	0.5295
0.05	0.95	0.7548	0.6518	0.5408
B) When $V = 1.55,\ v_0/c = 0.26,\ \beta = 0.01$				
		ϕ		
μ	ν	When $v_0 = 2.55$	When $v_0 = 2.60$	When $v_0 = 2.65$
0.01	0.99	0.4809	0.5058	0.5284
0.02	0.98	0.4946	0.5200	0.5436
0.03	0.97	0.5122	0.5398	0.5648
0.04	0.96	0.5295	0.5578	0.5843
0.05	0.95	0.5408	0.5710	0.5984
C) When $V = 1.55,\ v_0 = 2.55,\ v_0/c = 0.26$				
		ϕ		
μ	ν	When $\beta = 0.01$	When $\beta = 0.03$	When $\beta = 0.05$
0.01	0.99	0.4809	0.6144	0.6673
0.02	0.98	0.4946	0.5915	0.6466
0.03	0.97	0.5122	0.5902	0.6377
0.04	0.96	0.5295	0.5919	0.6332
0.05	0.95	0.5408	0.5971	0.6331

TABLE 5. Case II. Critical values of the electrostatic potential (ϕ_c) for $V > V_c$, $v_0 = 0.1,\ \mu = 1.00$ and $\beta = 0.00$.

	ϕ		
v_0/c	When $V = 1.67$	When $V = 1.72$	When $V = 1.78$
0.010	1.2969	1.3482	1.4096
0.012	1.3020	1.3539	1.4155
0.014	1.3080	1.3603	1.4230
0.016	1.3150	1.3680	1.4311
0.018	1.3229	1.3762	1.4403
0.020	1.3312	1.3857	1.4508

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IONSKO-AKUSTIČNI SOLITONI U SLABO-RELATIVISTIČKOJ PLAZMI S HLADNIM IONIMA I DVOTEMPERATURNIM ELEKTRONIMA

Analitički se proučava širenje ionsko-akustičnog solitonskog vala u slabo relativističkoj plazmi koja se sastoji od hladnih iona i vrućih elektrona na dvije temperature. Izvode se nužni i dovoljni uvjeti za postojanje ionsko-akustičnog solitona koji pokazuju važnost kako relativističkih efekata, tako i dviju elektronskih temperatura. Kritične vrijednosti solitonske amplitude i brzine ocijenjuju se numerički za niz vrijednosti koncentracije dvotemperaturnih elektrona i relativističke brzine strujanja.